SOME STUDY ON PERFECT FLUID SPACE TIME WITH AN ELECTROMAGNETIC FIELD

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Abstract- ***In this paper we study about the foregoing general constraint for two simple models has been evaluated. Firstly space-times filled with the mixture of charged spinless perfect fluid and an electromagnetic field is considered, and then the same with spinning fluid. Both cases are physically realistic models and are often used for example in the analysis of cosmological problems.***

Keywords- ***Perfect Fluid, Spinning Fluid, Electromagnetic Field, Realistic Models, Cosmology.***

1. INTRODUCTION

In recent years there has been growing interest in the study of the Einstein – Cartan theory of space time. The intrinsic spin of matter has been taken as the source of torsion of the space-time manifold in Einstein – Cartan theory. The discovery of pulsars gave a big impetus to the developments in relativistic astrophysics, particularly in the study of neutron stars. Since the rotating – neutron star models for a pulsar seems to be almost undisputed it is essential to study the possible internal structure of neutron stars with respect to gravitation and the associated geometry of space–time therein. Observations indicate a very high magnetic field associated with them in comparison with any other compact objects in the universe. It is not unlikely that this magnetic field might induce a spin polarization of the nucleons composing the fluid of neutron star . If the spins are aligned then it is probable that there would be a substantial non zero spin density which would then play, along with the mass density, a dynamical role in influencing the geometry of space-time containing the fluid. In general relativity as given by Einstein there is no way of considering the spin effects on the geometry of space-time. On the other hand, it is clear that one could study such configurations in the frame work of the Einstein – Cartan theory.

One of the problems facing the present day cosmologist in his quest for a good model to use, are several remaining basic problems of physics. Namely:

A. General relativity is a concept and theory of gravitation only. Although electromagnetic fields can be incorporated into the theory, the terms added are adhoc. There is still no satisfying underlying physical concept of charges and magnetic fields which is as aesthetically pleasing as the idea of distorted space-time being the underlying cause of gravitation.

B. Quantum theory and general relativity are still distinct, non-united theories. Since general relativity describes the behavior of particles in the macro-world, a way is needed to predict the gravitational behavior of particles as one transition into the micro-world of atoms and fundamental particles. For example, the FRW cosmological models predict that the universe, in the beginning, has zero size and infinite energy density. This prediction is almost impossible to reconcile with the expected behavior of elementary particles as the size of the universe approaches the size of a nucleus of smaller.

Even on the macroscopic scale, the property of intrinsic angular momentum or spin of a particle, one of the cornerstones of quantum mechanics, cannot be successfully incorporated into standard general relativity since additional degrees of freedom are necessary to add spin to the structure of space-time. Although spinning fluids can be added to the Stress-Energy-Momentum Tensor of the theory, it is not possible to incorporate the spin phenomenon itself into the structure of space-time. The concept of - “Torsion” was first developed by Cartan in rudimentary from, where it was proposed that the affine connection of general relativity be allowed to have an asymmetric part. However, Cartan’s original work contained only a vague idea as to the connection of “Torsion” with any physically real effects. It was not until the later work of various authors that it was shown that the energy momentum tensor of spinning fields must be anti-symmetric. The full theory of a space time containing Torsion was developed by Kibble and Sciama and further by Hehl et. al.

In Fact mass and spin are the two fundamental characters of an elementary particle system as per relativistic quantum mechanics. In Einstein’s theory of general relativity, it is not the spin but mass which plays a dynamic role. The curvature is due to the density of energy-momentum. An interesting link between the theory of gravitation and the theory of special relativity is established by introduction torsion and relating it to spin. By introducing torsion and relating this to the density of intrinsic angular-momentum, the Einstein- Cartan theory regains the analogy between mass and spin.

Eddington has mentioned the motion of an affine connection while discussing possible extensions of general relativity. He remarked that applications in micro-physics are conceivable but he could not develop his ideas. Elie Cartan in 1922 introduced torsion as the anti-symmetric part of an asymmetric affine connection and suggested a simple generalization of Einstein’s theory of gravitation. He put forth as a model of space-time, a four dimensional differentiable manifold with a metric tensor and a linear connection compatible with the metric, but not symmetric, in general. According to him, the torsion tensor of the connection should be linked to the density of intrinsic angular momentum of matter and it should disappear in matter free regions. Wagoner , Nordtvedt and Bergmen have also developed similar ideas independently without having link with those put forth by Cartan. The generalization done by Cartan, now known as Einstein's-Cartan theory, has only a slight deviation from the Einstein's theory, the field equations in empty space remain unchanged.

