COSMOLOGICAL ASPECTS OF THE EINSTEIN FIELD EQUATIONS IN HIGHER DIMENSIONAL SPACE TIME WITH SCIENCE AND TECHNOLOGY

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Abstract- ***The main purpose of this article is to present the cosmological consequences, favourable or not with modern technology. Cosmology occupies a key position in the family of sciences and technology in that it depends upon each of the others, to different degrees, and in illuminates them all in distinct ways. In this paper study of Einstein field equations in higher dimensional space-time provides an idea that our universe is much smaller at early stage of evolution as observed today. This paper has been a proliferation of works on higher dimensional space times both in localized and cosmological domains. This also provides the idea about study of physical situation at the early stages of the formation of the universe. Investigation on this topic is helpful to knowing behaviour of higher dimensional cosmological models for Static, Non Static, flat and non-flat models. It provides a natural way to make a search for exact solutions for constant deceleration parameter with different types of distributions of matter and for different type of symmetries of space time. The paper comprises one physicist's conjectures about each of these applied topics with approach of modern science and technology, suggesting the physical situation at the early stages of the formation and evolution of the universe, which can guide our search for viable solutions to real world predicaments confronting civilization today and play the role of pathfinder for modern technology in India.***

Keywords: ***Einstein field equations, Higher dimensional cosmological model, Static and Non Static models, Flat models, Deceleration parameter, Exact solutions.***

1. INTRODUCTION

General relativity explains gravity as the curvature of space time. It's all about geometry. The basic equation of general relativity is called Einstein's equation. In units where $c = 8\pi G = 1$, it says

(1.1) $$G_{ab} = T_{ab}$$

It looks simple, but what does it mean? Unfortunately, the beautiful geometrical meaning of this equation is a bit hard to find in most treatments of relativity. There are many nice popularizations that explain the philosophy behind relativity and the idea of curved space time, but most of them don't get around to explaining Einstein's equation in higher dimensional space times and showing how to work out its consequences.

There are also more technical introductions which explain Einstein's equation in detail, but here the geometry is often hidden under piles of tensor calculus. This is a pity, because in fact there is an easy way to express the whole content of Einstein's equation in plain English. In fact, after a suitable prelude, one can summarize it in a single sentence! One needs a lot of mathematics to derive all the consequences of this sentence, but it is still worth seeing and we can work out some of its consequences quite easily.

Many numerical codes now under development to solve Einstein's equations of general relativity in (3+1)-dimensional space-time employ the standard form of the field equations. This form involves evolution equations for the static spherically symmetric metric and extrinsic curvature tensor. In this paper we have obtained some exact static spherical solution of Einstein's field equation with cosmological constant $\wedge = 0$ and equation of state $p = \rho$ (taking suitable choice of g_{11} and g_{44}). which help to investigate the value of e^{α} & e^{β} respectively. Many previously known solutions are contained here in as a particular case. Various physical and geometrical properties have been studied. The explicit expressions for rotation, shear scalar of expansion and fluid velocity have also investigated. For different values of n we get many previously known solutions. Here $\wedge = 0$, this implies that Einstein element would degenerate into a line element of special relativity for flat space time.

The term ''Exact Solution'' generally refers to a solution that captures the entire mathematics and physics of a problem as opposed to one that is approximate, perturbative, etc. In General relativity, an exact solution is a Lorentzian manifold equipped with tensor fields modelling states of ordinary matter, such as a fluid, or classical non gravitational fields such as the electromagnetic field. Mathematical analysis dealing with limits and related theories, such as differentiation, integration, measure, relativity and analytic functions. Analysis may be distinguished from geometry; however, it can be applied to any space of mathematical objects that has a specific distance between objects (a metric space). The vast majority of relativity, classical mechanics, and quantum mechanics is based on applied analysis and differential equations in particular. These important differential equations include Einstein's field equations, metric equations and the Newton's second law. Hence mathematical analysis is also a major factor in study of relativity and other branches of science.

