Cycle tracking polynomial of a graph

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Abstract

We introduce a cycle tracking polynomial of a graph G. The cycle tracking polynomial of a graph G of order n is the polynomial $T(G, x)$ of G is defined as

$$T(G; x) = \sum_{i=\tau(G)}^{n} t(G, i)x^i$$

where $t(G, i)$ is the number of cycle tracking sets of G of size i, and $\tau(G)$ is the cycle tracking number of G. We obtain some properties of $T(G, x)$ and its coefficients. Also we compute this polynomial for some specific graphs.

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1 Introduction

The concept of cycle tracking set is introduced in [4] Let $G(V, E)$ be a graph. For $v \in V(G)$, the *cycle trace* (*simply trace*) of v is defined as the set of all vertices $u \in V$ such that u and v belong to same cycle of G and is denoted by $T_G(v)$. For $u, v \in V$, v is said to be *cycle traced* (*traced*) *by* u if $v \in T_G(u)$. A set S of vertices in a graph $G(V, E)$ is called a *cycle tracking set* if for every vertex $v \in V \setminus S$, there exists a vertex $u \in S$ such that $v \in T_G(u)$. A cycle tracking set is a *minimal cycle tracking set* if no proper subset $S' \subset S$ is a cycle tracking set. The *cycle tracking number*

$\tau(G)$ of a graph G is the minimum cardinality of a minimal cycle tracking set of G. The *upper cycle tracking number* $T(G)$ of a graph G is the maximum cardinality of minimal cycle tracking set of G. A cycle tracking set with minimum cardinality is called a $\tau - set$ of G.
A graph G(V,E) is said to be *transitive tracking* graph if for every $u, v, w \in V(G)$, $w \in T_G(u)$ and $u \in T_G(v)$ implies $w \in T_G(v)$ A graph G(V,E) is said to be *track connected* if for every pair of vertices $u, v \in V(G)$ there exist two internally disjoint paths connecting u and v. A vertex is said to be *trace free vertex* if it belongs to no cycle.
Through out this paper the letter G denotes a graph of order n.

2 Cycle tracking polynomial of a graph

Definition 2.1. Let G be a graph of order n. Let $T(G, i)$ be the family of all cycle tracking sets of a graph G with cardinality i and let $t(G, i) = |T(G, i)|$. Then the cycle tracking polynomial $T(G, x)$ of G is defined as

$$T(G; x) = \sum_{i=\tau(G)}^{n} t(G, i)x^i$$

where $\tau(G)$ is the cycle tracking number of G.

The path P_3 on three vertices has only one cycle tracking set with cardinality 3 ($\tau(G) = 3$) its tracking polynomial is then $T(P_3, x) = x^3$.In the case of the cycle C_n on $n(n \geq 0)$ vertices,
$T(C_n, x) = \binom{n}{1}x^1 + \binom{n}{2}x^2 + \binom{n}{3}x^3 + ... + \binom{n}{n}x^n = (1 + x)^n - 1.$

Theorem 2.1. *If a graph G consist of m components $G_1, G_2, ..., G_m$ then $T(G, x) = T(G_1, x)T(G_2, x)...T(G_m, x)$.*

Proof. It is enough to prove the theorem for n=2.
For $k \geq \tau(G)$, a cycle tracking set of k vertices in G arises by choosing a cycle tracking set of j vertices in G_1 for some j such that $\tau(G) \leq j \leq |V(G)|$ and a cycle tracking set of $k - j$ vertices in G_2. The number of ways of doing this over all $j = \tau(G_1), ..., |V(G_1)|$ is exactly the coefficient of x^k in $T(G_1, x)T(G_2, x)$. So $T(G, x) = T(G_1, x)T(G_2, x)$. □

Theorem 2.2. *Let G be a graph of order n. Then*

1. $t(G, n) = 1$
2. $t(G, i) = 0$ *if and only if* $i < \tau(G)$ *or* $i > n$
3. $T(G, x)$ *has no constant term.*
4. $T(G, x)$ *is a strictly increasing function on* $(0, \infty)$

