NANO-IDEAL SEMI-WEAKLY GENERALIZED MAPPINGS AND HOMEOMORPHISMS

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ABSTRACT:

This paper introduces several new mapping and continuity concepts in nano ideal topological spaces. We define and study nano ideal semi-weakly generalized closed maps and nano ideal semi-weakly generalized open maps, and establish their basic properties and relationships with classical nano-closed and nano-open maps. Two strengthened structural notions - nano ideal semi-weakly generalized homeomorphism and nano ideal semi-weakly generalized* homeomorphism are also developed and characterized. In addition, we investigate three refined continuity classes: totally $n\mathfrak{I}$-$\mathcal{Swg}$-continuous, strongly $n\mathfrak{I}$-$\mathcal{Swg}$-continuous, and contra $n\mathfrak{I}$-$\mathcal{Swg}$-continuous functions. Examples, counter examples, and hierarchy relations are provided to show the independence of these classes. The results extend the theory of generalized maps and continuity in nano ideal topological spaces.

Keywords:

Nano ideal topology; Semi-weakly generalized closed maps; Semi-weakly generalized open maps; $n\mathfrak{I}$-$\mathcal{Swg}$-homeomorphism; $n\mathfrak{I}$-$\mathcal{Swg}$*-homeomorphism; Totally $n\mathfrak{I}$-$\mathcal{Swg}$-continuous functions; Strongly $n\mathfrak{I}$-$\mathcal{Swg}$-continuous functions; Contra $n\mathfrak{I}$-$\mathcal{Swg}$-continuous functions; Generalized continuity; Ideal-based nano structures.

INTRODUCTION

Generalized closed sets and their associated mapping properties have played a central role in the development of modern topology. The foundational work of Levine [1] initiated the study of generalized closed sets, which later led to several weakened and strengthened forms such as semi-generalized closed sets [2] and weakly generalized closed sets [3, 8]. These developments created a rich background for formulating new continuity and homeomorphism concepts in topological structures.

Parallel to these classical extensions, nano topology emerged as a flexible framework for modelling approximation, granularity, and uncertainty, as introduced by Deekshitulu and Subramanian [4]. Further refinements in nano generalized closed sets, including nano semi-generalized and nano weakly generalized closed sets, were explored in [5], providing a foundation for the study of generalized structures in nano settings. Complementing this, ideal topology has contributed significantly to generalized closed mappings, as demonstrated in [6] and [7], where the interaction between ideal smallness and topological operators yields new mapping behaviours.

Motivated by the increasing interest in these hybrid frameworks, this paper introduces and investigates a new class of mappings based on nano ideal semi-weakly generalized closed sets. We first study nano ideal semi-weakly generalized closed maps and nano ideal semi-weakly generalized open maps, establishing their fundamental properties and clarifying their relationships with previously known mapping classes in nano and ideal topologies [4-7]. These concepts offer finer control over how generalized closedness behaves under images and preimages.

To strengthen these notions, we introduce two new equivalence structures: nano ideal semi-weakly generalized homeomorphism and nano ideal semi-weakly generalized* homeomorphism. These homeomorphisms extend the classical idea by relaxing the requirements on both the map and its inverse while retaining the essential features determined by nano ideal semi-weakly generalized closedness. Their characterizations, structural properties, and interrelations are discussed in detail, drawing inspiration from earlier generalized homeomorphism studies [1, 2, 3, 6, 7].

The paper also develops three new continuity classes totally $n\mathfrak{I}$-$\mathcal{Swg}$ -continuous, strongly $n\mathfrak{I}$-$\mathcal{Swg}$ -continuous, and contra $n\mathfrak{I}$-$\mathcal{Swg}$ -continuous functions. These functions refine ordinary nano ideal continuity by controlling the behaviour of semi-weakly generalized closed sets and their complements under a mapping. Several implications and non-implications among these continuity classes are established, supported by illustrative examples and counterexamples demonstrating the independence of these classes. The motivation for such refined classes is also aligned with recent advances in nano generalized structures, including computational and machine-learning-based approaches [9].

Overall, the results presented here contribute to the ongoing expansion of generalized mapping theory in nano ideal topological spaces. The newly introduced classes provide a unified foundation for future investigations in nano ideal separation axioms, generalized compactness, and applications involving approximation-based or granule-based structures [4, 5].

2. Preliminaries

In this section, we recall the basic notions of nano topology, ideal topology, and the operators used in nano ideal topological spaces. Throughout the paper, U denotes a nonempty set.

Definition 2.1:[4] Let $\mathcal{U}$ be a non-empty finite set of objects called the universe and $\mathcal{R}$ be an equivalence relation on $\mathcal{U}$, referred to as the indiscernibility relation. The pair $(\mathcal{U}, \mathcal{R})$ is said to be the approximation space. Let $\mathcal{X} \subseteq \mathcal{U}$. The lower approximation, upper approximation and boundary of the region of $\mathcal{X}$ with respect to $\mathcal{R}$ is defined as $L_{\mathcal{R}}(\mathcal{X}) = \cup_{\mathcal{X}\in\mathcal{U}}\{\mathcal{R}(x): \mathcal{R}(x) \subseteq \mathcal{X}\}$, $\mathcal{U}_{\mathcal{R}}(\mathcal{X}) = \cup_{\mathcal{X}\in\mathcal{U}}\{\mathcal{R}(x): \mathcal{R}(x) \cap \mathcal{X} \neq \emptyset\}andB_{\mathcal{R}}(\mathcal{X}) = \mathcal{U}_{\mathcal{R}}(\mathcal{X}) - L_{\mathcal{R}}(\mathcal{X})$where $\mathcal{R}(x)$ denotes the equivalence class determined by $\mathcal{X} \in \mathcal{U}$. Then, the nano topology (NT) $\tau_{\mathcal{R}}(\mathcal{X}) = \{\mathcal{U}, \emptyset, L_{\mathcal{R}}(\mathcal{X}), \mathcal{U}_{\mathcal{R}}(\mathcal{X}), B_{\mathcal{R}}(\mathcal{X})\}$ is defined on $\mathcal{U}$. The $\tau_{\mathcal{R}}(\mathcal{X})$ satisfies the following axioms:

(i) $\mathcal{U}$ and $\emptyset \in \tau_{\mathcal{R}}(\mathcal{X})$.

(ii) The union of the elements of any subcollection of $\tau_{\mathcal{R}}(\mathcal{X})$ is in $\tau_{\mathcal{R}}(\mathcal{X})$.

(iii) The intersection of the elements of any finite subcollection of $\tau_{\mathcal{R}}(\mathcal{X})$ is in $\tau_{\mathcal{R}}(\mathcal{X})$.

We call $(\mathcal{U}, \tau_{\mathcal{R}}(\mathcal{X}))$ is a nano topological space (briefly (($\mathcal{NTS}$)).

Definition 2.2:[5] Let $(\mathcal{U}, \tau_{\mathcal{R}}(\mathcal{X}))$ be a nano topological space. Subset $\mathcal{A}$ of $(\mathcal{U}, \tau_{\mathcal{R}}(\mathcal{X}))$ is referred to as a nano weakly generalized closed set (briefly $\mathcal{Nwg}$ CS) if $\mathcal{N}$Cl($\mathcal{N}$int ($\mathcal{A}$)) $\subseteq \mathcal{V}$, where $\mathcal{A} \subseteq \mathcal{V}$ and $\mathcal{V}$ is nano open. The complement of the $\mathcal{Nwg}$ closed set is an $\mathcal{Nwg}$ open set (briefly $\mathcal{Nwg}$ -OS). The family of all nano weakly generalized open sets is denoted by $\mathcal{NWGO}(\mathcal{U})$. We set $\mathcal{NWGO}(\mathcal{U}, x) = \{\mathcal{M} \in \mathcal{NWGO}(\mathcal{U})\ such\ that\ x \in \mathcal{M}\}$. Similarly, the family of all nano weakly generalized closed sets is denoted by $\mathcal{NWGC}(\mathcal{U})$. We set $\mathcal{NWGC}(\mathcal{U}, x) = \{\mathcal{M} \in \mathcal{NWGC}(\mathcal{U})\ such\ that\ x \in \mathcal{M}\}$. The $\mathcal{Nwg}$closure of a subset $\mathcal{A}$ of

U is denoted by $\mathcal{Nwg}$-Cl($\mathcal{A}$). Similarly, the $\mathcal{Nwg}$ interior of subset $\mathcal{A}$ of U is denoted by $\mathcal{Nwg}$-Int ($\mathcal{A}$).

Definition 2.3: [7] A subset A of a nano ideal topological space (U, N, I) is said to be nano-I-generalized closed (briefly, $n\mathfrak{I}$-g -closed) if A ∗ n ⊆ V whenever A ⊆ V and V is n-open.A subset A of a nano ideal topological space (U, N, I) is said to be nano-I-generalized open (briefly, nIg-open) if X − A is $n\mathfrak{I}$-g -closed.

Definition 2.4 [9] Let $\left(\mathcal{U}, (\tau_{\mathcal{R}}(\mathcal{X})\right)$ be a NTS. A subset $\mathcal{A}$ of $\left(\mathcal{U}, (\tau_{\mathcal{R}}(\mathcal{X})\right)$ is called a nano-semi-weakly generalized closed set (NSWG-CS), if $\mathcal{N}c\ell(\mathcal{N}\ int(\mathcal{A}) \subseteq \mathcal{V}$ whenever $\mathcal{A} \subseteq \mathcal{V}$ and $\mathcal{V}$ is nano-semi-open.