The present analysis deals with the exact solutions of the Einstein's field equations for the perfect fluid with variable gravitational and cosmological "constants" for a spatially homogeneous and anisotropic cosmological model. The Einstein's field equation has two parameters; the cosmological constants Λ and the gravitational constant G. Cosmological models with a cosmological constant are currently serious candidates to describe the dynamics of the Universe [15, 17, 22]. The rise of interest in the theory of General Relativity as a tool for studying the evolution and behaviour of various cosmological models has been rapid expensive. Since the early 1920's to the present, the Einstein's theory of relativity has been used extensively as a tool in the prediction and modelling of the cosmos. One reason for the prominence of modern relativity is its success in predicting the behaviour of large scale phenomena where gravitation plays a dominant role [6-8]. Various researcher in theory of relativity have focused their mind to the study of solution of Einstein's field equation with cosmological constant $\Lambda = 0$ and equation of state $p = \rho$. Solution of Einstein's field equation of state $p = \rho$ have been obtained by various authors e.g., Latelier [12], Letelier and Tabensky [13], Tabensky, R., et.al.[24] and Yadav[33]. Singh and Yadav [20] have also discussed the static fluid sphere with the equation of the state $p = \rho$. Further study in the line has been done by Yadav and Saini [30], which is more general than one due to Singh and Yadav [20]. Also in this case the relative mass m of a particle in the gravitational field related to its proper mass m_0 studied by Narlikar [14]. Schwarzschild [18] considered the perfect fluid spheres with homogeneous density and isotropic pressure in general relativity and obtained the solutions of relativistic field equations. Tolman [26] developed a mathematical method for solving Einstein's field equations applied to static fluid spheres in such a manner as to provide explicit solutions in terms of known analytic functions. A number of new solutions were thus obtained and the properties of three of them were examined in detail.

No stationary in homogeneous solutions to Einstein's equations for an irrotational perfect fluid have featured equations of state $p = \rho$ (Letelier [12], Letelier and Tabensky [13] and Singh and Yadav [20]). Solutions to Einstein's equations with a simple equations of state have been found in various cases, e.g. for $\rho + 3p = \text{constant}$ (Whittaker [29]) for $\rho = 3p$ (Klein [9]); for $p = \rho + \text{constant}$ (Buchdahl and Land [4], Allunt [1]) and for $\rho = (1+a)\sqrt{p} + ap$ (Buchdahl [2]). But if one takes, e.g. polytrophic fluid sphere $\rho = ap^{1+\frac{1}{n}}$ (Klein [10], Tooper [27], Buchdahl [3]) or a mixture of ideal gas radiation (Suhonen [23]), one soon has to use numerical methods. Yadav and Saini [30] have also studied the static fluid sphere with equation of state $p = \rho$ (i.e. stiff matter). Davidson [5] has presented a solution a non stationary analog to the case when $p = \frac{1}{3}\rho$. Tolman [26], Yadav and Purushottam [31], Thomas E Kiess [25], Karmer [11], Singh. et.al.[19], Raychaudhari[16], Walecka[28], Yadav, et.al.[34-35] and Yadav and Singh [32] are some of the authors [36-54] who have studied various aspects of interacting fields in the framework of Einstein's field equations for the perfect fluid with specified equation of state, general relativity and higher dimensional cosmological models.

In this paper we have obtained some exact static spherically symmetric solution of Einstein field equation for the static fluid sphere with cosmological constant $\Lambda = 0$ and equation of state $p = \rho$. It has been obtained taking suitable choice of g_{11} and g_{44} .For different values of n we get many previously known solutions. To overcome the difficulty of infinite density at the centre, it is assumed that distribution has a core of radius and constant density which is surrounded by the fluid with the specified equation of state. Many previously known solutions are contained here in as a particular case. Various physical and geometrical properties have been also studied. The main purpose of this article is to present the cosmological consequences, favourable or not with modern technology. Cosmology occupies a key position in the family of sciences and technology in that it depends upon each of the others, to different degrees, and in illuminates them all in distinct ways. In this paper study of Einstein field equations in higher dimensional space-time provides an idea that our universe is much smaller at early stage of evolution as observed today. The paper comprises one physicist's conjectures about each of these applied topics with approach of modern science and technology, suggesting the physical situation at the early stages of the formation and evolution of the universe, which can guide our search for viable solutions to real world predicaments confronting civilization today and play the role of pathfinder for modern technology in India.