5. *for any subgraph H of G,* $deg(T(G,x) \geq deg(T(H,x))$
6. *zero is a root of* $T(G,x)$ *with multiplicity* $\tau(G)$
7. $\tau(G) = n$ *if and only if* $T(G,x) = x^n$

Theorem 2.3. *[4] Let G be a graph of order n. Then* $\tau(G) = n$ *if and only if G(V,E) is a forest.*

Theorem 2.4. *Let G be a graph of order n. Then* $T(G,x) = x^n$ *if and only if G is a forest.*

Proof. $T(G,x) = x^n$ if and only if $V(G)$ is the only cycle tracking set for G. That is if and only if $\tau(G) = n$. That is if and only if G is a forest (by Theorem 2.3)

□

Theorem 2.5. *Let G be a graph of order n. Then* $T(G,x) = (1+x)^n - 1$ *if and only if G is track connected.*

Proof. If G is track connected then $\tau(G) = 1$ and every vertex traces all vertices of G. So coefficient of x is n and $t(G,p) = \binom{n}{p}$. So $T(C_n,x) = \binom{n}{1}x^1 + \binom{n}{2}x^2 + \binom{n}{3}x^3 + ... + \binom{n}{n}x^n = (1+x)^n - 1$.
Conversely if $T(G,x) = (1+x)^n - 1$, the coefficient of x is n. That is, every vertex traces all vertices of G. Hence G is track connected. □

Definition 2.2. [4]
Let G be a graph with exactly one cut vertex. Let v be the cut vertex of G and $G_1, G_2, ..., G_k$ be the components of $G\setminus\{v\}$. If the order of G_i is greater than or equal to three and the graphs induced by $V(G_i) \cup \{v\}, i = 1, 2, ..., k$ are track connected then G is called a *track connected floral graph*. And the graph induced by $V(G_i) \cup \{v\}, i = 1, 2, ..., k$ are called petals of G.

Theorem 2.6. *Let G be a track connected floral graph with k petals with each petal having* $m_1, m_2, ..., m_k$ *vertices respectively then*
$t(G,p) = \binom{m_1+m_2+...+m_k}{p-1}$ *when* $1 \leq p \leq k-1$ *and*

$$t(G,p) = \binom{m_1+m_2+...+m_k}{p-1} + \sum_{i_1=1}^{p-k+1}\binom{m_1}{i_1}\left[\sum_{i_2=1}^{p-k-i_1+2}\binom{m_2}{i_2}\left[...\sum_{i_j=1}^{p-k-i_1-i_2-...+j}\binom{m_j}{i_j}\left[...\left[\sum_{i_{k-1}=1}^{p-k-i_1-i_2-...-i_{k-1}+k-1}\binom{m_{k-1}}{i_{k-1}}\binom{m_k}{p-i_1-i_2-...-i_{k-1}}\right]...\right]\right]\right]$$

when $k \leq p \leq n$ *and*

$$T(G,x) = \sum_{p=1}^{k-1}\binom{m_1+m_2+...+m_k}{p-1}x^p + \sum_{p=k}^{n}\left[\binom{m_1+m_2+...+m_k}{p-1}+\right.$$

$$\sum_{i_1=1}^{p-k+1}\binom{m_1}{i_1}\left[\sum_{i_2=1}^{p-k-i_1+2}\binom{m_2}{i_2}\left[\ldots\sum_{i_j=1}^{p-k-i_1-i_2-\ldots+j}\binom{m_j}{i_j}\right.\right.$$
$$\left.\left.\left[\ldots\left[\sum_{i_{k-1}=1}^{p-k-i_1-i_2-\ldots-i_{k-1}+k-1}\binom{m_{k-1}}{i_{k-1}}\binom{m_k}{p-i_1-i_2-\ldots-i_{k-1}}\right]\ldots\right]\ldots\right]\right]x^p.$$