3. NANO IDEAL SEMI WEAKLY GENERALIZED CLOSED MAPS AND OPEN MAPS IN NANO IDEAL TOPOLOGICAL SPACES

In this section, we introduce the notions of nano ideal semi-weakly generalized closed sets and the associated classes of closed and open mappings in nano ideal topological spaces. These concepts extend nano weakly generalized and nano semi-weakly generalized structures to the ideal setting.

Definition 3.1: A mapping $\varphi: (\mathfrak{I}_{\mathbb{N}\mathcal{L}}(\mathfrak{U}).) \to \mathfrak{I}_{\mathcal{N}'\mathcal{L}}(\mathfrak{K})$ is called $n\mathfrak{I}$-$\mathcal{Swg}$ open (closed) map, if for every $\mathcal{A} \in \mathcal{N}$, the image $\varphi(\mathcal{A})$ *lies in* $n\mathfrak{I}$-$\mathcal{SwgO}(\mathfrak{B})$, ($\varphi(\mathcal{A}) \in n\mathfrak{I}$-$\mathcal{SwgC}(\mathfrak{B})$, for all $\mathcal{A} \in \mathcal{N}'$)).

Remark 3.2: The class of $n\mathfrak{I}$-$\mathcal{Swg}$ - closed map, is not necessarily stable under composition. Even when φ is nI–Swg closed and ψ is $n\mathfrak{I}$- closed, their composition need not preserve the $n\mathfrak{I}$-$\mathcal{Swg}$ closed property.

Example 3.3: Let $U = V = W = \{a, b, c, d\}$, and consider the nano-topological structures $\left(U, \tau_R(X)\right) = \{U, \emptyset, \{a\}, \{a, b, d\}, \{b, d\}\}$, $\left(V, \tau_{R'}(Y)\right) = \{V, \emptyset, \{b\}, \{a, b, c\}, \{a, c\}$ and $\left(W, \tau_{R''}(Z)\right) = \{W, \emptyset, \{c\}\{a, b, c\}\{a, b\}\}$.

Define a function $\varphi(a) = b, \varphi(b) = c, \varphi(c) = a, \varphi(d) = d$ and let $\psi: (\mathcal{V}, \mathcal{N}', \mathfrak{K}) \to \left(\mathcal{W}, \mathcal{N}^{"}, \mathfrak{L}\right)$ be the identity map. Individually, both φ and ψ satisfy the condition for $n\mathfrak{I}$-wg -closed mappings. However, their composition $\psi \circ \varphi: (\mathfrak{U}, \mathcal{N}, \mathfrak{I}) \to \left(\mathfrak{W}, \mathcal{N}^{"}, \mathfrak{L}\right)$ fails to be $n\mathfrak{I}$-$\mathcal{Swg}$ - closed. Indeed, the set $\{a, b\}$ forms an $n\mathfrak{I}$-$\mathcal{Swg}$-CS in $(\mathfrak{B}, \mathcal{N}', \mathfrak{K})$. Hence compositional closure does not hold in this instance.

Theorem 3.4: Let $\varphi: (\mathfrak{U}, \mathcal{N}, \mathfrak{I}) \to (\mathfrak{B}, \mathcal{N}', \mathfrak{K})$ be an $n\mathfrak{I}$- closed mapping and $\psi: (\mathfrak{B}, \mathcal{N}', \mathfrak{K}) \to \left(\mathfrak{W}, \mathcal{N}^{"}, \mathfrak{L}\right)$ be an $n\mathfrak{I}$-$\mathcal{Swg}$ - closed mapping. Then the composition $\psi \circ \varphi$ is an $n\mathfrak{I}$-$\mathcal{Swg}$ -closed mapping.

Proof: Take any nano-closed set $\mathcal{A}$ be in $(\mathfrak{U}, \mathcal{N}, \mathfrak{I})$. Because φ is $n\mathfrak{I}$-closed, $\varphi(\mathcal{A})$ is $n\mathfrak{I}$ -closed in $(\mathfrak{B}, \mathcal{N}', \mathfrak{K})$. Applying ψ yields $(\psi \circ \varphi)(\mathcal{A}) = \psi\left(\varphi(\mathcal{A})\right)$, which is $n\mathfrak{I}$-$\mathcal{Swg}$ closed, since ψ possesses this property. Therefore, the composition inherits th $n\mathfrak{I}$-Swg - closedness.

Theorem 3.5: Consider a mapping $\varphi: (\mathfrak{U}, \mathcal{N}, \mathfrak{I}) \to (\mathfrak{B}, \mathcal{N}', \mathfrak{K})$ with $\mathcal{N} \neq \{\mathcal{U}, \emptyset\}$ and $\mathcal{N}' = \{\mathcal{V}, \emptyset\}$. Then the following statements hold.

(i) φ is not $n\mathfrak{I}$- closed mapping.

(ii) φ is $n\mathfrak{I}$-g - closed mapping.

(iii) φ is $n\mathfrak{I}$-$\mathcal{Swg}$ - closed mapping.

Proof: (i) Since the only $n\mathfrak{I}$-closed subsets of $(\mathfrak{B}, \mathcal{N}', \mathfrak{K})$ are $\{\mathfrak{B}, \emptyset\}$ the image of a nontrivial closed set from $\mathfrak{U}$ cannot remain $n\mathfrak{I}$-closed set is not $n\mathfrak{I}$-closed in $\mathfrak{B}$. Hence, φ cannot be an $n\mathfrak{I}$-closed.

(ii) Let $\mathcal{A}$ be any $n\mathfrak{I} -$ closed subset $(\mathfrak{U}, \mathcal{N}, \mathfrak{I})$. The $n\mathfrak{I}$- open set containing $\varphi(\mathcal{A})$ is $\mathfrak{B}$. Thus, $cl_n^*\left(\varphi(\mathcal{A})\right) \subset \mathfrak{B}$. Implying that $\varphi(\mathcal{A})$ is $n\mathfrak{I}$-g- closed. Therefore $\varphi(\mathcal{A})$ is $n\mathfrak{I}$-g-closed.

(iii) For the same closed set $\mathcal{A}$, the only n $\mathfrak{J}$ –open set that contains $\varphi(\mathcal{A})$is again $\mathfrak{V}$. Hence, $cl_n^*\left(\mathcal{N}int(\varphi(\mathcal{A}))\right) \subset \mathfrak{V}$. Implying that $\varphi(\mathcal{A})$ is $n\mathfrak{J}$-$\mathcal{S}wg$- closed. Therefore $\varphi(\mathcal{A})$ is $n\mathfrak{J}$-$\mathcal{S}wg$- closed.

Theorem 3.6: Let $\varphi: (\mathfrak{U}, \mathcal{N}, \mathfrak{J}) \to (\mathfrak{V}, \mathcal{N}', \mathfrak{K})$ be any mapping. Let $\mathcal{A}$ be an $n\mathfrak{J}$-closed set in $(\mathfrak{U}, \mathcal{N}, \mathfrak{J})$. If $\mathfrak{V}$ is the unique $n\mathfrak{J}$-open set that contains $\varphi(\mathcal{A})$, then φ is an $n\mathfrak{J}$-$\mathcal{S}wg$- closed mapping.
Proof: Under the hypothesis, $\varphi(\mathcal{A})$ is a subset of $\mathfrak{V}$, and no proper $n\mathfrak{J}$ -open subset of $\mathfrak{V}$ contains $\varphi(\mathcal{A})$. Consequently, $cl_n^*(\mathcal{N}int(\varphi(\mathcal{A})) \subseteq \mathfrak{V}$. Thus, $\varphi(\mathcal{A})$ satisfies the defining condition of an $n\mathfrak{J}$- $\mathcal{S}wg$ -closed in $(\mathfrak{V}, \mathcal{N}', \mathfrak{K})$. Hence, φ is $n\mathfrak{J}$-$\mathcal{S}wg$- closed.

Theorem 3.7: Every $n\mathfrak{J}$-g- closed map is $n\mathfrak{J}$-$\mathcal{S}wg$- closed map, bur the converse does not hold.

Proof: Let $\varphi: (\mathfrak{U}, \mathcal{N}, \mathfrak{J}) \to (\mathfrak{V}, \mathcal{N}', \mathfrak{K})$ be a $n\mathfrak{J}$-closed map. Let $\mathcal{A}$ be a $n\mathfrak{J}$-closed subset of $(\mathfrak{U}, \mathcal{N}, \mathfrak{J})$. By assumption, $\varphi(\mathcal{A})$ is $n\mathfrak{J}$-$\mathcal{S}wg$- closed in $(\mathfrak{V}, \mathcal{N}', \mathfrak{K})$. Since every $n\mathfrak{J}$-$g-$ closed set is necessarily an $n\mathfrak{J}$-$\mathcal{S}wg$ closed set, $\varphi(A)$is $n\mathfrak{J}$-$\mathcal{S}wg$ closed. Thus, φ is $n\mathfrak{J}$-$\mathcal{S}wg$ closed.

The reverse implication fails in general, as demonstrated in Remark 3.8 and Example 3.9. Hence φ is $n\mathfrak{J}$-$\mathcal{S}wg$- closed.

