A Discrete Time Inventory System with Finite Backlog

Anilkumar M.P.[#1], K. P. Jose [*2]

[#] Department of Mathematics, T.M. Govt. College, Tirur-676502, Kerala, India

[*] PG & Research Department of Mathematics, St. Peter's College, Kolenchery-682311, Kerala, India

[1] anilkumarmp77@gmail.com

[2] kpjspc@gmail.com

Abstract **-**In this paper, we consider a discrete-time (s, S) inventory system with positive lead time. We assume that the arrival of customers constitutes a Bernoulli process and the lead time follows a geometric distribution. The maximum storage of inventory in the system is S. Whenever on-hand inventory level drops to pre-fixed level s due to demands, an order for replenishment is placed. A maximum of k demands (backlog) are allowed when the inventory level is zero. Various system chacteristics are obtained and a suitable cost function for minimum expected cost is also constructed. Matrix Analytic Method is used to analyze the system.

Keywords - Discrete-Time, Inventory, Matrix Analytic Method, Finite–Backlog, Numerical Experiments.

I. INTRODUCTION

The first noted work on modelling of inventory problems was done by Harris [4] in which Harris-Wilson economic lot size formula was derived. Later, Karlin and Scarf [6] made a systematic study of (s, S) inventory system. An algorithmic analysis for finding optimal (s, S) policy was done by Veinott and Wagner [11]. The inventory system with state dependent lead time and the dependence between replenishment rate and outstanding orders was considered by Gross and Harris [3]. Manoharan et al. [10] investigated (s, S) inventory system with unit demand and non-identically distributed inter-demand times having arbitrary distributed replenishment time. A review of the advances of deteriorating inventory literature since the early 1990s was done in Hui-Ming and Yu [5]. The models available in the relevant literature have been suitably classified by the shelf-life characteristic of the inventoried goods. They have further been sub-classified on the basis of demand variations and various other conditions or constraints. Lian and Liu [9] studied a discrete-time (s, S) perishable inventory model with geometric inter-demand times and batch demands with a zero lead time and allowing backlogs. They constructed a multi-dimensional Markov chain to model the inventory-level process and obtained a closed- form cost function. Numerical results that revealed in the work have some good properties of the cost function.

Lian et al. [8] discussed a discrete-time (s, S) inventory model with stored items have a random common lifetime with a discrete phase-type distribution. The arrival in batches follows a discrete phase-type renewal process, allowing backlogs. They obtained a closed-form expected cost function, by assuming lead-time zero. Numerical results demonstrate some properties of optimal ordering policies and cost functions. Abboud [1] introduced a production-inventory system as a discrete Markov chain in which machine produces an item at a constant rate. This constant rate is assumed to be greater than the demand rate, and the demand is assumed to be known and constant. While operating, the machine can fail, and upon failure it requires service. The machine times-to-failure and repair times are random, and during repairs, demand is back ordered as long as the back ordering level does not exceed a prescribed amount, after which demand is lost. Krishnamoorthy and Jose [7] analyzed and compared three production inventory systems with positive service time and retrial of customers in continuous environment. They obtained steady state analysis of the system. Deepthi [2] analysed Discrete-time inventory models with/without positive service time in her Ph.D. thesis. The author described the relevant models and obtained steady-state solution to the models. This paper is an extension of one of the models described in [2].

The rest of the paper is organized as follows. Section 2 provides mathematical modelling and transition analysis. Stability condition and steady state probability vector are derived in section 3. Relevant performance measures are included in section 4. Finally, Section 5 provides numerical experiments.

II. MATHEMATICAL MODELLING AND ANALYSIS

The following are the assumptions and notations used in this model.

Assumptions

i. Inter-arrival times of customers are geometrically distributed with paramete p.

ii. Service times are negligible.
iii. The maximum inventory level is S.
iv. The reorder level is s.
v. Up to k customer is allowed to enter in the system, when the inventory level is zero.
vi. Lead time is geometrically distributed with parameter q.
vii. Replenishment takes place at the end of slot boundaries.