2. THE FIELD EQUATIONS

We consider the static spherically symmetric metric given by

(2.1) $$ds^2 = e^{\beta}dt^2 - e^{\alpha}dr^2 - r^2d\theta^2 - r^2\sin^2\theta d\varphi^2$$

where α and β are functions of r only.

Taking cosmological constant Λ into account, we obtain the field equations

(2.2a) $$R^i_j - \frac{1}{2}R\delta^i_j + \Lambda\delta^i_j = -8\pi T^i_j$$

For $\boldsymbol{\Lambda = 0}$, (2.2a) gives

(2.2b) $$R^i_j - \frac{1}{2}R\,\delta^i_j = -8\,\pi T^i_j$$

For the metric (2.1) are (Tolman [26])

(2.3) $$-8\pi\,T^1_1 = e^{-\alpha}\left(\frac{\beta'}{r} + \frac{1}{r^2}\right) - \frac{1}{r^2}$$

(2.4) $$-8\pi\, T_2^2 = -8\pi\, T_3^3$$

$$= e^{-\alpha}\left(\frac{\beta''}{2} - \frac{\alpha'\beta'}{4} + \frac{\beta'^2}{4} + \frac{\beta'-\alpha'}{2r}\right)$$

(2.5) $$-8\pi\, T_4^4 = e^{-\alpha}\left(\frac{\alpha^1}{r} - \frac{1}{r_2}\right) + \frac{1}{r_2}$$

where a prime denotes differentiation with respect to r.

Through the investigation, we set velocity of light C and gravitational constant G to be unity. A Zeldovich fluid can be regarded as a perfect fluid having the energy momentum tensor.

(2.6) $$T_j^i = (\rho + p)u^i u_j - \delta_j^i\, p$$

Specified by the equation of state

(2.7) $$\boldsymbol{\rho = p}$$

we use co-moving co-ordinates so that

$$u^1 = u^2 = u^3 = 0 \text{ and } u^4 = e^{-\frac{\beta}{2}}$$

The non-vanishing components of the energy momentum tensor are

$$T_1^1 = T_2^2 = T_3^3 = -p \text{ and } T_4^4 = \rho$$

We can then write the field equations:-

(2.8) $$8\pi p = e^{-\alpha}\left(\frac{\beta'}{r} + \frac{1}{r^2}\right) - \frac{1}{r^2}$$

(2.9) $$8\pi p = e^{-\alpha}\left(\frac{\beta''}{2} - \frac{\alpha'\beta'}{4} + \frac{\beta''^2}{4} + \frac{\beta'-\alpha'}{2r}\right)$$

(2.10) $$8\pi\rho = e^{-\alpha}\left(\frac{\alpha'}{r} - \frac{1}{r^2}\right) + \frac{1}{r^2}$$

3. SOLUTION OF THE FIELD EQUATIONS

Using equations (2.7), (2.8) & (2.10), we have

(3.1) $$e^{-\alpha}\left(\frac{\beta'}{r} + \frac{1}{r^2}\right) - \frac{1}{r^2} = e^{-\alpha}\left(\frac{\alpha'}{r} - \frac{1}{r^2}\right) + \frac{1}{r^2}$$

From [3.1] we see that if β is known, α can be obtained, so we choose –

➤ ***<u>Case. I</u>***

(3.2) $$\mathbf{e^{\beta} = \boldsymbol{l}\, r^n,} \quad \text{(where } l \text{ is a constant)}$$

Using (3.2), equation (3.1) goes to the –

(3.3) $$\frac{de^{-\alpha}}{dr} + \frac{n+2}{r} e^{-\alpha} = \frac{2}{r}$$

Put $(\tau = e^{-\alpha})$ in the equation (3.3) is reduced to

(3.4) $$\frac{d\tau}{dr} + \frac{n+2}{r} \tau = \frac{2}{r}$$

This is a linear differential equation whose solution is given by

(3.5) $$\tau = \frac{2}{n+2} + \frac{C}{r^{n+2}}$$

or,

(3.6) $$e^{-\alpha} = \frac{2}{n+2} + \frac{C}{r^{n+2}}$$

where C is integration constant.