Proof. Case(1) $1 \le p \le k-1$
Then any cycle tracking set S contains the central vertex. So the central vertex together with p-1 vertices constitute a cycle tracking set S and it can be chosen in $\binom{m_1+m_2+\ldots+m_k}{p-1}$ ways.
Case(2) $k \le p \le n$
Here the central vertex together with p-1 vertices constitute a cycle tracking set S and a set of vertices S of cardinality p having at least one element from each leaf is also form a cycle tracking set and it can be chosen in

$$\binom{m_1+m_2+\ldots+m_k}{p-1}+\sum_{i_1=1}^{p-k+1}\binom{m_1}{i_1}\left[\sum_{i_2=1}^{p-k-i_1+2}\binom{m_2}{i_2}\left[\ldots\sum_{i_j=1}^{p-k-i_1-i_2-\ldots+j}\binom{m_j}{i_j}\right.\right.$$
$$\left.\left.\left[\ldots\left[\sum_{i_{k-1}=1}^{p-k-i_1-i_2-\ldots-i_{k-1}+k-1}\binom{m_{k-1}}{i_{k-1}}\binom{m_k}{p-i_1-i_2-\ldots-i_{k-1}}\right]\right]\right]\ldots\right] \text{ ways.}$$

□

Let G be a transitively tracked graph then its vertex set can be partitioned into $V_1, V_2, \ldots, V_k$ of cardinality $m_1, m_2, \ldots, m_k$ respectively such that the graph $\langle V_i \rangle$ induced by each V_i is maximal track connected subgraph of G. Then a set S of vertices which contains at least one element from each V_i form a cycle tracking set. So a cycle tracking set of G with cardinality p can be chosen in $\sum_{i_1=1}^{p-k+1}\binom{m_1}{i_1}\left[\sum_{i_2=1}^{p-k-i_1+2}\binom{m_2}{i_2}\left[\ldots\sum_{i_j=1}^{p-k-i_1-i_2-\ldots+j}\binom{m_j}{i_j}\right.\right.$
$\left.\left.\left[\ldots\left[\sum_{i_{k-1}=1}^{p-k-i_1-i_2-\ldots-i_{k-1}+k-1}\binom{m_{k-1}}{i_{k-1}}\binom{m_k}{p-i_1-i_2-\ldots-i_{k-1}}\right]\right]\right]\ldots\right]$ ways.

So $t(G,p)=\sum_{i_1=1}^{p-k+1}\binom{m_1}{i_1}\left[\sum_{i_2=1}^{p-k-i_1+2}\binom{m_2}{i_2}\left[\ldots\sum_{i_j=1}^{p-k-i_1-i_2-\ldots+j}\binom{m_j}{i_j}\right.\right.$
$$\left.\left.\left[\ldots\left[\sum_{i_{k-1}=1}^{p-k-i_1-i_2-\ldots-i_{k-1}+k-1}\binom{m_{k-1}}{i_{k-1}}\binom{m_k}{p-i_1-i_2-\ldots-i_{k-1}}\right]\right]\right]\ldots\right],$$
$k \le p \le n$.
The above discussion may be summarized as follows.

Theorem 2.7. *Let G is transitively traced and let $V(G)$ is partitioned into $V_1, V_2, \ldots, V_k$ of cardinality $m_1, m_2, \ldots, m_k$ respectively such that the graph $\langle V_i \rangle$ induced by each V_i is maximal track connected subgraph of G. Then*

$$T(G,x)=\sum_{p=k}^{n}\left[\sum_{i_1=1}^{p-k+1}\binom{m_1}{i_1}\left[\sum_{i_2=1}^{p-k-i_1+2}\binom{m_2}{i_2}\left[\cdots\sum_{i_j=1}^{p-k-i_1-i_2-\ldots+j}\binom{m_j}{i_j}\right.\right.\right.$$
$$\left.\left.\left.\left[\cdots\left[\sum_{i_{k-1}=1}^{p-k-i_1-i_2-\ldots-i_{k-1}+k-1}\binom{m_{k-1}}{i_{k-1}}\binom{m_k}{p-i_1-i_2-\ldots-i_{k-1}}\right]\right]\right]\cdots\right]\right]x^p.$$

