Advanced Vogel's approximation method to find feasible solution of Transportation Problem :A new method to calculate penalty costs for finding better solutions to the Transportation Problem

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Abstract

In Operation Research, obtaining significant result for Transportation Problems is very important now-a-days. Vogel's Approximation Method (VAM) is the very efficient algorithm to solve the transportation problem for feasible solution which is nearer to optimal solution. In this paper we identified a computational error in VAM and approach a logical development of VAM algorithm. The main concept of VAM is to determine penalty cost which obtains from the difference of smallest and next to smallest cost in each row or column and make maximum allocation in lowest cost cell of that row or column which have largest penalty. The difficulty arises when smallest cost and next to smallest cost have same magnitude. In that case we find a very logical concept to resolve this and developed a new algorithm "Advanced Vogel's Approximation Method (AVAM)" to find a feasible solution of transportation problem which is very close to optimal solution more than VAM.

Keywords:*AVAM, VAM, Penalty Cost, Feasible Solution, Error Estimation, Transportation Problem (TP) etc.*

I.Introduction

Transportation problem (TP) is a special kind of Linear Programming Problem (LPP) in which goods are transported from a set of sources to a set of destinations subject to the supply and demand of the sources and destination respectively such that the total cost of transportation is minimized.

In Operation research, TP is a special class of Linear Programming Problem (LPP) where Vogel"s Approximation Method (VAM) is known as efficient method to solve Transportation Problem. The concept penalty cost (difference of smallest and next to smallest cost in row or column) makes this method more effective more than other methods such as North West Corner Method (NWC) and Least Cost Method (LCM) .

1.1 Types of Transportation problems:

- **BalancedTransportation problem** :When both supplies and demands are equal then the problem is said to be a balanced transportation problem.
- **Unbalanced Transportation problem**: When the supply and demand are not equal then it is said to be an unbalanced transportation problem. In this type of problem, either a dummy row or a dummy column is added according to the requirement to make it a balanced problem. Then it can be solved similar to the balanced problem.
- **Methods to SolveLPP**: To find the initial basic feasible solution there are three methods:
  i. North-West Corner Cell Method.
  ii. Least Cost Cell Method.
  iii. Vogel's Approximation Method (VAM)

II.Vogel's Approximation Method :

The Vogel Approximation method is an iterative procedure for computing a basic feasible Solution of a transportation problem. This method is preferred over the two methods i.eNorth West Corner method and Least cost cell Method.

Step-1:

i) Check whether the given transportation problem is balanced or not. If the given probleis not balanced i.e. if supply and demand are not equal then add a dummy row or a dummy column depending on the necessity, to make it a balanced problem Then the problem can changed as balanced transportation problem.

ii) Identify the cells having minimum and next to minimum transportation cost in each row and write the absolute difference (Penalty) along the side of the table against the corresponding row.

iii) Identify the cells having minimum and next to minimum transportation cost in each column and write the absolute difference (Penalty) along the side of the table against the corresponding column.

Step-2:

If minimum cost appear in two or more times in a row or column then select these same costas a minimum and next to minimum cost and penalty will be zero.

Step-3:

i). Identify the row and column with the largest penalty, breaking ties arbitrarily. Allocate as much as possible to the variable with the least cost in the selected row or column. Adjust the supply and demand and cross out the satisfied row or column. If a row and column are satisfies simultaneously, only one of them is crossed out and remaining row or column is assigned a zero supply or demand.

ii). If two or more penalty costs have same largest magnitude, then select any one of them (or most top row or extreme left column).

Step-4:

i).If exactly one row or one column with zero supply or demand remains uncrossed out, Stop.

ii). If only one row or column with positive supply or demand remains uncrossed out, determine the basic variables in the row or column by the Least-Cost Method.

iii). If all uncrossed out rows or column have (remaining) zero supply or demand, determined the zero basic variables by the Least-Cost Method. Stop.

iv) Otherwise, go to step 1.

Vogel's Approximation Method:

Example : Solve the following transportation problem using Vogel's Approximation Method.

Source	Destination					**Supply**
	D1	D2	D3	D4	D5	
A1	4	4	9	8	13	**100**
A2	7	9	8	10	4	**80**
A3	9	3	7	10	6	**70**
A4	11	4	8	3	9	**90**
Demand	**60**	**40**	**100**	**50**	**90**	

Solution: For the given cost matrix,

Total supply = 100 + 80 + 70+ 90 = 340

Total demand = 60 + 40 + 100 + 50 + 90 = 340

Thus, the given problem is balanced transportation problem

Now, we can apply the Vogel's approximation method to minimize the total cost of transportation.