Let $I(m)$ be inventory level at an epoch m.Then $\{I(n); n = 0,1,2,3,..\}$ is a finite state discrete time Markov chain with state space $\{i; -k \le i \le S\}$. The transition probability matrix of the process is

$$P = \begin{array}{c c} & \begin{array}{ccccccc} -k & -k+1 & \cdots & s & s+1 & \cdots & S \end{array} \\ \begin{array}{c} -k \\ -k+1 \\ \vdots \\ s \\ s+1 \\ \vdots \\ S \end{array} & \begin{bmatrix} q & & & & & \cdots & q \\ p\bar{q} & \bar{p}\bar{q} & & & & \cdots & q \\ & \ddots & \ddots & & & \vdots & \vdots \\ & & p\bar{q} & \bar{p}\bar{q} & & \cdots & q \\ & & & p & \bar{p} & & \\ & & & & \ddots & \ddots & \\ & & & & & p & \bar{p} \end{bmatrix} \end{array}$$

III. Stability and Steady-State Analysis

Since the above Markov chain is irreducible and state space is finite, the system is obviously stable

Let $\mathbf{x} = (x_{-k}, x_{-k+1}, \dots, x_0, \dots, x_s, \dots x_{S-1}, x_S)$ be the steady state probability vector of P. $x_i's$ are given by the matrix equation $\mathbf{x}P = \mathbf{x}$, $\mathbf{xe} = 1$. On substitution, we obtain

$$x_i = \left(\frac{1-\bar{p}\bar{q}}{p\bar{q}}\right)^{i+k-1} x_{-k+1}, i = -k+2, \dots s,$$

$$x_{s+1} = \left(\frac{1-\bar{p}\bar{q}}{p\bar{q}}\right)^{i+k-1} x_s,$$

$$x_{s+1} = x_{s+2} = \cdots = x_S$$

On simplification

$$x_{-k} = \frac{p[p(1-q)]^{s+k}}{[p+(S-s)q][p+q-pq]^{s+k}},$$

$$x_i = \frac{pq[p(1-q)]^{s-i}}{[p+(S-s)q][p+q-pq]^{s-i+1}}, -k+1 \le i \le s$$

$$= \frac{q}{p+(S-s)q}, s+1 \le i \le S$$

IV. System Characteristics

We find some relevant system characteristics

i. Expected inventory level, EIL, is given by

$$EIL = \sum_{j=1}^{S} jx_j$$

ii. The Expected reorder rate, ERO, is given by

$$ERO = px_{s+1}$$

iii. Expected replenishment rate

$$ERR = q\sum_{j=0}^{s} y_j$$

iv. Expected replenishment quantity, ERQ, is given by

$$ERQ = \sum_{i=-k}^{s} q(S-i)x_i$$

v. Expected loss rate of customers

$$EL = px_{-k}$$

vi. Expected number of departure after completing the service, EDS, is given by

$$EDS = p\sum_{j=1}^{S} x_j$$

vii. Expected backlog, EB, is given by

$$EB = \sum_{i=-k}^{-1} (-i)x_i$$

V. NUMERICAL EXPERIMENTS

Define the expected total cost (ETC) of the system per unit time is given by

$$ETC = c_0ERO + c_1ERQ + c_2EIL + c_3EB + c_4EL + (c_5 - c_6)EDS$$

where,

c_0: setup cost/order
c_1: procurement cost/unit/unit time
c_2: holding cost/unit/unittime
c_3: backorder cost of demand/unit/unit time
c_4: cost due tol oss/unit/unittime
c_5: cost due to service/unit/unittime
c_6: revenue due to service/unit/unit time

VI. GRAPHICAL ILLUSTRATIONS

We draw the graphs of ETC by varying the parameters p and q and keeping other parameters fixed. From figures 1 and 2, one can observe that the minimum values of ETC are 1.36 and 2.66 at $p = 0.625$ and $q = 0.26$ respectively.

$S = 10; s = 3; k = 4; q = .3; C_0 = C_1 = C_2 = C_3 = C_4 = C_5 = 1; C_6 = 0.1$

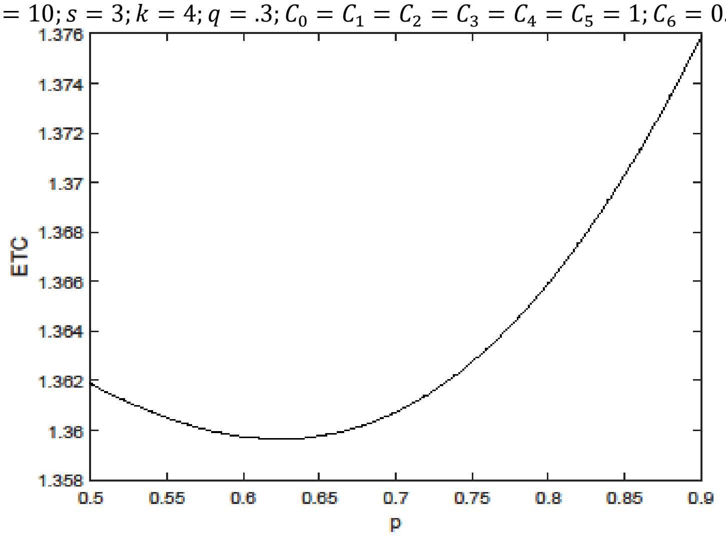

Fig.1: *p vs ETC*

$S = 10; s = 3; k = 4; p = .62; C_0 = C_1 = C_2 = C_3 = C_4 = 1; C_5 = 1.3; C_6 = 0.1$