Remark 3.8: Let $\varphi: (\mathfrak{U}, \mathcal{N}, \mathfrak{J}) \to (\mathfrak{V}, \mathcal{N}', \mathfrak{K})$ be any mapping, any suppose $\varphi(\mathcal{A}) \subseteq \mathcal{B}$, where $\mathcal{B}$ is an $n\mathfrak{J}$-open set in $(\mathfrak{V}, \mathcal{N}', \mathfrak{K})$ but $\mathcal{N}int(\varphi(\mathcal{A}))$ is empty , then $cl_n^*(\varphi(\mathcal{A})) \nsubseteq \mathcal{B}$. In this situation, φ still qualifies as an $n\mathfrak{J}$-$\mathcal{S}wg$- closed mapping, yet fails to be $n\mathfrak{J}$-g- closed. The following example illustrates this phenomenon.

Example 3.9: Let $\mathfrak{U} = \mathfrak{V} = \{a, b, c, d\}$ and consider the nano-topologies $(\mathfrak{U}, \mathcal{N}, \mathfrak{J}) = \{\mathfrak{U}, \emptyset, \{a, b, d\}\}$, and $(\mathfrak{V}, \mathcal{N}', \mathfrak{K}) = \{\mathfrak{U}, \emptyset, \{a, b, c\}\}$. Let $\varphi: (\mathfrak{U}, \mathcal{N}, \mathfrak{J}) \to (\mathfrak{V}, \mathcal{N}', \mathfrak{K})$ be the identity map. The map φ is $n\mathfrak{J}$-$\mathcal{S}wg$ - closed, since for each $n\mathfrak{J}$-g- closed in $\mathfrak{U}$, the image under φ satisfies the $n\mathfrak{J}$-$\mathcal{S}wg$ closedness condition. However, φ is not $n\mathfrak{J}$-g closed. Indeed $cl_n^*(\varphi(\{a\})) = cl_n^*(\{a\}) = \mathfrak{V}$ which is not contained in $\{a, b, c\}$. Thus $\{a\}$ is not $n\mathfrak{J}$-g - closed in $(\mathfrak{V}, \mathcal{N}', \mathfrak{K})$ and consequently φ fails to be $n\mathfrak{J}$-$\mathcal{S}wg$- closed.

Theorem 3.10: Every nano closed mapping is an $n\mathfrak{J}$-$\mathcal{S}wg$- closed mapping.

Proof: Let $\varphi: (\mathfrak{U}, \mathcal{N}, \mathfrak{J}) \to (\mathfrak{V}, \mathcal{N}', \mathfrak{K})$ be a nano closed. Let $\mathcal{A}$ be a nano-closed set of $(\mathfrak{U}, \mathcal{N}, \mathfrak{J})$. The image of $\mathcal{A}$ under the mapping φ is nano closed in $(\mathfrak{V}, \mathcal{N}', \mathfrak{K})$. Since every nano closed set is is $n\mathfrak{J}$-$\mathcal{S}wg$- closed. It follows that $\varphi(\mathcal{A})$ is $n\mathfrak{J}$-$\mathcal{S}wg$ - closed. Therefore φ is an $n\mathfrak{J}$-$\mathcal{S}wg$ – closed.

Theorem 3.11: Let $\varphi: (\mathfrak{U}, \mathcal{N}, \mathfrak{J}) \to (\mathfrak{V}, \mathcal{N}', \mathfrak{K})$ be a mapping. Then φ is $n\mathfrak{J}$-$\mathcal{S}wg$ closed if and only if the following condition holds. For every subset $\mathcal{A} \subseteq \mathfrak{V}$ and for each nano open set $\mathcal{B}$ containing $\mathfrak{U}$, such that $\varphi^{-1}(\mathcal{A}) \subseteq \mathcal{B}$ there exists an $n\mathfrak{J}$-$\mathcal{S}wg$ - open set $\mathcal{B}$ of $\mathfrak{V}$, such that $\mathcal{B} \subseteq \mathfrak{V}, \varphi^{-1}(\mathfrak{V}) \subset \mathfrak{U}$. $\mathcal{B}$ is an open set of $\mathfrak{V}$ ensuring that $\varphi^{-1}(\mathfrak{V}) \subset \mathfrak{U}$. In particular, the set $\mathcal{B} = \mathfrak{V} - \varphi(\mathfrak{U} - \mathcal{A})$ is an $n\mathfrak{J}$-$\mathcal{S}wg$- open set containing $\mathcal{B}$ such that $\varphi^{-1}(\mathcal{B}) \subset \mathcal{A}$.

Proof: Assume that $\varphi^{-1}$is $n\mathfrak{J}$-$\mathcal{S}wg$ - closed. Let $\mathcal{B}$ represent a subset of $\mathfrak{V}$ and $\mathcal{A}$ denote an open set of $\mathcal{U}$ such that $\varphi^{-1}(\mathfrak{V})$ is contained within $\mathfrak{U}$. Then let $\mathcal{B} = \mathfrak{V} - \varphi(\mathfrak{U} - \mathcal{A})$,which is a $n\mathfrak{J}$-$\mathcal{S}wg$ - open set that includes $\mathcal{B}$, ensuring that $\varphi^{-1}(\mathcal{B})$ is a subset of $\mathcal{A}$.

Conversely, suppose that $\mathcal{C}$ is closed subset of $\mathcal{N}$. Then we have $\varphi^{-1}(\mathfrak{V}) - \varphi(\mathcal{C}) \subset \mathfrak{U} - \mathcal{C}$ and $\mathfrak{U} - \mathcal{C}$ is nano open. By hypothesis there exists a $n\mathfrak{I}$-$\mathcal{Swg}$- open set $\mathcal{B}$ of $\mathfrak{V}$ such that $\mathfrak{V} - \varphi(\mathcal{C}) \subset \mathcal{B}$, and $\varphi^{-1}(\mathcal{B}) \subset \mathfrak{U} - \mathcal{C}$. This leads to the conclusion that $\mathcal{C}$ is a subset of $\mathfrak{U} - \varphi^{-1}(\mathcal{B})$. Consequently, we find that $\mathfrak{V} - \mathcal{B}$is a subset of $\varphi(\mathcal{C}) \subseteq \varphi\left(\mathfrak{U} - \varphi^{-1}(\mathcal{B})\right) \subset \mathfrak{V} - \mathcal{B}$ which shows that $\mathcal{C} = \mathfrak{V} - \mathcal{B}$. Thus, since $\mathfrak{V} - \mathcal{B}$ is $n\mathfrak{I}$-$\mathcal{Swg}$- closed set, it follows that $\varphi(\mathcal{C})$is $n\mathfrak{I}$-$\mathcal{Swg}$-closed. Therefore, φ is $n\mathfrak{I}$-$\mathcal{Swg}$ - closed.

4. NANO IDEAL SEMI WEAKLY GENERALIZED HOMEOMORPHISM AND NANO IDEAL SEMI WEAKLY GENERALIZED* HOMEOMORPHISM IN NANO IDEAL TOPOLOGICAL SPACES

In this section, we will introduce and investigate the concepts of $n\mathfrak{I}$-$\mathcal{Swg}$-homeomorphism and $n\mathfrak{I}$-$\mathcal{Swg}$ * within nano ideal topological spaces.

Definition 4.1: A bijective function $\varphi: (\mathfrak{U}, \mathcal{N}, \mathfrak{I}) \rightarrow (\mathfrak{V}, \mathcal{N}', \mathfrak{K})$ is said to be an $n\mathfrak{I}$-$\mathcal{Swg}$ homeomorphism if both the mapping φ and its inverse φ^{-1} are $n\mathfrak{I}$-$\mathcal{Swg}$ continuous.

Definition 4.2: A mapping $\varphi: (\mathfrak{U}, \mathcal{N}, \mathfrak{I}) \rightarrow (\mathfrak{V}, \mathcal{N}', \mathfrak{K})$ is called an $n\mathfrak{I}$-$\mathcal{Swg}$ *-

homeomorphism whenever φ and φ^{-1} are $n\mathfrak{I}$-$\mathcal{Swg}$ - irresolute.

Remark 4.3: For a bijective function $\varphi: (\mathfrak{U}, \mathcal{N}, \mathfrak{I}) \rightarrow (\mathfrak{V}, \mathcal{N}', \mathfrak{K})$. φ qualifies as an

$n\mathfrak{I}$-$\mathcal{Swg}$-homeomorphism function if it is both $n\mathfrak{I}$-$\mathcal{Swg}$-continuous and $n\mathfrak{I}$-$\mathcal{Swg}$-open.

Theorem 4.4: Every nano homeomorphism is also an $n\mathfrak{I}$-$\mathcal{Swg}$- homeomorphism.

Proof: Consider $\varphi: (\mathfrak{U}, \mathcal{N}, \mathfrak{I}) \rightarrow (\mathfrak{V}, \mathcal{N}', \mathfrak{K})$ as a nano homeomorphism. By its definition, φ is one-to-one and onto, as well as being a continuous nano open map. Since every nano continuous is $n\mathfrak{I}$-$\mathcal{Swg}$- continuous and every nano open map is $n\mathfrak{I}$-$\mathcal{Swg}$- open, it follows that φ is $n\mathfrak{I}$-$\mathcal{Swg}$ homeomorphism.