Theorem 2.8. *[4] Let G' be the graph formed by removing all cut edges of a graph G. Then a subset S of $v(G)$ is a cycle tracking set of G if and only if S is a cycle tracking set of G'.*

Theorem 2.9. *[4] Let G be a transitively tracked graph. Then the components of G obtained by deleting all cut edges of G are precisely the maximal track connected subgraph of G.*

Theorem 2.10. *Let G be transitively tracked and let $V(G)$ be partitioned into $V_1, V_2, ..., V_k$ such that the graph $\langle V_i\rangle$ induced by each V_i is maximal track connected subgraph of G. Then $T(G,x) = T(\langle V_1\rangle, x)T(\langle V_2\rangle, x)...T(\langle V_m\rangle, x)$.*

Proof. Let G be transitively tracked and let $V(G)$ be partitioned into $V_1, V_2, ..., V_k$ such that the graph $\langle V_i\rangle$ induced by each V_i is maximal track connected subgraph of G. Let G' be the graph formed by removing all cut edges of G. Then by Theorem 2.8 $T(G,x) = T(G',x)$ and by Theorems 2.9 and 2.1 $T(G,x) = T(\langle V_1\rangle, x)T(\langle V_2\rangle, x)...T(\langle V_m\rangle, x)$ □

Corollary 2.11. *Let G be transitively tracked and let $V(G)$ be partitioned into $V_1, V_2, ..., V_k$ of cardinality $m_1, m_2, ..., m_k$ respectively such that the graph $\langle V_i\rangle$ induced by each V_i is maximal track connected subgraph of G. Then $T(G,x) = ((x+1)^{m_1} - 1)((x+1)^{m_2} - 1)...((x+1)^{m_k} - 1)$*

Theorem 2.12. *For a graph G, $t(G,1) = 1$ if and only if G is a track connected floral graph.*

Proof. Let G be any graph with $t(G,1) = 1$. Then there exists one and only one cycle tracking set S with $|S| = 1$. That is there exist a vertex $v \in V$ such that $T_G(v) = V$ and no other vertex can trace G. And since $\tau(G) = 1$ G must be a track connected floral graph [4]. □

Remark 2.1. For any graph G,

$$t(G,1)=\begin{cases}1 & if\ G\ is\ track\ connected\ floral\ graph\\ |V| & if\ G\ is\ track\ connected\\ 0 & otherwise.\end{cases}$$

Theorem 2.13. *Let G be a graph of order n with r trace free vertices. If $T(G;x) = \sum_{i=\tau(G)}^{n} t(G,i)x^i$ is its cycle tracking polynomial, then $r = n - t(G, n-1)$.*

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Proof. Suppose that $A \subset V(G)$ is the set of all trace free vertices. Then by hypothesis, $|A| = r$. For a vertex $v \in V(G)$, the set $V(G) \setminus \{v\}$ is a cycle tracking set of G if and only if $v \in V(G) \setminus A$. Therefore $t(G, n-1) = |V(G \setminus A)| = n - r$. Hence the theorem. □

Theorem 2.14. *Let $G(V, E)$ be a graph of order n. Then $t(G, 1) = |\{v \in V(G) : T_G(v) = V(G)\}|$.*

Proof. For every $v \in V(G)$, $\{v\}$ is a cycle tracking set if and only if v traces all vertices. ie; $T_G(v) = V(G)$. □

3 Cycle tracking polynomial for some graphs

In this section we find the cycle tracking polynomial for some graphs.