➢ ***Case. II***

(3.7) $$\mathbf{e^{\beta} = \mathit{l}r^{n-1}}$$ **(*for getting a generalised value*)**

(where l is constant)

Using (3.7), in equation (3.1) we get

(3.8) $$\frac{de^{-\alpha}}{dr} + \frac{n+1}{r} e^{-\alpha} = \frac{2}{r}$$

Put $\tau = e^{-\alpha}$ in equation (3.8) is reduced to

(3.9) $$\frac{d\tau}{dr} + \frac{n+1}{r} \tau = \frac{2}{r}$$

Solution of this linear differential equation is

(3.10) $$\tau = \frac{2}{n+1} + \frac{C}{r^{n+1}}$$

or

(3.11) $$e^{-\alpha} = \frac{2}{n+1} + \frac{C}{r^{n+1}}$$

➢ **So we get a generalised value for this (i.e. $\mathbf{e^{\beta} = \mathit{l}r^{n}}$):-**

(3.12) $$\mathbf{e^{-\alpha} = \frac{2}{k} + \frac{C}{r^{k}}}$$ **or**

(3.13) $$\mathbf{e^{\alpha} = \frac{kr^{k}}{2r^{k} + kC}}$$

(where $k = n + 2$, n = power of r and C = integral constant).

Hence, using the equation (3.6) the metric (2.1) yields:-

(3.14) $$\mathbf{ds^2 = lr^n dt^2 - \left(\frac{2}{n+2} + \frac{C}{r^{n+2}}\right)^{-1} dr^2 - r^2(d\theta^2 + sin^2\theta . d\varphi^2)}$$

Absorbing the constant l in the co-ordinates differentials dt and putting C = 0, the metric (3.14) goes to the form:-

(3.15a) $$\mathbf{ds^2 = r^n dt^2 - \frac{n+2}{2} dr^2 - r^2(d\theta^2 + sin^2\theta . d\varphi^2)}$$

or

(3.15b) $$\mathbf{ds^2 = r^n dt^2 - \frac{k}{2} dr^2 - r^2(d\theta^2 + sin^2\theta . d\varphi^2)}$$

The non-zero components of Reimann- christoffel curvature tensor R_{hijk} for the metric [3.15] are

(3.16) $$\sin^2\theta R_{2424} = R_{3434} = \frac{n+2}{2} r^n . \sin^2\theta = \frac{k}{2} r^n \sin^2\theta = R_{2323}$$

We see that $R_{hijk \to 0}$ as $r \to \infty$

Hence it follows that the space time is asymptotically Homaloidal.

For the metric [3.15] the fluid velocity v' is given by

(3.17) $$v^1 = v^2 = v^3 = 0 \; ; \quad v^4 = r^{-n/2} = \frac{1}{r^{n/2}}$$

The scalar of expansion $\Theta = v^j_i$ is identically zero (i.e. $\Theta = 0$). The non-vanishing components of the tensor of rotation ω_{ij} is defined by

(3.18) $$\omega_{ij} = v_{i,j} - v_{j,i}$$

(3.19) $$\omega_{14} = -\omega_{41} = -\frac{n}{2} r^{n/2 - 1} = -\frac{n}{2} r^{\frac{n-2}{2}}$$

The components of the shear tensor σ_{ij} defined by

(3.20) $$\sigma_{ij} = \frac{1}{2}\left(v_{ij} + v^{ij}\right) - \frac{1}{3} H_{ij}$$

With the projection tensor

(3.21) $$H_{ij} = g_{ij} - v_i v_j$$

(3.22) $$\sigma_{14} = \sigma_{41} = \frac{n}{2} r^{\frac{n}{2} - 1} = \frac{n}{2} r^{\frac{n-2}{2}}$$

- **(Particular case) :-**

If we choose

(3.23) $$\mathbf{e^\beta = \mathit{l} r^{5/4}} \; , \qquad \text{(where } l \text{ is constant)}$$

Using (3.23) in equation (3.1) goes to form:-

(3.24) $$\frac{de^{-\alpha}}{dr} + \frac{13}{4r}e^{-\alpha} = \frac{2}{r}$$

Substituting $\tau = e^{-\alpha}$, the equation (3.24) is reduced to,

(3.25) $$\frac{d\tau}{dr} + \frac{13}{4r}\tau = \frac{2}{r}$$

which is a linear differential equation whose solution is given by:-

(3.26) $$\tau = \frac{8}{13} + \frac{C}{r^{13/4}}$$

or

(3.26a) $$e^{-\alpha} = \frac{8}{13} + \frac{C}{r^{13/4}}$$

where C is integration constant.