Remark 4.5: The converse of the above theorem is not necessarily true.

Example 4.6: Consider the nano ideal topological spaces $(\mathfrak{U}, \mathcal{N}, \mathfrak{I})$ and $(\mathfrak{V}, \mathcal{N}', \mathfrak{K})$.Let $\mathfrak{U} = \{\hbar_1, \hbar_2, \hbar_3, \hbar_4\}$, $\mathfrak{U}/\mathcal{R} = \left\{\{\hbar_1\}, \{\hbar_3\}, \{\hbar_2, \hbar_4\}\right\}, \mathcal{X} = \{\hbar_1, \hbar_2\} \subseteq \mathfrak{U}, \mathcal{N} = \{\, \mathcal{U}, \emptyset, \{\hbar_1\}, \{\hbar_1, \hbar_2, \hbar_4\}, \{\hbar_2, \hbar_4\}\,\}$ and ideal $\mathfrak{I} = \{\varphi, \{\hbar_1\}\}$, and let $\mathfrak{V} = \{k_1, k_2, k_3, k_4,\}, \mathcal{V}/\mathcal{R}' = \left\{\{k_1\}, \{k_2, k_3\}, \{k_4\}\right\}, \mathcal{Y} = \{k_1, k_3\} \subseteq \mathfrak{V}, \mathcal{N} = \{\, \mathcal{U}, \emptyset, \{k_1\}, \{k_2, k_3\}, \{k_1, k_2, k_3\}\,\}$ and ideal $\mathfrak{K} = \{\varphi, \{k_1\}\}$. Define a function $\varphi: (\mathfrak{U}, \mathcal{N}, \mathfrak{I}) \rightarrow (\mathfrak{V}, \mathcal{N}', \mathfrak{K})$, such that $\varphi(\hbar_1) = k_2, \varphi(\hbar_2) = k_3, \varphi(\hbar_3) = k_4, \varphi(\hbar_4) = k_1$. The function$\varphi$ is $n\mathfrak{I}$-$\mathcal{Swg}$ - homeomorphism. But not nano homeomorphism.

Theorem 4.7: Every $n\mathfrak{I}$-g-homeomorphism is an $n\mathfrak{I}$-$\mathcal{Swg}$- homeomorphism.

Proof: The proof of the theorem follows from the fact that every $n\mathfrak{I}$-g - continuous function is also $n\mathfrak{I}$-$\mathcal{Swg}$ -continuous function and every $n\mathfrak{I}$-g -open map is $n\mathfrak{I}$-$\mathcal{Swg}$ - open map.

Remark 4.8: The converse of the above theorem is not necessarily true, as demonstrated in the following example.

Example 4.9: In example 4.6 φ is not a $n\mathfrak{I}$-g - homeomorphism but qualifies as an $n\mathfrak{I}$-$\mathcal{Swg}$– homeomorphism.

Theorem 4.10: Let $\varphi: (\mathfrak{U}, \mathcal{N}, \mathfrak{I}) \to (\mathfrak{V}, \mathcal{N}', \mathfrak{K})$ be a bijective $n\mathfrak{I}$-Swg -continuous function. Then the following are equivalent

(i) φ is $n\mathfrak{I}$-Swg - open map.
(ii) φ is $n\mathfrak{I}$-Swg - homeomorphism.
(iii) φ is $n\mathfrak{I}$-Swg - closed map.

Proof: (i)⇒(ii) By the assumption and (i) φ is bijective $n\mathfrak{I}$-Swg -continuous function and $n\mathfrak{I}$-Swg - open map. By the definition of $n\mathfrak{I}$-Swg - homeomorphism φ is $n\mathfrak{I}$-Swg-homeomorphism.

(ii)⇒(iii) Since φ is $n\mathfrak{I}$-Swg - homeomorphism, it is 1-1, onto, $n\mathfrak{I}$-Swg -continuous function and $n\mathfrak{I}$-Swg - open map. Let $\mathcal{A}$ be a $n\mathfrak{I}$- closed set in $(\mathfrak{U}, \mathcal{N}, \mathfrak{I})$. Then $\varphi(\mathfrak{U} - \mathcal{A})$ is $n\mathfrak{I}$-Swg - open in $(\mathfrak{V}, \mathcal{N}', \mathfrak{K})$. $\varphi(\mathfrak{U} - \mathcal{A}) = \varphi(\mathfrak{U}) - \varphi(\mathcal{A}) = \mathfrak{V}$ is $n\mathfrak{I}$-wg-open. Hence $\varphi(\mathcal{A})$ is $n\mathfrak{I}$-Swg- closed map in $(\mathfrak{V}, \mathcal{N}', \mathfrak{K})$.

(iii)⇒(i) Let $\mathcal{A}$ be a $n\mathfrak{I}$-wg-open set in $(\mathfrak{U}, \mathcal{N}, \mathfrak{I})$. Then $\varphi(\mathfrak{U} - \mathcal{A})$ is $n\mathfrak{I}$-Swg - closed in $(\mathfrak{V}, \mathcal{N}', \mathfrak{K})$. That is $\varphi(\mathcal{A})$ is $n\mathfrak{I}$-Swg - open map in $(\mathfrak{V}, \mathcal{N}', \mathfrak{K})$. Therefore φ is $n\mathfrak{I}$-Swg-open map.

Theorem 4.11: Any $n\mathfrak{I}$-Swg *-homeomorphism is also an $n\mathfrak{I}$-Swg-homeomorphism.

Proof: The proof of this theorem follows directly from its definition and by referencing Theorem 4.8.

Remark 4.12: The converse of this theorem is not necessarily true, as $n\mathfrak{I}$-Swg - continuous functions might not be a $n\mathfrak{I}$-Swg - irresolute.

Example 4.13: In example 4.5.10, the image of open sets corresponds to is $n\mathfrak{I}$-Swg - open sets in $(\mathfrak{V}, \mathcal{N}', \mathfrak{K})$. This indicates that φ is an $n\mathfrak{I}$-Swg -open function, thus making φ an $n\mathfrak{I}$-Swg -homeomorphism, even though φ is not an $n\mathfrak{I}$-Swg - irresolute. Since the set $\varphi^{-1}(\{b\}) = \{a\}$ is not $n\mathfrak{I}$-αg - closed in $(\mathfrak{U}, \mathcal{N}, \mathfrak{I})$, φ is not an $n\mathfrak{I}$-Swg *-homeomorphism.

5.TOTALLY $n\mathfrak{I}$-Swg -CONTINUOUS FUNCTIONS STRONGLY $n\mathfrak{I}$-Swg -CONTINUOUS FUNCTIONS IN NANO IDEAL TOPOLOGICAL SPACES

In this segment, the concept of $n\mathfrak{I}$-Swg -totally continuous functions in the context of nano ideal topological spaces is introduced, along with a discussion of certain properties associated with them.

Definition 5.1: A function $\varphi: (\mathcal{U}, \mathcal{N}, \mathfrak{I}) \to (\mathcal{V}, \mathcal{N}', \mathfrak{K})$ is referred to as an

(i) $n\mathfrak{I}$-Swg -totally continuous function if the preimage of every $n\mathfrak{I}$-Swg -open subset of $(\mathcal{V}, \mathcal{N}', \mathfrak{K})$ is clopen in $(\mathcal{U}, \mathcal{N}, \mathfrak{I})$.
(ii) A function is considered totally $n\mathfrak{I}$-Swg -continuous function if the preimage of every nano closed subset of $(\mathcal{U}, \mathcal{N}, \mathfrak{I})$ constitute a $n\mathfrak{I}$-Swg - clopen set within $(\mathcal{V}, \mathcal{N}', \mathfrak{K})$.
(iii) A function is labeled strongly $n\mathfrak{I}$-Swg-continuous if the preimage of every $n\mathfrak{I}$-Swg -open set from $(\mathcal{V}, \mathcal{N}', \mathfrak{K})$ is open in $(\mathcal{U}, \mathcal{N}, \mathfrak{I})$.

Theorem 5.2: A function $\varphi: (\mathcal{U}, \mathcal{N}, \mathfrak{I}) \to (\mathcal{V}, \mathcal{N}', \mathfrak{K})$is $n\mathfrak{I}$-Swg -totally continuous if and only if the preimage of every $n\mathfrak{I}$-Swg -closed subset of $(\mathcal{V}, \mathcal{N}', \mathfrak{K})$ is nano clopen in $(\mathcal{U}, \mathcal{N}, \mathfrak{I})$.

Proof: Let $\mathcal{A}$ be an arbitrary $n\mathfrak{J}$-$\mathcal{Swg}$ -closed set in $\mathcal{V}$. Then $\mathcal{A}^c$ is $n\mathfrak{J}$-$\mathcal{Swg}$ -open in $\mathcal{V}$. According to the definition, $\varphi^{-1}(\mathcal{A}^c)$ is nano clopen in $\mathcal{U}$. However, $\varphi^{-1}(\mathcal{A}^c) = (\varphi^{-1}(\mathcal{A}))^c$ is also nano clopen in $\mathcal{U}$. This indicates that $\varphi^{-1}(\mathcal{A})$ is nano clopen in $\mathcal{U}$. Conversely, if $\mathcal{B}$ is $n\mathfrak{J}$-$\mathcal{Swg}$-open in $\mathcal{V}$, then $\mathcal{B}^c$ *is* $n\mathfrak{J}$-$\mathcal{Swg}$ -closed in $\mathcal{V}$. By hypothesis $\varphi^{-1}(\mathcal{B}^c)$ is nano clopen in $\mathcal{U}$. But $\varphi^{-1}(\mathcal{B}^c) = (\varphi^{-1}(\mathcal{A}))^c = (\varphi^{-1}(\mathcal{B}))^c$ which is nano clopen in $\mathcal{U}$, which implying that φ^{-1} (V) is nano clopen in $\mathcal{U}$. Thus, the preimage of every $n\mathfrak{J}$-$\mathcal{Swg}$ -open set in $\mathcal{U}$ is nano clopen in $\mathcal{U}$. As a results, φ is $n\mathfrak{J}$-$\mathcal{Swg}$ -totally continuous.