Step 1: Identify the least and second least cost in each row and column and then write the corresponding absolute differences of these values.

For example, in the first row, 4 and 4 are the least and second least values, their absolute difference is 0.

These row and column differences are called penalties.

Step 2: Now, identify the maximum penalty and choose the least value in that corresponding row or column. Then, assign the min(supply, demand).

Here, the maximum penalty is **5** and the least value in the corresponding column is **3**.

For this cell, min(supply, demand) = min(90, 50) = 50

Allocate 50 in that cell and strike the corresponding column since in this case demand will be satisfied, i.e., 50 – 50 = 0

Source	**Destination**					**Supply**	**Row Penalty**
	D1	D2	D3	D4	D5		
A1	4	4	9	8	13	**100**	0
A2	7	9	8	10	4	**80**	3
A3	9	3	7	10	6	**70**	3
A4	11	4	8	3 **50**	9	**90** **40**	1
Demand	**60**	**40**	**100**	**50**	**90**		
Column Penality	3	1	1	**5**	2		

Step 3: Now, find the absolute row and column differences for the remaining rows and columns.

Then repeat step 2.

Here, the maximum penalty is 4 and the least cost in that corresponding row is 4. Also, the min(supply, demand) = min(40, 40) = 40

Thus, allocate 40 for that cell and write down the new supply and demand for the corresponding row and column.

Supply = 40 – 40 = 0

Demand = 40 – 40 = 0

If row and column are satisfied simultaneously then crossed out one of them and set zero supplyor demand in remaining row or column.

Here both Supply and Demand is 0, strike the second column and set zero supply.

Source	Destination					Supply	Row Penality	Row Penality
	D1	D2	D3	D4	D5			
A1	4	4	9	8	13	**100**	0	0
A2	7	9	8	10	4	**80**	3	3
A3	9	3	7	10	6	**70**	3	3
A4	11	4 40	8	3 **50**	9	**90** **40** **0**	1	4
Demand	**60**	**40**	**100**	**50**	**90**			
Column Penality	3	1	1	**5**	2			
Column Penality	3	1	1	**-**	2			

Step 4: Repeat the above step, i.e., step 3. This will give the below result.

In this step, the first column crossed out and the min(supply, demand) = min(100, 60) = 60 is assigned for the cell with value 4.

Source	Destination					Supply	R.P	R.P	R.P
	D1	D2	D3	D4	D5				
A1	4 **60**	4	9	8	13	**100** **40**	0	0	**5**
A2	7	9	8	10	4	**80**	3	3	3
A3	9	3	7	10	6	**70**	3	3	1
A4	11	4 40	8	3 **50**	9	**90** **40** **0**	1	4	1
Demand	**60**	**40**	**100**	**50**	**90**				
Column Penality	3	1	1	**5**	2				
Column Penality	3	1	1	-	2				
Column Penality	3	-	1	-	2				

Source	Destination					Supply	R.P	R.P	R.P	R.P
	D1	D2	D3	D4	D5					
A1	4 **60**	4	9 **40**	8	13	**100** **40**	0	0	**5**	**4**
A2	7	9	8	10	4	**80**	3	3	3	4
A3	9	3	7	10	6	**70**	3	3	1	1
A4	11	4 40	8	3 **50**	9	**90** **40** **0**	1	4	1	1
Demand	**60**	**40**	**100**	**50**	**90**					

			60			
Column Penality	3	1	1	5	2	
Column Penality	3	1	1	-	2	
Column Penality	3	-	1	-	2	
Column Penality	-	-	1	-	2	

Step 5: Again repeat step 3, as we did for the previous step

In this case, we got 4 as the maximum penalty and 9 as the least cost of the corresponding row.So allocate min(supply, demand) = min(40, 60) = 40 for that cell. And crossed out firstrow.

Step 6: Now, again repeat step 3 by calculating the absolute differences for the remaining rows and columns.

Source	Destination					Supply	R.P	R.P	R.P	R.P	R.P
	D1	D2	D3	D4	D5						
A1	4 **60**	4	9 **40**	8	13	**100** **40**	0	0	**5**	**4**	**-**
A2	7	9	8	10	4 **80**	**80**	3	3	3	4	**4**
A3	9	3	7	10	6	**70**	3	3	1	1	1
A4	11	4 40	8	3 **50**	9	**90** **40** **0**	1	4	1	1	1
Demand	**60**	**40**	**100** **60**	**50**	**90** **10**						
Column Penality	3	1	1	**5**	2						
Column Penality	3	1	1	**-**	2						
Column Penality	3	-	1	**-**	2						
Column Penality	-	-	1	**-**	2						

In this case, we got 4 as the maximum penalty and 4 as the least cost of the corresponding row.Soallocate min(supply, demand) = min(80, 90) = 80 for that cell. And crossed out Second row.