Definition 3.1. [?] A firefly graph $F_{s,t,n-2s-2t-1}$ ($s \geq 0, t \geq 0$ and n–$2s - st - 1 \geq 0$) is a graph of order n that consists of s triangles, t pendent paths of length 2 and $n - 2s$–$2t$–1 pendant edges sharing a common vertex.

Theorem 3.1. $\tau(F_{s,t,n-2s-2t-1}) = n - 2s$.

Proof. The graph $F_{s,t,n-2s-2t-1}$ has $n - 2s - 1$ trace free vertices and the common vertex traces all s triangles. So the $n - 2s - 1$ trace free vertices together with the common vertex form a τ-set. Hence $\tau(F_{s,t,n-2s-2t-1}) = n - 2s$. □

Theorem 3.2. $t(F_{s,t,n-2s-2t-1}, p) = \binom{2s}{p-n+2s}$ *if* $n-2s \leq p \leq n-s-2$ *and*

$$t(F_{s,t,n-2s-2t-1}, p) = \binom{2s}{p-n+2s} + \sum_{i_1=1}^{p-n+s+2} \binom{2}{i_1} \left[\sum_{i_2=1}^{p-n+s-i_1+3} \binom{2}{i_2} \left[\dots \sum_{i_j=1}^{p-n+s-i_1-i_2-\dots+j+1} \binom{2}{i_j} \left[\dots \left[\sum_{i_{s-1}=1}^{p-n+s-i_1-i_2-\dots-i_{s-1}+s} \binom{2}{i_{s-1}} \binom{2}{p-n+2s+1-i_1-i_2-\dots-i_{s-1}} \right] \dots \right] \dots \right] \right]$$

if $n - s - 1 \leq p \leq n$.

Proof. since $\tau(F_{s,t,n-2s-2t-1}, x) = n - 2s$. case(1)*if* $n - 2s \leq p \leq n - s - 2$ Then any cycle tracking set S contains the common vertex. So the central vertex together with $n - 2s - 1$ trace free vertices and $p - n + 2s$ other vertices constitute a cycle tracking set S and it can be chosen in $\binom{2s}{p-n+2s}$ ways. case(2)*if* $n - s - 1 \leq p \leq n$

Here the central vertex together with $n - 2s - 1$ trace free vertices and $p - n + 2s$ other constitute a cycle tracking set S and a set of vertices S of cardinality p having at least one element from each triangle is also form a cycle tracking set and it can be chosen in

$$\binom{2s}{p-n+2s} + \sum_{i_1=1}^{p-n+s+2} \binom{2}{i_1} \left[\sum_{i_2=1}^{p-n+s-i_1+3} \binom{2}{i_2} \left[\dots \sum_{i_j=1}^{p-n+s-i_1-i_2-\dots+j+1} \binom{2}{i_j} \right.\right.$$
$$\left.\left.\left[\dots \left[\sum_{i_{s-1}=1}^{p-n+s-i_1-i_2-\dots-i_{s-1}+s} \binom{2}{i_{s-1}} \binom{2}{p-n+2s+1-i_1-i_2-\dots-i_{s-1}} \right] \dots \right]\right]\right]$$
ways. □

Theorem 3.3. $T(F_{s,t,n-2s-2t-1}, x) = \sum_{p=n-2s}^{n-s-2} \binom{2s}{p-n+2s} + \sum_{p=n-s-2}^{n} \left[\binom{2s}{p-n+2s} \right.$
$$+ \sum_{i_1=1}^{p-n+s+2} \binom{2}{i_1} \left[\sum_{i_2=1}^{p-n+s-i_1+3} \binom{2}{i_2} \left[\dots \sum_{i_j=1}^{p-n+s-i_1-i_2-\dots+j+1} \binom{2}{i_j} \right.\right.$$
$$\left.\left.\left.\left[\dots \left[\sum_{i_{s-1}=1}^{p-n+s-i_1-i_2-\dots-i_{s-1}+s} \binom{2}{i_{s-1}} \binom{2}{p-n+2s+1-i_1-i_2-\dots-i_{s-1}} \right] \dots \right]\right]\right]\right]$$

Definition 3.2. A Lollipop graph $L_{n,m}$ is obtained by joining K_n to a path P_m of length m with a bridge.