Theorem 5.3: Any $n\mathfrak{J}$-$\mathcal{Swg}$-totally continuous function is also totally nano continuous.

Proof: Let $\varphi: (\mathcal{U}, \mathcal{N}, \mathfrak{J}) \to (\mathcal{V}, \mathcal{N}', \mathfrak{K})$ be $n\mathfrak{J}$-$\mathcal{Swg}$ -totally continuous. Consider $\mathcal{A}$ as a nano open subset of $\mathcal{V}$. Since all nano open set is $n\mathfrak{J}$-$\mathcal{Swg}$ -open, $\mathcal{A}$ qualifies as $n\mathfrak{J}$-$\mathcal{Swg}$ -open in $\mathcal{V}$, because φ is $n\mathfrak{J}$-$\mathcal{Swg}$ -totally continuous, it follows that $\varphi^{-1}(\mathcal{A})$ is nano clopen in $\mathcal{U}$. Therefore, φ is totally nano continuous.

Remark 5.4: The reverse of this theorem may not hold true, as demonstrated by the following example.

Example 5.5: Let us examine the nano ideal topological spaces $(\mathcal{U}, \mathcal{N}, \mathfrak{J})$ and $(\mathcal{V}, \mathcal{N}', \mathfrak{K})$. Consider $\mathcal{U} = \{\mathfrak{s}_1, \mathfrak{s}_2, \mathfrak{s}_3\}$, where $\mathcal{N}$ consists of $\{ \mathcal{U}, \emptyset, \{\mathfrak{s}_1\}, \{\mathfrak{s}_2, \mathfrak{s}_3\} \}$ and the ideal $\mathfrak{J} = \{\emptyset, \{\mathfrak{s}_1\}\}$. Similarly let $\mathcal{V} = \{\mathscr{k}_1, \mathscr{k}_2, \mathscr{k}_3\}$, with $\mathcal{N}'$being $\{ \mathcal{V}, \emptyset, \{\mathscr{k}_1\}\}$ and the ideal $\mathfrak{K} = \{\emptyset, \{\mathscr{k}_1\}\}$. Define a function $\varphi: (\mathcal{U}, \mathcal{N}, \mathfrak{J}) \to (\mathcal{V}, \mathcal{N}', \mathfrak{K})$, such that $\varphi(\mathfrak{s}_1) = \mathscr{k}_1, \varphi(\mathfrak{s}_2) = \mathscr{k}_2, \varphi(\mathfrak{s}_3) = \mathscr{k}_3$. Therefore φ is totally nano continuous not $n\mathfrak{J}$-$\mathcal{Swg}$ -totally continuous, since $\varphi^{-1}(\{\mathscr{k}_1, \mathscr{k}_2\}) = \{\mathfrak{s}_1, \mathfrak{s}_2\}$ is not nano clopen in $\mathcal{U}$.

Theorem 5.6: Let $\varphi: (\mathcal{U}, \mathcal{N}, \mathfrak{J}) \to (\mathcal{V}, \mathcal{N}', \mathfrak{K})$be a function, then the following are equivalent.

(i) φ is $n\mathfrak{J}$-$\mathcal{Swg}$ -totally continuous.

(ii) for each $\in \mathcal{U}$ and each $n\mathfrak{J}$-$\mathcal{Swg}$ -open set $\mathcal{B}$ in $\mathcal{V}$ with$\varphi(u) \in \mathcal{B}$, there is a nano clopen set $\mathcal{A}$ in $\mathcal{U}$ such that $u \in \mathcal{A}$ and $\varphi(\mathcal{A}) \subseteq \mathcal{B}$

Proof: (i) ⇒(ii). If φ is $n\mathfrak{J}$-$\mathcal{Swg}$ -totally continuous and $\mathcal{B}$ be any $n\mathfrak{J}$-$\mathcal{Swg}$ -open set in $\mathcal{V}$ containing φ (u) so that $u \in \varphi^{-1}(\mathcal{B})$. Since $\varphi$is $n\mathfrak{J}$-$\mathcal{Swg}$-totally continuous, $\varphi^{-1}(\mathcal{B})$ is nano clopen in $\mathcal{U}$. Let $\mathcal{A} = \varphi^{-1}(\mathcal{B})$, then $\mathcal{A}$ is nano clopen in $\mathcal{U}$and $u \in \mathcal{A}$. Also $\varphi(\mathcal{A}) = \varphi(\varphi^{-1}(\mathcal{B})) \subseteq \mathcal{B}$. This implies that $\varphi(\mathcal{A}) \subseteq \mathcal{B}$.

(ii)⇒(i). Let $\mathcal{B}$ be a $n\mathfrak{J}$-$\mathcal{Swg}$-open set in $\mathcal{V}$. Let $u \in \varphi^{-1}(\mathcal{B})$ be any arbitrary point. This implies $\varphi(u) \in \mathcal{B}$. By (ii) there is a nano clopen set $\mathcal{K} \subseteq \mathcal{U}$ such that $\varphi(\mathcal{K}) \subseteq \mathcal{B}$, which implies $\mathcal{K} \subseteq \varphi^{-1}(\mathcal{B}), u \in \mathcal{K} \subseteq \varphi^{-1}(\mathcal{B})$. This implies $\varphi^{-1}(\mathcal{B})$is nano clopen neighbourhood of each of its points. Hence it is nano clopen set in $\mathcal{U}$. Hence φ is $n\mathfrak{J}$-$\mathcal{Swg}$-totally continuous.

Theorem 5.7: If a function $\varphi: (\mathcal{U}, \mathcal{N}, \mathfrak{J}) \to (\mathcal{V}, \mathcal{N}', \mathfrak{K})$ is $n\mathfrak{J}$-$\mathcal{Swg}$-totally continuous then φ is nano continuous, however, the converse is not necessarily true.

Proof: Let $\mathcal{B}$ be a nano open set in $\mathcal{V}$. Therefore, $\mathcal{B}$ is also $n\mathfrak{J}$-$\mathcal{Swg}$-open in $\mathcal{V}$. Given that φ is $n\mathfrak{J}$-$\mathcal{Swg}$-totally continuous, $\varphi^{-1}(\mathcal{B})$ is both open and closed in $\mathcal{U}$. This implies that φ is nano continuous. The converse of this theorem is not guaranteed, as illustrated by the following example.

Example 5.8: Let $\mathcal{U} = \mathcal{V} = \{\mathfrak{s}_1, \mathfrak{s}_2, \mathfrak{s}_3, \mathfrak{s}_4\}$, with the nano topology defined as

$\mathcal{N} = \{\mathcal{U}, \emptyset, \{\mathfrak{s}_1, \mathfrak{s}_3\}, \{\mathfrak{s}_4\}, \{\mathfrak{s}_1, \mathfrak{s}_2, \mathfrak{s}_3\}\}, \, and \; \mathfrak{J} = \{\emptyset, \{\mathfrak{s}_1\}\}. Let$

$\mathcal{N}' = \{\mathcal{V}, \emptyset, \{\mathfrak{s}_1\}, \{\mathfrak{s}_2, \mathfrak{s}_4\}, \{\mathfrak{s}_1, \mathfrak{s}_2, \mathfrak{s}_4\}\} and\ \mathfrak{K} = \{\emptyset, \{\mathfrak{s}_1\}\}$.We define $\varphi: (\mathcal{U}, \mathcal{N}, \mathfrak{I}) \rightarrow (\mathcal{V}, \mathcal{N}', \mathfrak{K})$ by $\varphi(\mathfrak{s}_1) = \mathfrak{s}_4, \varphi(\mathfrak{s}_2) = \mathfrak{s}_3, \varphi(\mathfrak{s}_3) = \mathfrak{s}_4$ and $\varphi(\mathfrak{s}_4) = \mathfrak{s}_2$. In this case, φ is nano continuous but not $n\mathfrak{I}$-$\mathcal{Swg}$ -totally continuous. Because the subset $\{\mathfrak{s}_2\}$ is $n\mathfrak{I}$-$\mathcal{Swg}$ -open in $(\mathcal{V}, \mathcal{N}', \mathfrak{K})$ but $\varphi^{-1}(\{\mathfrak{s}_2\}) = \{\mathfrak{s}_4\}$ is not nano closed in $(\mathcal{U}, \mathcal{N}, \mathfrak{I})$.

Theorem 5.9: The composition of two $n\mathfrak{I}$-$\mathcal{Swg}$-totally continuous functions is $n\mathfrak{I}$-$\mathcal{Swg}$-totally continuous.

