SOME ASPECTS OF GENERAL RELATIVITY WITH EINSTEIN-CARTAN THEORY AND THEIR APPLICATIONS

Abhishek Kumar Singh
(*PhD, M.Sc., Gold-Medalist*)
P.G. Department of Mathematics, Magadh University, Bodhgaya-824234, Bihar (INDIA)
Email-dr.aksingh2909@gmail.com

Abstract- ***This paper provides the brief knowledge about General relativity and Einstein-Cartan theory with their applications. As a matter of fact the general relativistic solution for the gravitational field of the sun takes into account just the sun itself. But Einstein wanted to apply his theory of general relativity to obtain the cosmological models of the universe. The study of Einstein-Cartan theory and cosmological model play an important role in the study of universe and the study is more interesting as these models contain isotropy in special cases and permit arbitrary small anisotropy levels at some instant of time. These models have also played an important role in the study of universe and its early stages of evolution during phase transition. Different cosmological models have significant contribution in the evolution of galaxies and stellar bodies. In future it play important role in the field of science and technology.***

Keywords- ***Einstein-Cartan theory, Cosmological model, anisotropy, galaxies and stellar bodies.***

1. INTRODUCTION

Much interest has been shown by many researchers in the theory of General Relativity as a tool for studying the evolution and behaviour of various cosmological models. Since the early 1920's to the present, the Einstein Theory of Relativity has been used extensively as a tool in the prediction and modelling of the cosmos. One reason for the prominence of modern relativity is it success in predicting the behaviour of large- scale phenomena where gravitation plays a dominant role. In recent years there has been growing interest in the study of the Einstein – Cartan theory of space time. The intrinsic spin of matter has been taken as the source of torsion of the space-time manifold in Einstein – Cartan theory. The discovery of pulsars gave a big impetus to the developments in relativistic astrophysics, particularly in the study of neutron stars. Since the rotating – neutron star models for a pulsar seems to be almost undisputed [4] it is essential to study the possible internal structure of neutron stars with respect to gravitation and the associated geometry of space–time therein. Observations indicate a very high magnetic field associated with them in comparison with any other compact objects in the universe. It is not unlikely that this magnetic field might induce a spin polarization of the nucleons composing the fluid of neutron star [19]. If the spins are aligned then it is probable that there would be a substantial non zero spin density which would then play, along with the mass density, a dynamical role in influencing the geometry of space-time containing the fluid. In general relativity as given by Einstein there is no way of considering the spin effects on the geometry of space-time. On the other hand, it is clear that one could study such configurations in the frame work of the Einstein – Cartan theory [5-8]

Einstein himself was one of the first to apply his theory to the behaviour of the cosmos itself. He was spurred on by the theory's early success in predicting the residual advance of the perihelion of mercury not predicted by Newtonian theory; the "bending" of light as it grazes the sun; and the gravitational red- shift. The original equations of General Relativity predicted an expanding universe, a conflict with the generally held view of De Sitter [18] that the universe was essentially static. Instead of holding to the predictions of his model, Einstein modified his gravitational equations by adding to the so- called "gravitational constant". Einstein later called the new addition of the constant to be "the biggest blunder of my life". The constant was necessary to obtain stationary (non-expanding) solutions to the field equations and thus model a universe of constant radius. In 1929, the astronomer E. Hubble discovered that the spectrum of distant galaxies was red-shifted by an amount directly proportional to the apparent distance of the galaxy from Earth. If the red- shift was caused by the phenomenon known as the Doppler Effect, a phenomenon directly attributable to an object's velocity, then this meant that the galaxies were indeed moving away from one another at high-speed. Of course, there is still some argument over this interpretation because distances of far galaxies cannot be computed by geometrical calculations (e.g.-Parallax). The calculation of the distance of galaxies relies on a method of comparing the brightness of certain classes of stars (known as Cepheid's) which, in theory, have the some properties everywhere in the universe. If this is true, then a current astronomical observation leads one to conclude that the universe is indeed expanding, and not static. In the 1920's, various authors returned to the original Einstein field Equations. Friedman , Robertson and Walker developed cosmological models (known here after as the FRW cosmological models) which predicted not only an expanding universe but also a singularity of time or time of infinite density and infinitesimally small size of the universe at its birth (approximately 3.8×10^{17} seconds ago in the FRW models). This concept of a universe which begins with a "bang" has many conceptual problems today remains unsolved. However, it is safe to say that General Relativity remains the cosmological model par excellence and stills the best model and framework that has ever been developed to model and visualize the evolutionary nature of the universe.

In this paper provides the brief knowledge about General relativity and Einstein-Cartan theory with their applications. As a matter of fact the general relativistic solution for the gravitational field of the sun takes into account just the sun itself. But Einstein wanted to apply his theory of general relativity to obtain the cosmological models of the universe. The study of Einstein-Cartan theory and cosmological model play an important role in the study of universe and the study is more interesting as these models contain isotropy in special cases and permit arbitrary small anisotropy levels at some instant of time. These models have also played an important role in the study of universe and its early stages of evolution during phase transition. Different cosmological models have significant contribution in the evolution of galaxies and stellar bodies. In future it play important role in the field of science and technology.