Theorem 3.4. $\tau(L_{n,m}) = m + 1$

Proof. Since a vertex in K_n can trace all vertices in it and all vertices of P_m are trace free vertices we need at least $m + 1$ vertices to trace $L_{n,m}$. Hence $\tau(L_{n,m}) = m + 1$. □

Theorem 3.5. $T(L_{n,m}, x) = ((1 + x)^n - 1)x^m$

Definition 3.3. A Tadpole $T_{(n,l)}$ is a graph obtained by attaching a path P_l to one of the vertices of the cycle C_n by a bridge.

Theorem 3.6. $\tau(T_{(n,l)}) = l + 1$

Proof. Since a vertex in C_n can trace all vertices in it and all vertices of P_l are trace free vertices we need at least $m + 1$ vertices to trace $T_{(n,l)}$ and hence $\tau(T_{(n,l)}) = l + 1$ □

Theorem 3.7. $T(T(n, l)) = ((1 + x)^n - 1)x^l$

Definition 3.4. For a positive integer n, a helm graph, denoted by H_n is obtained from the Wheel W_n by joining a pendant vertex to each vertex in the outer circle of W_n by means of an edge.

Theorem 3.8. $\tau(H_n) = n$

Proof. Since H_n contains $n - 1$ pendant vertices, all these vertices belong to every cycle tracking set. Since W_n is track connected, a vertex of W_n together with the pendant vertices form a minimal cycle tracking set. So $\tau(H_n) = n$. □

Theorem 3.9. $T(H_n) = ((1+x)^n - 1)x^{n-1}$

Definition 3.5. For a positive integer $n > 3$, a web graph $WB_n, n > 3$ is obtained by joining the pendent vertices of a helm H_n to form a cycle and then adding a single pendent edge to each vertex of this outer cycle.
The web graph WB_n has $3n-2$ vertices and $3(n-1)$ edges.

Theorem 3.10. $\tau(WB_n) = n$

Theorem 3.11. $T(WB_n)) = ((1+x)^{2n-1} - 1)x^{n-1}$

Definition 3.6. A friendship graph F_n is the one point union of n copies of the cycle C_3.

Since F_n is a track connected floral graph with n petals each having 3 vertices we have:

Theorem 3.12. $\tau(F_n) = 1$

Using Theorem2.6:

Theorem 3.13. $t(F_n, i) = \begin{cases} \binom{2n}{i-1} & 1 \leq i \leq n-1 \\ \binom{2n}{i-1} + \binom{n}{i-n}2^{2n-i} & n \leq i \leq 2n \end{cases}$

and $T(F_n, x) = x + 2nx^2 + ... + \binom{2n}{i-1}x^i + ... + \binom{2n}{n-2}x^{n-1}$
$+\left[\binom{2n}{n-1} + 2^n\right]x^n + ... + \left[\binom{2n}{j-1} + \binom{n}{j-n}2^{2n-j}\right]x^j + ... + x^{2n+1}.$

Definition 3.7. An armed crown $C_n \bigodot P_m$ is a graph obtained by attaching path P_m to every vertex of the cycle C_n.

Theorem 3.14. $\tau(C_n \bigodot P_m) = mn + 1$

Proof. Since a vertex in C_n can trace all vertices in it and the remaining mn vertices are trace free vertices. So we need at least $mn+1$ vertices to trace $C_n \bigodot P_m$. Hence $\tau(C_n \bigodot P_m) = mn + 1$ □

Theorem 3.15. $T(C_n \odot P_m) = ((1+x)^n)x^{mn}$

The corona of two graphs G_1 and G_2 is the graph $G = G_1 \circ G_2$ formed from one copy of G_1 and $|V(G_1)|$ copies of G_2, where the i^th vertex of G_1 is adjacent to every vertex in the i^th copy of G_2 [?].