Such analysis may be related to recent discoveries in astronomy. It may be conceived that torsion produces significant effects inside those objects which, as the neutron stars, have built-in strong magnetic fields. These magnetic fields may be associated with a substantial average value of the density of spin. It may be concluded that intrinsic angular-momentum may influence or even prevent the occurrence of singularities in gravitational collapse and cosmology. The above idea is supported by the out-come of Kopczynski on the geometry of universe filled with a spherically-symmetric distribution of mass and spin.

2. CHARGED SPINLESS PERFECT FLUID

In general relativity (GR) the matter influences the geometry of space-time through its energy momentum tensor. The mathematical model is a four-dimensional differential manifold M equipped with a Lorenzian metric g_{ab} and with a metric affine connection Γ^{c}_{ab} chosen to be symmetric. The dynamical equation, which links the geometrical Ricci-tensor R_{ab} with the metric energy-momentum tensor σ_{ab} is the will known Einstein equation.

(2.1) $$R_{ab} = 8\pi\left(\sigma_{ab} - \frac{1}{2} g_{ab}\sigma\right)$$

For Charged Spinless Perfect Fluid the metric energy-momentum tensor is simply the sum of those of perfect fluid and electromagnetic field [24].

(2.2) $$\sigma_{ab} = \sigma^{(f)}_{ab} + \sigma^{(em)}_{ab}$$

$$\sigma^{(f)}_{ab} = \rho(1+e)w_a w_b + p(w_a w_b + g_{ab}),$$

$$\sigma^{(em)}_{ab} = F_{ac}F^{c}_{b} - \frac{1}{4} g_{ab}F_{cd}F^{cd}$$

Where the usual definitions were used except that here w^a denotes the unit for velocity of the fluid. The spin density S_{ab} (spin per unit mass) is zero, so the torsion [3]

(2.3) $$S^{c}_{ab} = 4\pi p S_{ab} w^{c}$$

Vanishes and the Einstein equation (2.1) arises. Here we accept the view that the mass should be conserved, so the trace of the torsion S^{b}_{ab} vanishes .

In the evaluation of the above expression a distinction should be made between the cases when the considered geodesics are time like or when they are null.

A. Time like geodesics ($u^a u_a$: = -1):

From expressions (2.1) and (2.2) it follows that

(2.4) $$R_{44} = 8\pi\left[\rho(1+\in)\left\{(u^a w_a)^2 - \frac{1}{2}\right\} + p\left\{(u^a w_a)^2 + \frac{1}{2}\right\}\right]$$

$$+4\pi\sum_{k=1}^{3}\left\{E_k^2 + B_k^2\right\}$$

Where the following definitions of the electric and magnetic fields measured by the geodetic observers were also used

(2.5) $$E_1 = F_{14} = B_1 = \frac{1}{2}\sum_{j,k=1}^{3} e_{ijk}F_{jk}$$

Let us suppose that density ρ, the internal energy E and pressure p are all non-negative. Because u^a and w^a are both time like unit vectors, it follows that

(2.6) $$R_{44} \geq 4\pi\left\{\rho(1+\in)+3p\right\} + 4\pi\sum_{k=1}^{3}\left\{E_k^2 + B_k^2\right\} \geq 0$$

Using above proposition we can get the following restriction on the rate of the divergence of the above quantities.

(2.7) $$\sigma_{ab}^{(f)} = \rho(1+e)w_a w_b + p(w_a w_b + g_{ab})$$

(2.8) $$27 \geq \underset{t \to t_o}{\text{Lim}} \inf\left[4\pi\left\{\left\{\rho(1+\in)+3p\right\} + \sum_{k=1}^{3}\left\{{E_k}^2 + {B_k}^2\right\}\right\}\right.$$

$$\left. + (t-t_o)^2\right] \geq 0$$

Every component in (2.8) is non-negative, so all the following bounds for the given physically measurable quantities are valid along maximal in complete time like geodesics.