Proof: Let $\varphi: (\mathcal{U}, \mathcal{N}, \mathfrak{I}) \rightarrow (\mathcal{V}, \mathcal{N}', \mathfrak{K})$ and $\psi: (\mathcal{V}, \mathcal{N}', \mathfrak{K}) \rightarrow (\mathcal{W}, \mathcal{N}'', \mathfrak{L})$ be any two $n\mathfrak{I}$-wg-totally continuous functions. If $\mathcal{B}$ is an $n\mathfrak{I}$-$\mathcal{Swg}$-open set in $\mathcal{W}$, then since ψ is $n\mathfrak{I}$-$\mathcal{Swg}$ -totally continuous, $\psi^{-1}(\mathcal{B})$ is nano clopen and thus open in $\mathcal{V}$. Since every nano open set is $n\mathfrak{I}$-$\mathcal{Swg}$ -open, $\psi^{-1}(\mathcal{B})$ is $n\mathfrak{I}$-$\mathcal{Swg}$ -open in $\mathcal{V}$. Furthermore, because φ is $n\mathfrak{I}$-$\mathcal{Swg}$ -totally continuous, $\varphi^{-1}(\psi^{-1}(\mathcal{B})) = (\psi \circ \varphi)^{-1}(\mathcal{B})$ is nano clopen in $\mathcal{U}$. Thus, $\psi \circ \varphi$ is $n\mathfrak{I}$-$\mathcal{Swg}$ -totally continuous.

Theorem 5.10: If a function $\varphi: (\mathcal{U}, \mathcal{N}, \mathfrak{I}) \rightarrow (\mathcal{V}, \mathcal{N}', \mathfrak{K})$ is $n\mathfrak{I}$-$\mathcal{Swg}$ -totally continuous, then φ is strongly $n\mathfrak{I}$-$\mathcal{Swg}$ -continuous, however, the converse is not true.

Proof: The proof follows directly from the definition.

Example 5.11: Let $\mathcal{U}$=$V = \{\mathfrak{s}_1, \mathfrak{s}_2, \mathfrak{s}_3, \mathfrak{s}_4\}$ with the nano topology defined as

$\mathcal{N} = \{\mathcal{U}, \emptyset, \{\mathfrak{s}_1, \mathfrak{s}_3\}, \{\mathfrak{s}_4\}, \{\mathfrak{s}_1, \mathfrak{s}_2, \mathfrak{s}_3\}\},\ and\ I = \{\emptyset, \{\mathfrak{s}_1\}\}$. Let

$\mathcal{N}' = \{\mathcal{V}, \emptyset, \{\mathfrak{s}_1\}, \{\mathfrak{s}_2, \mathfrak{s}_4\}, \{\mathfrak{s}_1, \mathfrak{s}_2, \mathfrak{s}_4\}\}\ and\ \mathfrak{K} = \{\emptyset, \{\mathfrak{s}_1\}\}$. We define $\varphi: (\mathcal{U}, \mathcal{N}, \mathfrak{I}) \rightarrow (\mathcal{V}, \mathcal{N}', \mathfrak{K})$ by $\varphi(\mathfrak{s}_1) =, \varphi(\mathfrak{s}_2) = \varphi(\mathfrak{s}_3) = \mathfrak{s}_4$, and $\varphi(\mathfrak{s}_4) = \mathfrak{s}_1$. In this example φ is a strongly $n\mathfrak{I}$-$\mathcal{Swg}$ -continuous function but not $n\mathfrak{I}$-$\mathcal{Swg}$ -totally continuous. Since $\varphi^{-1}(\{\mathfrak{s}_1\}) = \{\mathfrak{s}_4\}$ is not nano closed in $(\mathcal{U}, \mathcal{N}, \mathfrak{I})$.

Theorem 5.12: Let $\mathcal{U}$ be a discrete topological space and $\mathcal{V}$ is any ideal space and $\varphi: (\mathcal{U}, \mathcal{N}, \mathfrak{I}) \rightarrow (\mathcal{V}, \mathcal{N}', \mathfrak{K})$ is a function. If φ is strongly $n\mathfrak{I}$-$\mathcal{Swg}$ -continuous it follows that φ is $n\mathfrak{I}$-$\mathcal{Swg}$ -totally continuous.

Proof: Let $\mathcal{B}$ be $n\mathfrak{I}$-$\mathcal{Swg}$ -open in $\mathcal{V}$. Since φ is strongly $n\mathfrak{I}$-$\mathcal{Swg}$ -continuous, $\varphi^{-1}(\mathcal{B})$ is open in $\mathcal{U}$. Furthermore, as $\mathcal{U}$ is a discrete space, $\varphi^{-1}(\mathcal{B})$ is also nano closed in $\mathcal{U}$. Which means φ is $n\mathfrak{I}$-$\mathcal{Swg}$ -totally continuous.

Theorem 5.13: Any function that is totally $n\mathfrak{I}$-$\mathcal{Swg}$-continuous function is also $n\mathfrak{I}$-$\mathcal{Swg}$-continuous.

Proof: The proof is evident from the definition.

Remark 5.14: The following example demonstrates that the converse of the above statements need not be true.

Example 5.15: Let $\mathcal{U}$=$V = \{\mathfrak{s}_1, \mathfrak{s}_2, \mathfrak{s}_3\}$ the nano topology defined as $\mathcal{N} = \{\mathcal{U}, \emptyset, \{\mathfrak{s}_1\}\{\mathfrak{s}_1, \mathfrak{s}_2,\}\}$,$I = \{\emptyset, \{\mathfrak{s}_2\}\}, \mathcal{N}' = \{\mathcal{V}, \emptyset, \{\mathfrak{s}_2\}, \{\mathfrak{s}_1, \mathfrak{s}_2\}\}\ and\ \mathfrak{K} = \{\emptyset, \{\mathfrak{s}_1\}\}$. The function $\varphi: (\mathcal{U}, \mathcal{N}, \mathfrak{I}) \rightarrow (\mathcal{V}, \mathcal{N}', \mathfrak{K})$ is defined by $\varphi(\mathfrak{s}_1) = \mathfrak{s}_2, \varphi(\mathfrak{s}_2) = \mathfrak{s}_1, \varphi(\mathfrak{s}_3) = \mathfrak{s}_3$. In this instance, φ is $n\mathfrak{I}$-wg -continuous but not totally $n\mathfrak{I}$-$\mathcal{Swg}$ -continuous because $\{\mathfrak{s}_2\}$ is open in $\mathcal{V}$ but $\varphi^{-1}(\{\mathfrak{s}_2\}) = \{\mathfrak{s}_1\}$ which is $n\mathfrak{I}$-$\mathcal{Swg}$ -open and not $n\mathfrak{I}$-$\mathcal{Swg}$ -closed in $(\mathcal{U}, \mathcal{N}, \mathfrak{I})$.

Theorem 5.16: Any function that is $n\mathfrak{I}$-$\mathcal{S}wg$ -totally continuous function is also totally $n\mathfrak{I}$-$\mathcal{S}wg$ -continuous.

Proof: Assume $\varphi: (\mathcal{U}, \mathcal{N}, \mathfrak{I}) \to (\mathcal{V}, \mathcal{N}', \mathfrak{K})$ is $n\mathfrak{I}$-$\mathcal{S}wg$ -totally continuous. Let $\mathcal{A}$ be any open subset of $\mathcal{V}$. Since every open set is $n\mathfrak{I}$-$\mathcal{S}wg$ open, $\mathcal{A}$ is $n\mathfrak{I}$-$\mathcal{S}wg$ -open in $\mathcal{V}$ and φ is $n\mathfrak{I}$-$\mathcal{S}wg$ -totally continuous, it follows $\varphi^{-1}(\mathcal{A})$ is nano clopen in $\mathcal{U}$. Then $\varphi^{-1}(\mathcal{A})$ is $n\mathfrak{I}$-$\mathcal{S}wg$-clopen in $\mathcal{U}$. This establishes the theorem.

Remark 5.17: The converse of the previous theorem may not hold true, as demonstrated in the following example.

Example 5.18: Let $\mathcal{U}$=$V = \{\mathfrak{s}_1, \mathfrak{s}_2, \mathfrak{s}_3\}$ the nano topology defined as $\mathcal{N} = \{\mathcal{U}, \emptyset, \{\mathfrak{s}_1\}\{\mathfrak{s}_1, \mathfrak{s}_3\}\}, \mathfrak{I} = \{\emptyset, \{\mathfrak{s}_1\}\}, The\ set\ \ \mathcal{N}'\ is\ defined\ as\ \{\mathcal{V}, \emptyset, \{\mathfrak{s}_1\}\}\ and\ \mathfrak{K}\ \ \{\emptyset, \{\mathfrak{s}_1\}\}$. we define a function $\varphi: (\mathcal{U}, \mathcal{N}, \mathfrak{I}) \to (\mathcal{V}, \mathcal{N}', \mathfrak{K})$ such that $\varphi(\mathfrak{s}_1) = \mathfrak{s}_1, \varphi(\mathfrak{s}_2) = \mathfrak{s}_2, \varphi(\mathfrak{s}_3) = \mathfrak{s}_3$, Clearly the inverse image of every open set is $n\mathfrak{I}$-$\mathcal{S}wg$ -clopen in $\mathcal{U}$. Thus φ is totally $n\mathfrak{I}$-$\mathcal{S}wg$ -continuous. But φ is not $n\mathfrak{I}$-$\mathcal{S}wg$ -totally continuous, since for the $n\mathfrak{I}$-$\mathcal{S}wg$ -open set $\{\mathfrak{s}_1, \mathfrak{s}_2\}$, the inverse $\varphi^{-1}(\{\mathfrak{s}_1, \mathfrak{s}_2\}) = \{\mathfrak{s}_1, \mathfrak{s}_2\}$ is not clopen in $(\mathcal{U}, \mathcal{N}, \mathfrak{I})$

6. CONTRA $n\mathfrak{I}$-$\mathcal{S}wg$ -CONTINUOUS FUNCTIONS IN NANO IDEAL TOPOLOGICAL SPACES

In this section, we introduce a new class of functions referred to as contra $n\mathfrak{I}$-$\mathcal{S}wg$ -continuous functions, and we derive some of their characterizations and properties. Additionally, we explore their connections with other existing types of functions.