Hence the metric (2.1) yields

(3.27) $$\mathbf{ds^2 = \mathit{l} r^{5/4} dt^2 - \left(\frac{8}{13} + \frac{C}{r^{13/4}}\right)^{-1} dr^2 - r^2(d\theta^2 + \sin^2\theta . d\varphi^2)}$$

Absorbing the constant l in the co-ordinate differential dt and put $C = 0$ the metric (3.27) goes to the form –

(3.28) $$\mathbf{ds^2 = r^{5/4} dt^2 - \frac{13}{8} dr^2 - r^2(d\theta^2 + \sin^2\theta . d\varphi^2)}$$

The non-zero component of Reimann-christoffel curvature tensor R_{hijk} for the metric (3.28) is

(3.29) $$\sin^2\theta R_{2424} = R_{3434} = \frac{13}{8} r^{13/4} \sin^2\theta = R_{2323}$$

For the metric [3.28] the fluid velocity v' is given by

(3.30) $$v^1 = v^2 = v^3 = 0, \quad v^4 = r^{-5/8} = \frac{1}{r^{5/8}}$$

In the usual notation, we have the rotation and shear tensor same as equation (3.18, 3.20, 3.21 and 3.22) which gives results for metric (3.28) as:-

(3.31) $$\Theta = 0,\ \omega_{14} = -\omega_{41} = \frac{-5}{8} r^{-3/8} = \frac{-5}{8r^{3/8}}$$

and

(3.32) $$\sigma_{14} = \sigma_{41} = \frac{5}{8} r^{-3/8} = \frac{5}{8r^{3/8}} \quad .$$

➢ ***Case. III***

(3.33) $\mathbf{e^{-\alpha} = a}$ **,**

where a is constant.

Using (3.33), equation (3.1) goes to the –

(3.34) $\beta' - \alpha' + \frac{2}{r}\left[1 - \frac{1}{a}\right] = 0$

Since $e^{-\alpha} = a$, is constant, then $\alpha' = 0$ and hence (3.34) reduces to

(3.35) $\beta' + \frac{2}{r}\left[1 - \frac{1}{a}\right] = 0$

Now (3.35) integrate w.r.t r we get-

(3.36) $e^{\beta} = Ar^{2\left(1-\frac{1}{a}\right)}$

where A is a integration constant.

If we consider $\mathbf{a = 3}$,then we get -

(3.37) $e^{\beta} = Ar^{4/3}$

Hence, using the equation (3.37) the metric (3.1) yields:-

(3.38 $\mathbf{ds^2 = Ar^{4/3}dt^2 - 1/3(dr^2) - r^2(d\theta^2 + \sin^2\theta . d\varphi^2)}$

Absorbing the constant A in the co-ordinates differentials dt the metric (3.38) goes to the form :-

(3.39) $\mathbf{ds^2 = r^{4/3}dt^2 - 1/3(dr^2) - r^2(d\theta^2 + \sin^2\theta . d\varphi^2)}$

The non-zero components of Reimann-christoffel curvature tensor R_{hijk} for the metric (3.39) are:-

(3.40) $\sin^2\theta R_{2424} = R_{3434} = \frac{-1}{2} r^2 \sin^2\theta = R_{2323}$

we see that $R_{hijk} \to 0$ as $r \to \infty$

Hence it follows that the space time is asymptotically Homaloidal.