(2.9) $$\frac{27}{4\pi} \geq \underset{t \to t_o}{\text{Lim}} \inf\left\{\begin{array}{l}\rho(t-t_o)^2 \\ \rho\in(t-t_o)^2 \\ 3p(t-t_o)^2\end{array}\right\} \geq 0$$

$$\sqrt[3]{\frac{3}{4\pi}} \geq \underset{t \to t_o}{\text{Lim}} \inf\left\{\begin{array}{l}|E_k|(t-t_o) \\ |B_k|(t-t_o)\end{array}\right\} \geq 0 \qquad \forall\ k \in \{1,2,3\}$$

B. Null geodesics ($u^a u_a = 0$) :

Here the electric and magnetic vectors cannot be defined but the following equation continues to be valid:

(2.10)
$$R_{44} = 8\pi\left\{\sigma_{ab}^{(t)} + \sigma_{ab}^{(cm)}\right\}u^a u^b \geq \sigma_{ab}^{(f)} u^a u^b$$

$$= 8\pi\left[\left\{\rho(1+\in)+p\right\}\left(u^a w_a\right)^2\right] \geq 0$$

We can choose a preferred affine parameter along the null geodesics, which satisfies the inequality

(2.11)
$$\lim_{t \to t_o} \inf\left(u^a w_a\right)^2 \geq 2$$

From above prop. we can get that,

(2.12)
$$18 \geq \lim_{t \to t_0} \inf \; 8\pi\left[\left\{\rho(1+\in)+p\right\}\left(u^a w_a\right)^2 (t-t_0)^2\right]$$

$$\geq \lim_{t \to t_0} \inf \; 8\pi\left[\inf\left\{\rho(1+\in)+p\right\}(t+t_0)^2 \inf\left(u^a w_a\right)^2\right]$$

$$\geq \lim_{t \to t_0} \inf \; 8\pi\left[\left\{\rho(1+\in)+p\right\}(t-t_0)^2\right] \geq 0$$

Where the special choice of the affine parameter (2.11) was used in the last step. Every component in (2.12) was supposed to be non-negative, so the next bounds are valid along maximal incomplete null geodesics.

(2.13)
$$\frac{9}{4\pi} \geq \lim_{t \to t_o} \inf \begin{Bmatrix} \rho(t-t_o)^2 \\ \rho\varepsilon(t-t_o)^2 \\ p(t-t_o)^2 \end{Bmatrix} \geq 0$$

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In this case the metric energy-momentum tensor has the following form [3].

(3.1)
$$\sigma_{ab} = \rho E w_a w_b + \left(g_{ab} + w_a w_b\right)p + \nabla^c\left(\rho w\left(aS_b\right)c\right)$$

$$+\left[\left(w^c P_c^d + w^d\right)w\left(a - P_a^d s_b\right)d\right]$$

$$+F_{ab}F_b^c - \frac{1}{4}g_{ab}F_{cd}F^{cd}$$

where the definitions

$$E = 1 + \in - \frac{X}{2} S_{ab} F^{ab}$$

$$P_{ab} = \rho\left(XF_{ab} - 2\frac{\partial \varepsilon}{\partial S^{ab}} \right)$$

Where used (X magnetic susceptibility). The torsion is supposed to be traceless and given by expression (2.2) . After substitution into the dynamical equation (2.1 and 2.3) the following formula remains for the Ricci-tensor:

(3.2)
$$R_{ab} = 8\pi\left(\sigma_{ab} - \frac{1}{2} g_{ab}\sigma \right) + (4\pi\rho)^2 \left(2S_{ac}S^c_b + S^2 w_a w_b\right)$$

where $S^2 = S_{ab}S^{ab}$. Just as in the case of spinless fluid, Kannar [32] has evaluated this expression along time-like and null maximal incomplete geodesics.

A. Time like geodesics ($u^a u_a$ = -1):

From the above expression (3.2) it follows that

(3.3)
$$R_{44} = 8\pi\left(\sigma_{ab} - \frac{1}{2} g_{ab}\sigma \right) u^a u^b$$

$$+ \ (4\pi\rho)^2 \left[2\left(u^a s_{ac}\right)\left(u^b s^c_b\right) + s^2 \left(u^a w_a\right)^2 \right]$$

$$\geq \ 8\pi\left(\sigma_{ab} - \frac{1}{2} g_{ab}\sigma \right) u^a u^b + (4\pi\rho s)^2$$

In the last step the fact that both u^a and w^a are time like unit vectors and that vector $u^a S^c_b$ is space like is utilized. We can suppose that expression (3.1) of the energy-momentum tensor satisfies the strong energy-condition (2.2). Because of the positive contribution of the electromagnetic field this may hold also in the case when the fluid itself would violate it, only the field has to be strong enough. Then form prop (2.1) we can get the following restrictions on the divergency rate of the physical quantities of the system.