Step 7: If only one row or column with positive supply or demand remains uncrossed out, determine the basic variables in the row or column by the Least-Cost Method.

Source	Destination					Supply	R.P	R.P	R.P	R.P	R.P	R.P
	D1	D2	D3	D4	D5							
A1	4 **60**	4	9 **40**	8	13	**100** **40**	0	0	**5**	**4**	**-**	**-**
A2	7	9	8	10	4 **80**	**80**	3	3	3	4	**4**	**-**
A3	9	3	7 **60**	10	6 **10**	**70** **60**	3	3	1	1	1	1
A4	11	4 **40**	8 **0**	3 **50**	9	**90** **40** **0**	1	4	1	1	1	1
Demand	**60**	**40**	**100** **60**	**50**	**90** **10**							
Column Penality	3	1	1	**5**	2							
Column Penality	3	1	1	**-**	2							
Column Penality	3	-	1	**-**	2							
Column Penality	-	-	1	**-**	2							
Column Penality	-	-	1	**-**	**3**							

Total Transportation Cost = 4(60) +9(40) +4(80) +7(60) +6(10) + 4(40) +8(0) +3(50)

= 240 + 360 + 320 + 420 + 60 + 160 + 0 + 150

= 1710

III. Procedure for Advanced Vogel's Approximation Method (AVAM)

In AVAM, when smallest cost appear in two or more times in a row or column then penalty determined by difference of two minimum cost taken one of them as a minimum and following smallest cost other than equal smallest costs as a next to minimum. For example, if**6, 5, 5, 10, 9** are the costs of a row or column then select **5** as a smallest cost and select **6** as a next to smallest cost instead of **5** again and penalty will be 1. In that case penalty is not zero and if this penalty has the largest magnitude then probability of the chance of taking larger cost in next iteration will be decreased because of at least one more smallest cost remains.

Procedure for **AVAM** is as follows:

Step-1: Check whether the given transportation problem is balanced or not. If then problem is not balanced i.e. if supply and demand are not equal then add a dummy row or a dummy column depending on the necessity, to make it a balanced problem Then the problem can changed as balanced transportation problem.

Step-2:

i). Identify the smallest and next to smallest cost of each row and column and calculate the difference between them which is called by penalty.

ii). If smallest cost appear two or more times in a row or column then select one of them as a smallest and following smaller cost other than equal smallest costs as a next to smallest cost.

iii). If there is no more cost other than equal smallest costs ie..all costs are same then select smallest and next to smallest as same and penalty will be zero

Step-3:

i). Select lowest cost of that row or column which has largest penalty and allocate maximum possible amount . If the lowest cost appear in two or more cells in that row or column then choose the extreme left or most top lowest cost cell.

ii). If two or more penalty costs have same largest magnitude, then select any one of them (or select most top row or extreme left column).

Step-4: Adjust the supply and demand and cross out the satisfied row or column.

If row and column are satisfied simultaneously then crossed out one of them and set zero

Supply or Demand in remaining row or column.

Step-5:

i). If exactly one row or one column with zero supply or demand remains uncrossed out, Stop.

ii). If only one row or column with positive supply or demand remains uncrossed out, determine the basic variables in the row or column by the Least-Cost Method.

iii). If all uncrossed out rows or column have (remaining) zero supply or demand, determined the zero basic variables by the Least-Cost Method. Stop.

iv). Otherwise go to Step 2

ADVANCED VOGEL'S APPROXIMATION METHOD:

Example-5 : Solve the following transportation problem using Advanced Vogel's approximation method , Vogel's Approximation Method and MODI METHOD.

Source	**Destination**					**Supply**
	D1	D2	D3	D4	D5	
A1	4	4	9	8	13	**100**
A2	7	9	8	10	4	**80**
A3	9	3	7	10	6	**70**
A4	11	4	8	3	9	**90**
Demand	**60**	**40**	**100**	**50**	**90**	

Solution:

Step:1: : For the given cost matrix,

Total supply = 100 + 80 + 70+ 90 = 340

Total demand = 60 + 40 + 100 + 50 + 90 = 340

Thus, the given problem is balanced transportation problem

Now, we can apply the Advanced Vogel's approximation method to minimize the total cost of transportation.

Identify the smallest and next to smallest cost of each row and column and calculate

the row penality. and column penality.