For the metric (3.39) the fluid velocity v' is given by

(3.41) $v^1 = v^2 = v^3 = 0$; $v^4 = 1/r = r^{-1}$

The scalar of expansion $\Theta = v^j_i$ is identically zero (i.e., $\Theta = 0$). The non-vanishing components of the tensor of rotation ω_{ij} is defined by-$\omega_{ij} = v_{ij} - v_{ji}$, we get

(3.42) $\omega_{14} = -\omega_{41} r = r^0 = 1$

The components of the shear tensor σ_{ij} defined by $\sigma_{ij} = \frac{1}{2}\left(v_{ij} + v^{ij}\right) - \frac{1}{3}H_{ij}$, with the projection tensor $H_{ij} = g_{ij} - v_i v_j$ are

(3.43) $$\sigma_{14} = \sigma_{41} = \frac{1}{2}r^0 = \frac{1}{2}.$$

4. SOLUTION FOR THE PERFECT FLUID CORE

Pressure and density for the metric (3.14-15a, 3.28) are

(4.1) $$8\pi p = 8\pi\rho = \frac{n+1}{r^2}\left[\frac{2}{n+2} + \frac{C}{r^{n+2}}\right] - \frac{1}{r^2}$$

If we consider $C = 0$, then equation (4.1) reduces to

(4.2) $$8\pi p = 8\pi\rho = \frac{n+1}{r^2}\left[\frac{2}{n+2}\right] - \frac{1}{r^2}$$

(4.3) $$8\pi p = 8\pi\rho = \frac{18}{13r^2}$$

It follows from (4.1-4.3) that the density of the distribution tends to infinity as r tends to zero. In order to get rid of singularity at $r = 0$ in the density we visualize that the distribution has a core of radius r_o and constant ρ_o. The field inside the core is given by Schwarzschild internal solution.

(4.4a) $$e^{-\lambda} = 1 - \frac{r^2}{R^2}$$

(4.4b) $$e^{\nu} = \left[\bar{L} - \bar{M}\left(1 - \frac{r^2}{R^2}\right)\right]^2$$

(4.4c) $$8\pi p = \frac{1}{R^2}\left[\frac{3\bar{M}\left(1-\frac{r^2}{R^2}\right)-\bar{L}}{\bar{L}-\bar{M}\left(1-\frac{r^2}{R^2}\right)^{\frac{1}{2}}}\right]$$

where $\bar{L}, \bar{M}$ are constants and $R^2 = \frac{3}{8\pi\rho}$.

The continuity condition for the metric (3.14) and (4.4a-4b-4c) at the boundary gives

(4.5a) $$R^2 = \frac{r_0^2}{\left(\frac{n}{n+2} - \frac{c}{r_0^{n+2}}\right)}$$

(4.5b) $$\bar{L} = r_0^{n/2} + \frac{nR^2}{2r_o^{2-\frac{n}{2}}}\left(1 - \frac{r_0^2}{R^2}\right)$$

(4.5c) $$\bar{M} = \frac{nR^2}{2r_o^{2-\frac{n}{2}}}\left(1 - \frac{r_0^2}{R^2}\right)^{1/2}$$

(4.5d) $$C = r_0^{n+2}\left(\frac{n}{n+2} - \frac{r_o^2}{R^2}\right)$$

and the density of the core

(4.6) $$\rho_o = \frac{3}{8\pi r^2}\left(\frac{n}{n+2} - \frac{C}{r_0^{n+2}}\right)$$

which complete the solution for the perfect fluid core of radius r_0 surrounded by considered fluid. The energy condition $T_{ij}u^iu_j > 0$ and the Hawking and Penrose condition (Hawking and Penrose, 1970).

$$(T_{ij} - \frac{1}{2}g_{ij}T)u^iu_j > 0,$$

Both reduces to $\rho > 0$, which is obviously satisfied.

For different value of n, solution obtained above in case I and case II provide many previously known solutions. For $n = 2$ and by suitable adjustment of constant we get the solution due to Singh and Yadav [20] and Yadav and Saini [30]. Also for n = 3 we get solution due to Yadav et.al [33].