2. A BRIEF REVIEW OF GENERAL RELATIVITY AND EINSTEIN-CARTAN THEORY

Special and General Relativity [20-21] which was given by Einstein in 1905 and 1915 respectively and which remain among Einstein's greatest achievements in Physics changed man's view and approach in formulating and describing the universe and the physical laws which govern various phenomena. In fact Minkowski's four dimensional space time continuums is the starting point in the evolution of Einstein's general and gravitational theory. Minkowski utilizing the principles of the special theory of relativity and the four dimensional geometry of Riemann, was able, in 1908 to arrive at new concept of a four dimensional space time continuum which may be regarded as a geometrical interpretation of the special theory.

In Minkowski space-time continuum the line element is given by:

$$\text{(A)} \qquad ds^2 = -dx^2 - dy^2 - dz^2 + c^2dt^2$$

$$= dx^2 + dy^2 + dz^2 - c^2dt^2$$

The modified form of above equation in tensor form is

$$\text{(B)} \qquad ds^2 = g_{ij}\,dx^i dy^j$$

In regions far from gravitational field, the general theory of relativity reduces to special one and the Minkowski space-time continuum holds in regions far from gravitational field also.

The element (B) represents the curved geometry. Thus according to general theory of relativity the space is curved in a gravitational field. Since the space is curved in gravitational of space in a gravitational field is Riemannian, for the idea was originally developed by Riemann. Thus free motion in a gravitational field or in a non- Euclidean space is not straight, but curvilinear. The theory of gravitation also deals with non- inertial systems. Due to this reason, the theory of gravitation is called "general theory of relativity" The laws of nature remain covariant independent of the frame of Reference. This statement is called the principle of general covariance. According to these principle laws of nature by means of equation in the covariant form, which are independent of the co-ordinate systems.

But in order to introduce the effect of gravitational field in the relativistic theory of gravitation, Einstein gave another, principle known as the principle of equivalence which states that a system which is stationary in a gravitational field of strength g is physically equivalent to a system which is in gravitational free space but accelerated in the opposite direction with an acceleration g.

The Einstein's theory of general relativity became a subject of much interest for scientists only a few decades ago. A number of new relativistic theories of gravitation were put forward by Cartan, Brans and Dicke, Bergamann, Wagoner, Nordtvedt, Sen and Dunn. Their predictions with the observational data and the available experimental results are compared with those of the older theories. Throne and Will] have undertaken a systematic study of what they call "metric theories of gravitation". These are the theories which may be formulated in terms of Riemannian geometry of space-time, possible with supplementary structures added to it. The total stress energy tensor of matter is assumed to satisfy a differential conservation law determined by the Riemannian linear conservation of space-time. As a matter of fact Einstein- Cartan theory begins with Sciama , and Kibble . It was further developed Trautman , Adamovicz, Kerlick, Kuchowicz, Hehl , Hehl et. al. , Tafel , Stewart and Hajicek, Kopczynski, Raychaudhuri and Prasanna . Since the predictions of Einstein- Cartan theory differs from those of general relativity only for matter filled regions. Therefore besides cosmology, an important application field of Einstein- Cartan theory is relativistic astrophysics. It deals with the theories of stellar objects similar to neutron stars with some alignment of spins of constituent particles and under the conditions when torsion produces some observable effects. So it has become desirable to understand the implications of the Einstein- Cartan theory for finite dimensions such as fluid spheres with non-zero pressure. Keeping this view Prasanna, Kerlics , Kuchowioz , Skinner and Webb studied the problem of static fluid spheres in Einstein- Cartan theory.

The Einstein’s field equations with incoherent matter for the case of homogeneous space-time, i.e., for metrics allowing a four parametric simply transitive group of motion was solved by Istvan Ozsvath in 1964. With the use of spinor- technique Istvan Ozsvath developed two sets of new solutions. Misra, Pandey and Shrivastava analyzed the Einstein-Maxwell equation for a stationary gravitational field in 1972 and also for an axially symmetric stationary gravitational field in 1973. Assuming that the space is filled with charged incoherent matter, they established that if the Lorentz force disappears everywhere, the charge density maintains a constant ratio to the mass density. This constant ration may take arbitrary values. In 1975, Prasanna has described briefly the Einstein- Cartan equations with special reference to a perfect fluid distribution following work of Trautman . He has obtained three solutions, adopting Hehl’s approach and Tolman’s technique. He has found that a space-time metric similar to the Schwarzschild interior solution will no longer represent a homogeneous fluid sphere in the presence of spin density. Further the field equations of general relativity with spin and torsion (U_4 theory) to describe correctly the gravitational properties of matter on a macrophysical level were considered by Hehl, Heyde and Kerlick. By an averaging procedure one can arrive, at a macroscopic field equation, which under normal matter densities coincides with Einstein’s equation of conventional general relativity. They have shown how the singularity theorems of Penrose and Hawking must be modified to apply in U_4 theorems and all known cosmological models in U_4 theory which prevent singularities have also shown to violate an energy condition of a singularity theorem.