Definition 6.1: A function $\varphi: (\mathcal{U}, \mathcal{N}, \mathfrak{I}) \to (\mathcal{V}, \mathcal{N}', \mathfrak{K})$ is called contra $n\mathfrak{I}$-$\mathcal{S}wg$ -continuous on $\mathcal{U}$, if the inverse image of every nano open set in $\mathcal{V}$ is $n\mathfrak{I}$-$\mathcal{S}wg$ - closed set in $\mathcal{U}$.

Theorem 6.2: A function $\varphi: (\mathcal{U}, \mathcal{N}, \mathfrak{I}) \to (\mathcal{V}, \mathcal{N}', \mathfrak{K})$ is classified as a contra $n\mathfrak{I}$-$\mathcal{S}wg$ -continuous function, if and only if inverse image of every nano closed set in $\mathcal{V}$ is $n\mathfrak{I}$-$\mathcal{S}wg$-open set in $\mathcal{U}$.

Proof: Let $\varphi: (\mathcal{U}, \mathcal{N}, \mathfrak{I}) \to (\mathcal{V}, \mathcal{N}', \mathfrak{K})$, and let $\mathcal{B}$ represent any nano closed set in $\mathcal{V}$. Since φ is contra $n\mathfrak{I}$-$\mathcal{S}wg$ -continuous function, we know that $\varphi^{-1}(\mathcal{V} - \mathcal{B}) = \mathcal{U} - \ \varphi^{-1}(\mathcal{B})$ is a nano closed set in $\mathcal{U}$. Thus $\varphi^{-1}(\mathcal{B})$ must be $n\mathfrak{I}$-$\mathcal{S}wg$-open set in $\mathcal{V}$.

Conversely, $\mathcal{B}$ as a nano open set in $\mathcal{V}$. By our assumption $\varphi^{-1}(\mathcal{V} - \mathcal{B})$ is $n\mathfrak{I}$-$\mathcal{S}wg$-open set. Since $\varphi^{-1}(\mathcal{V} - \mathcal{B}) = \mathcal{U} - \varphi^{-1}(\mathcal{B})$, it follows that $\varphi^{-1}(\mathcal{B})$ is an $n\mathfrak{I}$-$\mathcal{S}wg$-closed set in $\mathcal{U}$. Therefore φ is confirmed to be contra $n\mathfrak{I}$-$\mathcal{S}wg$-continuous.

Theorem 6.3: For a function $\varphi: (\mathcal{U}, \mathcal{N}, \mathfrak{I}) \to (\mathcal{V}, \mathcal{N}', \mathfrak{K})$, the following statements are equivalent.

(i) The inverse image of every nano closed set in $\mathcal{V}$ is $n\mathfrak{I}$-$\mathcal{S}wg$-open in $\mathcal{U}$.

(ii) For each $u \in \mathcal{U}$ and any nano closed set $\mathcal{B}$ in $\mathcal{V}$ with $\varphi(u) \in \mathcal{B}$ there exist an $n\mathfrak{I}$-$\mathcal{S}wg$ -open set in $\mathcal{U}$ such that $\varphi(\mathcal{A}) \subseteq \mathcal{B}$.

Proof: (i) → (ii), Let $\mathcal{B}$ be nano closed set in $\mathcal{V}$ such that $\varphi(u) \in \mathcal{B}, u \in \mathcal{U}$. Let $\mathcal{A} = \varphi^{-1}(\mathcal{B})$.
(ii) → (i) , Let $\mathcal{B}$ be any nano closed in $\mathcal{V}$, $u \in \mathcal{U}, \varphi(u) \in \mathcal{B}$. There exists an $n\mathfrak{I}$-wg open set $\mathcal{P}$ such that $\varphi(\mathcal{P}) \subseteq \mathcal{B}, \varphi^{-1}(\mathcal{B}) = \cup \{\mathcal{P}, u\ \in \varphi^{-1}(\mathcal{B}) \in nO(u)\}, \varphi^{-1}(\mathcal{B})$ is $n\mathfrak{I}$-$\mathcal{S}wg$-open set in $\mathcal{U}$.

Remark 6.4: The contra $n\mathfrak{I}$-$\mathcal{S}wg$ -continuous function and $n\mathfrak{I}$-$\mathcal{S}wg$ -continuous function are distinct from one another.

Example 6.5: Let $\mathcal{U} = \{\hbar_1, \hbar_2, \hbar_3, \hbar_4, \hbar_5\}$ with $\mathcal{U}/\mathcal{R} = \{\{\hbar_1, \hbar_3\}, \{\hbar_2\}, \{\hbar_4\}, \{\hbar_5\}\}$ and $\mathcal{X} = \{\hbar_1, \hbar_2\}, \mathfrak{I} = \{\emptyset, \{\hbar_1\}\}$. Then $\mathcal{N} = \{\mathcal{U}, \emptyset, \{\hbar_2\}, \{\hbar_1, \hbar_3\}, \{\hbar_1, \hbar_2, \hbar_3\}\}$. Let $\mathcal{V} = \{\hbar_1, \hbar_2, \hbar_3, \hbar_4, \hbar_5\}$ with $\mathcal{V}\ /\ \mathcal{N}' = \{\{\hbar_1\},\ \{\hbar_2, \hbar_4\},\ \{\hbar_3, \hbar_5\}\}$ and $\mathcal{Y} = \{\hbar_3, \hbar_5\}, \mathfrak{K} = \{\emptyset, \{\hbar_1\}\}$. Then the nano topology is $\mathcal{N}' = \{\mathcal{V}, \emptyset, \{\hbar_3, \hbar_5\}\}$. Define $\varphi: (\mathcal{U}, \mathcal{N}, \mathfrak{I}) \to (\mathcal{V}, \mathcal{N}', \mathfrak{K})$

as $\varphi(\hbar_1) = \hbar_3, \varphi(\hbar_2) = \hbar_5, \varphi(\hbar_3) = \hbar_1, \varphi(\hbar_4) = \hbar_2$,and$\varphi(\hbar_5) = \hbar_2$. Then φ is contra $n\mathfrak{I}$-Swg -continuous function. Since $\varphi^{-1}(\{\hbar_3, \hbar_5\}) = \{\hbar_1, \hbar_2\}$ is not $n\mathfrak{I}$-Swg -closed set in $\mathcal{U}$ and φ is not contra $n\mathfrak{I}$-Swg -continuous function.

Example 6.6: Let $\mathcal{U} = \{\hbar_1, \hbar_2, \hbar_3, \hbar_4, \hbar_5\}$ with $\mathcal{U}/\mathcal{R}$=$\{\{\hbar_1, \hbar_3\}, \{\hbar_2\}, \{\hbar_4\}, \{\hbar_5\}\}$ and $\mathcal{X} = \{\hbar_1, \hbar_2\}, \mathfrak{I} = \{\emptyset, \{\hbar_1\}\}$. Then the nano topology is $\mathcal{N} = \{\mathcal{U}, \emptyset, \{\hbar_2\}, \{\hbar_1, \hbar_3\}, \{\hbar_1, \hbar_2, \hbar_3\}\}$. Let $\mathcal{V} = \{\hbar_1, \hbar_2, \hbar_3, \hbar_4, \hbar_5\}$ with $\mathcal{V} / \mathcal{N}' = \{\{\hbar_1\}, \{\hbar_2\}, \{\hbar_3, \hbar_4\}, \{\hbar_5\}\}$ and $\mathcal{Y} = \{\hbar_1, \hbar_2\}, \mathfrak{K} = \{\emptyset, \{\hbar_1\}\}$. Then the nano topology is $\mathcal{N}' = \{\mathcal{V}, \emptyset, \{\hbar_1, \hbar_2\}\}$. Define $\varphi: (\mathcal{U}, \mathcal{N}, \mathfrak{I}) \to (\mathcal{V}, \mathcal{N}', \mathfrak{K})$ as $\varphi(\hbar_1) = \hbar_3, \varphi(\hbar_2) = \hbar_4, \varphi(\hbar_3) = \hbar_5, \varphi(\hbar_4) = \hbar_1$,and$\varphi(\hbar_5) = \hbar_2$, then φ is contra $n\mathfrak{I}$-wg -continuous function. Since $\varphi^{-1}(\{\hbar_1, \hbar_2\}) = \{\hbar_3, \hbar_4\}$ is not $n\mathfrak{I}$-Swg -open φ is not $n\mathfrak{I}$-Swg -continuous function.