5. DISCUSSION

In this paper we have obtained some exact static spherical solution of Einstein's field equation with cosmological constant $\Lambda = 0$ and equation of state $p = \rho$. We have shown that when cosmological constant $\Lambda = 0$, then in the absence of electromagnetic field pressure and density become equal and conversely if pressure and density are equal there is no electromagnetic field. Our assumption is $\mathbf{e^{\beta} = \mathit{l}r^{n}}$, $\mathbf{e^{\beta} = \mathit{l}r^{n-1}}$, $\mathbf{e^{\beta} = \mathit{l}r^{5/4}}$ and $\mathbf{e^{-\alpha} = a}$, which investigate a generalised value problem. It describe several important cases, e.g.- relativistic model, fluid velocity, rotation, shear tensor, scalar of expansion. It also helpful to investigates solution for the perfect fluid core. The main purpose of this article is to present the cosmological consequences, favourable or not with modern technology. Cosmology occupies a key position in the family of sciences and technology in that it depends upon each of the others, to different degrees, and in illuminates them all in distinct ways. In this paper study of Einstein field equations in higher dimensional space-time provides an idea about our universe and its physical & geometrical behaviour.

6. CONCLUSION

The cosmology group is interested in gravity in extra or higher dimensions, and in particular in the modifications that can be brought to Einstein theory both in infrared and ultraviolet regimes(very large and very small distances).This research in the domain of high-energies where gravity becomes comparable to the other fundamental forces is motivated on the one hand by theories proposing the unification of gravity with the other interactions (such as string theories living in more than 4 dimensions), and on the other hand, by recent observations showing that our universe is accelerating. Therefore, either the main part of our Universe is not made of observable matter or gravity itself is modified at these scales. The group studies theories which generalises Einstein theory at 4 dimensions (exact solutions, properties, stability).We work actively on brane universes where our 4-dimensional universe is a subspace of the whole space-time.

An important aspect of the cosmological models is that it can provide a natural explanation for the mysterious dark matter, which contributes nearly thirty times as much as luminous matter like stars, galaxies etc to the total energy content of the universe. The study of cosmological models in higher dimensional space plays an important role in the study of universe. The study is more interesting as these models contain isotropic special cases and permit arbitrary small anisotropy levels at some instant of time. At the very early stages of evolution of the universe, during phase transitions, it is believed that the symmetry of universe is broken spontaneously. It can give rise to topologically stable defects such as domain walls, strings, and monopoles. In these three cosmological structures, cosmic strings are most interesting because they are believed to give rise to density perturbations which lead to the formation of galaxies. It is very interesting to study the gravitational effect that arises from string using Einstein's field equations in four and higher dimensions. The study of higher dimensional space provides an idea that our universe is much smaller at early stage of evolution as observed today.

7. APPLICATIONS AND LIMITATION

- In this paper we get a generalised value of e^{α} and metric. Now we can easily obtained the metric for any given value of n, where n is the power of r.
- It helpful to investigates solution for different types of distributions of matter and for different type of symmetries of space time.
- To study of staler body and radiation.
- To study of cosmological models for $p = \rho$ and relativistic model, fluid velocity, rotation, shear tensor, scalar of expansion.

If we consider n = -2 for equations (3.14), then the value of $e^{-\alpha}$ *is undefined and* e^{α} *become a constant value.*

8. FUTURE PROSPECTS

The investigation on this topic can be further taken up in different directions:

- This topic has been a proliferation of works on higher dimensional cosmological model with Einstein Field Equations of state for $p = \rho$ in localized and cosmological domains.
- It is important in a natural way to make a search for exact solutions of theories of gravitation for constant deceleration parameter with different types of distributions of matter and for different type of symmetries of space time. our model represents a vacuum universe (for $\rho = 0$, $p = 0$) and also an anisotropic one (for $\rho = -\gamma$, $p = \gamma$).
- This also helpful to provide the idea about study of physical situation at the early stages of the formation of the universe.

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➢ Nomenclature

C integration constant.
g_{ij} fundamental tensor
H_{ij} projection tensor
l constants
$\overline{L}$ constants
$\overline{M}$ constants
p pressure
r_o radius of perfect fluid core
R_{ij} Ricci tensor
R_{hijk} curvature tensor
T_{ij} energy-momentum tensor
u^i N-dimensional velocity vector

➢ Greek Symbols

α, β function of r
Λ cosmological constant
δ^i_j Kronecker delta
ν fluid velocity
ω_{ij} tensor of rotation
φ_i displacement vector
ρ_o constant density of the core
ρ energy density
σ_{ij} shear tensor
Θ scalar of expansion