For first row here smallest cost 4 appears two times in a row so select 4 as a smallest cost and 8 as a next to smallest cost. Take absolute difference between these two = 8 – 4 = 4 as first row penality.

For second row smallest is 4 and next smallest is 7. Second Rowpenality is 7- 4 = 3.

Simillarly for third row penalty is 6 – 3 = 3 and fourth row penality is 4 – 3 = 1 .

Simillarly the same process also applicable to column penalties.

Select lowest cost of the row or column which has largest penalty and allocate maximum possible amount .

If two or more penalty costs have same largest magnitude, then select any one of them (or select most top row or extreme left column).

Adjust the supply and demand and cross out the satisfied row or column. If row and column are satisfied simultaneously then crossed out one of them and set zero supply or demand in remaining row or column.

If exactly one row or one column with zero supply or demand remains uncrossed out.

If only one row or column with positive supply or demand remains uncrossed out, determine the basic variables in the row or column by the Least-Cost Method.

Step:1: : For the given cost matrix,

Total supply = 100 + 80 + 70+ 90 = 340

Total demand = 60 + 40 + 100 + 50 + 90 = 340

Thus, the given problem is balanced transportation problem

Now, we can apply the Advanced Vogel's approximation method to minimize the total cost of transportation.

Step-2: Here 4 Column has thelargest penalty **5** among all row and column penalties. So allocate maximum possible amount **50** at lowest cost **3.**

Source	**Destination**					**Supply**	**Row Penality**
	D1	D2	D3	D4	D5		
A1	4	4	9	8	13	**100**	4
A2	7	9	8	10	4	**80**	3
A3	9	3	7	10	6	**70**	3
A4	11	4	8	3 **50**	9	**90** **40**	1
Demand	**60**	**40**	**100**	**50**	**90**		
Column Penality	3	1	1	**5**	2		

Step-3: In this first row has maximum penalty **5**. . Here least cost **4** appears in two cells. In case of tie select the cell which has maximum allocation.

Allocate min (100,60) = 60 at least cost cell which has maximum allocation.

Source	**Destination**					**Supply**	**Row Penalty**	**Row Penalty**
	D1	D2	D3	D4	D 5			
A1	4 **60**	4	9	8	13	**100** **40**	4	**5**
A2	7	9	8	10	4	**80**	3	3
A3	9	3	7	10	6	**70**	3	3
A4	11	4	8	3 **50**	9	**90** **40**	1	1
Demand	**60**	**40**	**100**	**50**	**90**			
Column Penalty	3	1	1	**5**	2			
Column Penality	3	1	1	**-**	2			

Step-4: Here first row has maximum penalty **4**.

Allocate min (40,40) = 40 at least cost cell. If row and column are satisfied simultaneously then crossed out one of them and set zero supply or demand in remaining row or column.

Here crossed out Second Column and set zero supply in first row.

Source	**Destination**					**Supply**	**Row Penality**	**Row Penality**	**Row Penality**
	D1	D2	D3	D4	D5				
A1	4 **60**	4 **40**	9	8	13	**100** **40** **0**	4	**5**	**4**
A2	7	9	8	10	4	**80**	3	3	3
A3	9	3	7	10	6	**70**	3	3	3
A4	11	4	8	3 **50**	9	**90** **40**	1	1	1
Demand	**60**	**40**	**100**	**50**	**90**				
Column Penality	3	1	1	**5**	2				
Column Penality	3	1	1	-	2				
Column Penality	-	-	1	-	2				

Step 5: Here maximum penalty is **3.** Allocate min (80,90) = 40 at least cost cell value 4 and crossed out second row.

Repeat this process until we have only one row or column with positive supply or demand .

In such a case apply Least Cost Method.

Source	Destination					Supply	R.P	R.P	R.P	R.P
	D1	D2	D3	D4	D5					
A1	4 60	4 40	9	8	13	100 40 0	4	5	4	1
A2	7	9	8	10	4 80	80	3	3	3	3
A3	9	3	7	10	6	70	3	3	3	3
A4 1. N 2.	11	4	8	3 50	9	90 40	1	1	1	1
Demand	60	40	100	50	90 10					
C.P	3	1	1	5	2					
C.P	3	1	1	-	2					
C.P	-	-	1	-	2					
C.P	-	-	1	-	2					

Source	Destination					Suppl y	R.P	R.P	R.P	R.P	R.P	R.P
	D1	D2	D3	D4	D5							
A1	4 60	4 40	9	8	13	100 40 0	4	5	4	1	1	1
A2	7	9	8	10	4 80	80	3	3	3	3	-	-
A3	9	3	7 60	10	6 10	70 60	3	3	3	3	3	4
A4	11	4	8	3 50	9	90 40	1	1	1	1	1	1
Demand	60	40	100	50	90 10							
C.P	3	1	1	5	2							
C.P	3	1	1	-	2							