Theorem 6.7: Every nano ideal contra continuous function is contra $n\mathfrak{I}$-Swg -continuous function.

Proof: Let $\varphi: (\mathcal{U}, \mathcal{N}, \mathfrak{I}) \to (\mathcal{V}, \mathcal{N}', \mathfrak{K})$ be a nano contra continuous function and $\mathcal{A}$ be nano open set in $\mathcal{V}$. Since φ is nano contra continuous function $\varphi^{-1}(\mathcal{A})$ is Nano closed set in $\mathcal{U}$. Thus $\varphi^{-1}(\mathcal{A})$ is $n\mathfrak{I}$-Swg-closed in $\mathcal{U}$. Hence φ is Contra $n\mathfrak{I}$-Swg -continuous function.

Remark 6.8: Converse of the above theorem need not be as shown in the following example.

Example 6.9: Let $\mathcal{U} = \{\hbar_1, \hbar_2, \hbar_3, \hbar_4, \hbar_5\}$,
$\mathcal{U}/\mathcal{R} = \{\{\hbar_1\}, \{\hbar_3\}, \{\hbar_2\}, \{\hbar_4\}, \{\hbar_5\}\}$, then the nano topology is
$\mathcal{N} = \{\mathcal{U}, \emptyset, \{\hbar_1, \hbar_2, \hbar_3\}\}$ and $\mathfrak{I} = \{\emptyset, \{\hbar_1\}\}$. Let $\mathcal{V} = \{\hbar_1, \hbar_2, \hbar_3, \hbar_4, \hbar_5\}$ with$\mathcal{V} / \mathcal{N}' = \{\{\hbar_2\}, \{\hbar_1, \hbar_3\}, \{\hbar_4, \hbar_5\}\}$ and $\mathcal{Y} = \{\hbar_1, \hbar_2, \hbar_3\}$. Then the Nano topology is $\mathcal{N}' = \{\mathcal{V}, \emptyset, \{\hbar_1, \hbar_2\}\}$ and$\mathfrak{K} = \{\emptyset, \{\hbar_1\}\}$. Define $\varphi: (\mathcal{U}, \mathcal{N}, \mathfrak{I}) \to (\mathcal{V}, \mathcal{N}', \mathfrak{K})$as $\varphi(\hbar_1) = \hbar_3, \varphi(\hbar_2) = \hbar_4, \varphi(\hbar_3) = \hbar_5, \varphi(\hbar_4) = \hbar_1$,and$\varphi(\hbar_5) = \hbar_2$, then φ is Contra $n\mathfrak{I}$-Swg -continuous function but not Swg- contra continuous function. Since $\varphi^{-1}(\{\hbar_1, \hbar_2\}) = \{\hbar_3, \hbar_4\}$ is $n\mathfrak{I}$-Swg -closed but not $n\mathfrak{I}$-nano closed in $\mathcal{U}$.

Theorem 6.10: A function $\varphi: (\mathcal{U}, \mathcal{N}, \mathfrak{I}) \to (\mathcal{V}, \mathcal{N}', \mathfrak{K})$ is Contra Swg-continuous function if the only nano open set containing the inverse image of every nano open set $\mathcal{A}$ of $\mathcal{V}$ is $\mathcal{U}$.

Proof: Let $\mathcal{A}$ be a Nano open set $\mathcal{V}$ and $\mathcal{U}$ is the only Nano open set such that $\varphi^{-1}(\mathcal{A}) \subseteq \mathcal{U}$. Then $cl_n^*(\mathcal{N}int(\mathcal{A})) \subseteq \mathcal{U}$. (i.e.) $\varphi^{-1}(\mathcal{A})$ is $n\mathfrak{I}$-Swg-closed in $\mathcal{U}$. φ is Contra $n\mathfrak{I}$-Swg-continuous function.

Corollary 6.11: Let $\varphi: (\mathcal{U}, \mathcal{N}, \mathfrak{I}) \to (\mathcal{V}, \mathcal{N}', \mathfrak{K})$and $\mathcal{N}int(\varphi^{-1}(\mathcal{A})) = \emptyset$ for every nano open set $\mathcal{A}$ of $\mathcal{V}$ then φ is Contra $n\mathfrak{I}$-Swg -continuous function.

Theorem 6.12: Let $\varphi: (\mathcal{U}, \mathcal{N}, \mathfrak{I}) \to (\mathcal{V}, \mathcal{N}', \mathfrak{K})$ be a Contra $n\mathfrak{I}$-Swg -continuous function then $\varphi\left(\mathcal{N}int_{Swg}(\mathcal{A})\right) \subseteq cl_n^*(\varphi(\mathcal{A}))$ for every subset $\mathcal{A} \subseteq \mathcal{U}$.

Proof: Let $\mathcal{A} \subseteq \mathcal{U}$ then $cl_n^*(\varphi(\mathcal{A}))$ is a Nano closed set in $\mathcal{V}$. Since φ is Contra $n\mathfrak{I}$-Swg -continuous $\varphi^{-1}\left(cl_n^*(\varphi(\mathcal{A}))\right)$ is $n\mathfrak{I}$-Swg -open set in $\mathcal{U}$ and $\mathcal{N}int_{Swg}\left(\varphi^{-1}(cl_n^*(\varphi(\mathcal{A}))\right) = \varphi^{-1}\left(cl_n^*(\varphi(\mathcal{A}))\right)$. Then $\varphi(\mathcal{A}) \subseteq cl_n^*(\varphi(\mathcal{A})), \mathcal{N}int_{Swg}(\mathcal{A}) \subseteq$
$\mathcal{N}int_{Swg}\left(\varphi^{-1}\left(cl_n^*(\varphi(\mathcal{A}))\right)\right), \mathcal{N}int_{Swg}(\mathcal{A}) \subseteq \varphi^{-1}\left(cl_n^*(\varphi(\mathcal{A}))\right), \varphi\left(\mathcal{N}int_{Swg}(\mathcal{A})\right) \subseteq$
$cl_n^*(\varphi(\mathcal{A}))$.

Remark 6.13: As demonstrated in the following example, the converse of the previous theorem need not be true.

Example 6.14: In example 6.8 $\varphi\left(\mathcal{N}int_{Swg}(\mathcal{A})\right) \subseteq cl_n^*(\mathcal{A})$ but φ is not contra $n\mathfrak{I}$-Swg -continuous function.

Corollary 6.15: Let $\varphi: (\mathcal{U}, \mathcal{N}, \mathfrak{I}) \to (\mathcal{V}, \mathcal{N}', \mathfrak{K})$be a $n\mathfrak{I}$-wg -continuous function then $\varphi\left(\mathcal{N}int_{nSwg}(\varphi^{-1}(\mathcal{A}))\right) \subseteq cl_n^*(\mathcal{A})$ for every subset $\mathcal{A} \subseteq \mathcal{V}$.

Theorem 6.16: Let $\varphi: (\mathcal{U}, \mathcal{N}, \mathfrak{I}) \to (\mathcal{V}, \mathcal{N}', \mathfrak{K})$ be a $n\mathfrak{I}$-$\mathcal{S}w\mathcal{g}$ -continuous function then $\varphi^{-1}(\varphi^{-1}(\mathcal{A})) \subseteq cl^{*}_{\mathcal{S}w\mathcal{g}}(\varphi^{-1}(\mathcal{A}))$ for every subset $\mathcal{A} \subseteq \mathcal{V}$.

Proof: Let $\mathcal{A} \subseteq \mathcal{V}$ then $\mathcal{N}int(\mathcal{A})$ is a Nano open set in $\mathcal{V}$. Since φ is Contra $n\mathfrak{I}$-$\mathcal{S}w\mathcal{g}$ -continuous $\varphi^{-1}\left(\mathcal{N}int(\mathcal{A})\right)$ is $n\mathfrak{I}$-$\mathcal{S}w\mathcal{g}$ -closed set in $\mathcal{U}$ and $cl^{*}_{n\mathcal{S}w\mathcal{g}}(\varphi^{-1}\left(\mathcal{N}int(\mathcal{A})\right))) = \varphi^{-1}(\mathcal{N}int(\mathcal{A})).\mathcal{N}int(\mathcal{A}) \subseteq \mathcal{A}$,
$\varphi^{-1}\left(\mathcal{N}int(\mathcal{A})\right) \subseteq \varphi^{-1}(\mathcal{A})$,$cl^{*}_{n\mathcal{S}w\mathcal{g}}\varphi^{-1}(\mathcal{N}int(\mathcal{A})))) \subseteq cl^{*}_{n\mathcal{S}w\mathcal{g}}(\varphi^{-1}(\mathcal{A}))$,
$(\varphi^{-1}(\mathcal{N}int(\mathcal{A}))) \subseteq cl^{*}_{n\mathcal{S}w\mathcal{g}}(\varphi^{-1}(\mathcal{A}))$.

Remark 6.17: The converse of the above theorem need not be true as shown in the following example.

Example 6.18: In example 6.9, φ is not Contra $n\mathfrak{I}$-$\mathcal{S}w\mathcal{g}$ -continuous function, but $\varphi^{-1}(\mathcal{N}int(\mathcal{A})))) \subseteq cl^{*}_{nSwg}(\varphi^{-1}(\mathcal{A}))$ for every subsete $\mathcal{A} \subseteq \mathcal{V}$.

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