(3.4)
$$27 \geq \lim_{t \to t_o} \inf\left\{ \left[8\pi\left(\sigma_{ab} - \frac{1}{2} g_{ab}\sigma \right) u^a u^b + (4\pi\rho s)^2 \right] (t - t_o)^2 \right\}$$

$$\geq \lim_{t \to t_o} \inf\left\{ 8\pi\left(\sigma_{ab} - \left(\frac{1}{2}\right) g_{ab}\sigma \right) u^a u^b (t - t_o)^2 \right.$$

$$\left. (4\pi\rho s)^2 (t - t_o)^2 \right\} \geq 0$$

From the bottom line of the above inequality it follows that.

(3.5) $$\frac{3\sqrt{3}}{4\pi} \geq \lim_{t \to t_0} \inf\{\rho s(t-t_0)\} \geq 0$$

Supposing that density is non-negative. This inequality means that the volume spin density (spin per volume) ρs cannot blow up faster than the minus first power of the affine parameter. It follows from the equations of motion of the fluid [3] that the spin density (spin per mass) is constant along the geodesics, which are being considered. But if there were a cross section of the flow lines available, where s were constant, then it would be the same everywhere and therefore also along the geodesics considered. If we impose this homogeneity conditions, then from (3.5) it follows that

(3.6) $$\frac{3\sqrt{3}}{4\pi s} \geq \lim_{t \to t_0} \inf\{\rho(t-t_0)\} \geq 0$$

Because s is supposed to be constant, it means that in the case of homogenous spinning fluid density cannot blow up faster than the minus first power of the affine parameter. This divergence is defined weaker than in the case of spinless fluid, where the minus second power was the limit.

B. Null geodesics ($u^a u_a = 0$):

Just like in the time like case it follows from the above dynamical equation (2.1) that

(3.7) $$R_{44} = 8\pi\sigma_{ab}u^a u^b + (4\pi\rho)^2$$

$$\left[2\left(u^a S_{ac}\right)\left(u^b S_b^c\right) + S^2\left(u^a w_a\right)^2\right]$$

$$\geq 8\pi\sigma_{ab}u^a u^b + (4\pi\rho s)^2\left(u^a w_a\right)^2 \geq 0$$

As in the spinless case, we shall choose a preferred an affine parameter along the null geodesics, which satisfy the earlier inequality (2.10). From prop. (2.11) similar bounds are arrived at as in the time like case i.e.

(3.8) $$18 \geq \lim_{t \to t_0} \inf \left\{\left[8\pi\sigma_{ab}u^a u^b + (4\pi\rho s)^2\left(u^a w_a\right)^2\right](t-t_0)^2\right\}$$

$$\lim_{t \to t_0} \inf \left\{8\pi\sigma_{ab}u^a u^b (t-t_0)^2\right\}$$

$$\lim_{t \to t_0} \inf\left\{\left[(4\pi\rho s)^2 (t-t_0)\right]\inf\left(u^a w_a\right)^2\right\} \geq 0$$

Here it is supposed that the weak energy condition is satisfied by the energy impulse tensor of the system, that is

(3.9) $$\sigma_{ab}u^a u^b \geq 0$$

for arbitrary causal vector v^a. With the help of the parameter choice (3.9) it can be deduced from the bottom line of the above inequality that

(3.10) $$\frac{3\sqrt{2}}{4\pi} \geq \lim_{t \to t_o} \inf\{\rho s(t - t_o)\} \geq 0$$

Which is a restriction of the volume spin density similar to inequality (3.6) in the time-like case. Under identical conditions to those above the following limit on the density appears:

(3.11) $$\frac{3\sqrt{2}}{4\pi s} \geq \lim_{t \to t_o} \inf\{\rho(t - t_o)\} \geq 0$$

The divergence rates are the same as in the time-like case.

4. CONCLUDING REMARKS

Thus in the presence of spin the fluid density cannot blow up so fast as in the spinless case. The results (3.6)-(3.7) or (3.10)-(3.11) are highly independent of the concrete form of the metric energy-momentum tensor, which only has to satisfy the strong or the weak energy condition. The expression (2.3) and its property of tracelessness, which related the space time torsion to the matter variables, appeared to be crucial points.

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