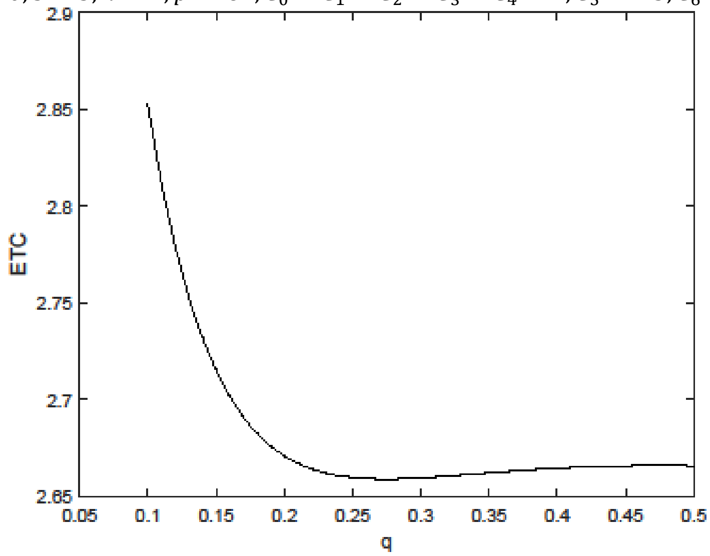

Fig.2: *q vs ETC*

CONCLUDING REMARKS

We discussed a discrete-time inventory model with finite backlog. In this model, inter-arrival time and lead-time are geometrically distributed. A maximum of k backlogs are allowed in the system. We obtained steady state probability vector. Relevant performance measures are calculated. Numerical Experiments are discussed to highlight the important results of the model. One can generalize this model by considering discrete MAP arrivals or discrete Phase type service time or both.

ACKNOWLEDGEMENT

Anilkumar M. P. acknowledges the financial support of University Grant Commission (UGC) of India under Faculty Development Programme (FDP) F .No. FIP/12th Plan/KL CA 064 TF 04, dated: 27.03.2017.

REFERENCES

[1] N. Abboud. A Discrete-Time Markov Production-Inventory model with machine breakdowns. *Computers & Industrial Engineering*, 39(1-2):95–107, 2001.

[2] C. Deepthi and A. Krishnamoorthy. *Discrete Time Inventory Models with/without Positive Service Time*. PhD thesis, Cochin University of Science and Technology, 2013.

[3] D. Gross and C. M. Harris. *Continuous-review (s, S) inventory models with state-dependent lead times*. *Management Science*, 19(5): 567–574, 1973.

[4] F. Harris. Operations and costs (factory management series), 18-52. *AW Shaw Co, Chicago*, 1915.

[5] W. Hui-Ming and J. Yu. *A deteriorating inventory model with a temporary price discount. International Journal of Production Economics*, 53(1):81–90, 1997.

[6] S. Karlin and H. Scarf. Inventory models and related stochastic processes. *Studies in the mathematical theory of inventory and production*, 1:319, 1958.

[7] A. Krishnamoorthy and K.P. Jose. Three production inventory systems with service, loss and retrial of customers. *Int. J. Inf. Manag. Sci*, 19(3):367–389, 2008.

[8] Z. Lian, L. Liu, and M. F. Neuts. A discrete-time model for common life time inventory systems. *Mathematics of Operations Research*, 30(3):718–732, 2005.

[9] Z. Lian and L. Liu. A discrete-time model for perishable inventory systems. *Annals of Operations Research*, 87:103–116, 1999.

[10] M. Manoharan, A. Krishnamoorthy, and T. Madhusoodanan. On the (s, S) inventory policy with independent non-identically distributed inter-arrival demand times and random lead time. *Cahiers du Centre d'études de recherché opérationnelle*, 29(3-4):239–248, 1987.

[11] A.F. Veinott Jr. and H. M. Wagner. Computing optimal (s, S) inventory policies. *Management Science*, 11(5):525–552, 1965.