Theorem 3.16. *For any graph G of order n, $\tau(G \circ K_1) = n + \tau(G)$.*

Theorem 3.17. $T(G \circ K_1, x) = T(G, x)x^n$

Theorem 3.18. *Let G be a graph with n vertices and H be a connected graph with $m(m > 1)$ vertices. Then $\tau(G \circ H) = n$.*

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In particular if $G = K_1$, then $\tau(K_1 \circ H) = 1$

Corollary 3.19. *For a connected graph H of cardinality $m(m > 1)$, $T(K_1 \circ H, x) = (1 + x)^{m+1} - 1$.*

Proof. Since $K_1 \circ H$ is a track connected graph with $m + 1$ vertices, $T(K_1 \circ H, x) = (1 + x)^{m+1} - 1$ □

Theorem 3.20. *Let G be a graph with n vertices and H be a connected graph with $m(m > 1)$ vertices. Then $T(G \circ H, x) = [(1 + x)^{m+1} - 1]^n$.*

Proof. It is enough to prove for n=2
For $k \geq \tau(G \circ H) = 2$, a cycle tracking set of k vertices in $G \circ H$ is chosen by selecting j $(1 \leq j \leq k - 1)$ vertices from first copy of $K_1 \circ H_m$ and $k - j$ vertices from second copy of $K_1 \circ H_m$. The number of way of doing this over all $k = 2, 3, ..., mn$ is exactly the coefficient of x^k in $[(1 + x)^{m+1} - 1]^2$. So $T((G \circ H, x) = [(1 + x)^{m+1} - 1]^2$. □

References

[1] S. Alikhani, Y.H. Peng, Dominating sets and domination polynomial of cycles, Glob. J. Pure Appl. Math. 4 (2) (2008) 151–162.

[2] S. Alikhani, Y.H. Peng, Dominating sets and domination polynomials of paths, Int. J. Math. Math. Sci., 2009, Article ID 542040.

[3] Harary F,"Graph Theory",Adison-Wesley.

[4] Jalsiya M.P. and Raji Pilakat, "An efficient approach to circuit analysis through introduction of cycle tracking sets in a graph", communicating.

[5] Jalsiya M.P. and Raji Pilakat, "Changing and unchanging cycle tracking", communicating.

[6] Jalsiya M.P. and Raji Pilakat, "Cycle tracking polynomial of a graph", communicating.

[7] Jalsiya M.P. and Raji Pilakat, "Independent and Irredundant Cycle Tracking Sets of a Graph : An efficient approach to electrical circuit analysis", to appear,Far East Journal of Mathematical Sciences (FJMS),Volume 111, Number 2, 2019, Pages 225-238.

[8] Jalsiya M.P. and Raji Pilakat, "Mixed Circuit Domination Number", International Journal of Research in Advent Technology, Vol.6, No.10, October 2018.

[9] Jalsiya M.P. and Raji Pilakat, "Spectra of cycle tracking matrix of a graph", communicating.

[10] Jalsiya M.P. and Raji Pilakat, "Tracing Matrix of a Graph", communicating.

[11] Jalsiya M.P. and Raji Pilakat, "Transitively Tracked Graphs", communicating.

[12] Jalsiya M.P. and Raji Pilakat, "Total cycle tracking set", communicating.

[13] Teresa W. Haynes, Stephen T. Hedetniemi and Peter J. Slater, "Domination in Graphs-Advanced Topics", Marcel Dekker, Inc.1998.

[14] Teresa W. Haynes, Stephen T. Hedetniemi and Peter J. Slater, Fundamentals of Domination in Graphs, Marcel Dekker, Inc.1998.

[15] D.B.West,"Introduction to Graph Theory" ,2nd ed. Pearson Education.