C.P	-	-	1	-	2		
C.P	-	-	1	-	2		
C.P	-	-	1	-	3		
C.P	-	-	1	-	-		

Source	**Destination**					**Supply**	**R.P**	**R.P**	**R.P**	**R. P**	**R.P**	**R. P**	**R.P**
	D1	D2	D3	D4	D5								
A1	4 **60**	4 **40**	9	8	13	**100** **40** **0**	4	**5**	**4**	1	1	1	1
A2	7	9	8	10	4 **80**	**80**	3	3	3	**3**	-	-	-
A3	9	3	7 **60**	10	6 **10**	**70** **60**	3	3	3	3	3	**4**	-
A4	11	4	8 **40**	3 **50**	9	**90** **40**	1	1	1	1	1	1	**1**
Demand	**60**	**40**	**100** **40**	**50**	**90** **10**								
C.P	3	1	1	**5**	2								
C.P	3	1	1	-	2								
C.P	-	-	1	-	2								
C.P	-	-	1	-	2								
C.P	-	-	1	-	3								
C.P	-	-	1	-	-								
C.P	-	-	1	-	-								

TOTAL TRANSPORTATION **COST** = 4(60)+4(40)+4(80)+7(60)+6(10)+8(40)+3(50)=1670

IV. Least Cost Method:

The Least Cost Method, also known as the Minimum Cost Method or Matrix Minimum Method, is an algorithm used in operations research to find an initial basic feasible solution for a transportation problem in linear programming. The goal is to minimize the total transportation cost by allocating resources starting with the cells that have the lowest transportation costs.

4.1. Identify the Cell with the Minimum Cost:

Find the cell with lowest in the transportation table.

4.2. Allocate Resources:

Allocate the maximum possible quantity to this cell, considering the supply and demand constraints of the corresponding row and column.

4.3. Eliminate Row or Column:

If either the supply or demand for the allocated cell is fully utilized, eliminate that row or column from further consideration.

4.4. Repeat:

Repeat steps 1-3 for the reduced transportation table until all supply and demand is satisfied.

Source	**Destination**					**Supply**
	D1	D2	D3	D4	D5	
A1	4	4	9	8 /	13	**100**
A2	7	9	8	10 /	4	**80**
A3	9	3	7	10 /	6	**70**
A4	11	4	8	3 / **50**	9	**90** **40**
Demand	**60**	**40**	**100**	**50** **0**	**90**	

Source	Destination				Supply
	D1	D2	D3	D5	
A1	4	4 /	9	13	**100**

A2	7	9 /	8	4	**80**
A3	9	3 / **40**	7	6	**70** **30**
A4	11	4 /	8	9	**90** **40**
Demand	**60**	**40** **0**	**100**	**90**	

Source	Destination			Supply
	D1	D3	D5	
A1	4	9	13	**100**
A2	7 /	8 /	4 / **80**	**80** **0**
A3	9	7	6	**70** **30**
A4	11	8	9	**90** **40**
Demand	**60**	**100**	**90** **10**	

Source	Destination			Supply
	D1	D3	D5	
A1	4 / **60**	9	13	**100** **40**
A3	9 /	7	6	**70** **30**
A4	11 /	8	9	**90** **40**

Demand	**60** **0**	**100**	**90** **10**

Source	Destination		Supply
	D3	D5	
A1	9	13 /	**100** **40**
A3	7	6 / **10**	**70** **30** **20**
A4	8	9 /	**90** **40**
Demand	**100**	**90** **10** **0**	

Source	Destination	Supply
	D3	
A1	9 40	**100** **40** **0**
A3	7 20	**70** **30** **20** **0**
A4	8 40	**90** **40**
Demand	**100** **80** **40** **0**	

Total Transportation Cost = 3(50) + 3(40) + 4(80) + 4(60) + 6(!0) + 7(20) + 8(40) + 9(40)

= 150 + 120 + 320 + 240 + 60 + 140 + 320 +360

= 1710

Result Analysis: We observed in the above example that Advanced Vogel's Approximation Method (AVAM) gives the lower feasible solution other than **Vogel's Approximation Method (VAM) and Least Cost Method.**

Conclusion : In this paper we fixed the computational error of Vogel"s Approximation Method (VAM) and proposed a new algorithm named "Advanced Vogel"s Approximation Method (AVAM)". From the above example it is shown that AVAM gives the lower feasible solution than VAM .and **Least Cost Method**.

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