Kopczynski studied and developed the Einstein- Cartan theory of gravitation. Especially homogeneous cosmological models of Bianchi types VI and VII based on Einstein- Cartan theory were considered by Tsoubelis [. Som and Bedran got a glass of solutions that represents a static incoherent spherical dust distribution in equilibrium under the influence of torsion and spin. Krori et. al. gave a singularity free solutions for a static changed fluid sphere in Einstein- Cartan theory. The junction conditions in Einstein- Cartan theory were discussed by Arkuszewski et. al. Raychaudhuri and Banerji constructed a specific solution corresponding to a collapsing sphere and showed that it bounces at a radius greater than the Schwarzschild radius. Banerji has pointed out that Einstein- Cartan sphere must bounce outside the Schwarzschild radius if it bounces at all. Nduka generalized the Prasanna work by considering a static charged fluid sphere in Einstein- Cartan theory. Singh and Yadav studied the static fluid sphere in Einstein- Cartan theory and obtained the solutions in an analytic from by the method of equadratur. Hence a wide scope created in this field.

Here we deal its application with one example-

We take metric in the form :

$$ds^2 = e^{\nu}\, dt^2 - e^{\lambda}\, dr^2 - r^2\,(d\theta^{\,2} + \sin^2\theta d\Phi^2) \tag{2.1}$$

where λ and ν are the function of r only.

Thus the Einstein-Maxwell field equations are [Adler et. al [1]]

$$8\pi\rho = -E - e^{-\lambda}\left(\frac{1}{r^2} - \frac{\lambda'}{r}\right) + \frac{1}{r^2}$$

[

or $$8\pi\rho = -\left[E + e^{-\lambda}\left(\frac{1}{r^2} - \frac{\lambda'}{r}\right) - \frac{1}{r^2}\right] \tag{2.2}$$

$$8\pi p \Rightarrow E + e^{-\lambda}\left(\frac{1}{r^2} + \frac{\upsilon'}{r}\right) - \frac{1}{r^2} \tag{2.3}$$

$$8\pi p = -E - e^{-\lambda}\left[\frac{1}{4}\upsilon'\lambda' - \frac{1}{4}\upsilon'^2 - \frac{1}{2}\upsilon'' - \frac{1}{2}\left(\frac{\upsilon' - \lambda'}{r}\right)\right] \tag{2.4}$$

Where $$E = -F^{41}F_{41} \tag{2.5}$$

and $$4\pi\sigma = \left[\frac{\partial F^{41}}{\partial r} + \frac{2}{r}F^{41} + \frac{\lambda'\upsilon'}{2}F^{41}\right]e^{\upsilon/2} \tag{2.6}$$

By the use of above equation 2.2 to 2.4 we get expression for p, ρ and E as

$$16\pi p = e^{-\lambda}\left[\frac{3\upsilon'}{2r} - \frac{\lambda'}{2r} + \frac{\upsilon''}{2} + \frac{(\upsilon')^2}{4} + \frac{\lambda'\upsilon'}{4} + \frac{1}{r^2}\right] \tag{2.7}$$

$$16\pi\rho = e^{-\lambda}\left[\frac{5\lambda'}{2r} - \frac{\upsilon''}{2} + \frac{\lambda'\upsilon'}{4} - \frac{(\upsilon')^2}{4} + \frac{\upsilon'}{2r} - \frac{1}{r^2}\right] \tag{2.8}$$

$$2E = e^{-\lambda}\left[\frac{\upsilon''}{2} - \frac{\lambda'\upsilon'}{4} + \frac{(\upsilon')^2}{4} - \frac{\upsilon'}{2r} - \frac{\lambda'}{2r} - \frac{1}{r^2}\right] + \frac{1}{r^2} \tag{2.9}$$

3. SOLUTION OF THE FIELD EQUATIONS

We have four equations (2.2)–(2.4) and (2.6) in six variables $(\rho, E, p, \lambda, \nu, \sigma)$. Hence the two variable are free. We take λ and ν as the two free variables. Here we choose.

Let,

$$\lambda = ar^3 + br^2 + c \tag{3.1}$$

$$\nu = lr^2 + mr + n \tag{3.2}$$

Where a, b, c, l, m, n are constant.

Then equation (2.6) to (2.9) give us

$$16\pi p = e^{-(ar^3+br^2+c)} \left[\left\{\frac{16l-4b-6ar(1+lr^2)+4lr^2(l-b)+2mr(2l-b)+m(m-3ar^2)}{4}\right\} + \frac{3m}{2r} + \frac{1}{r^2}\right] - \frac{1}{r^2} \tag{3.3}$$

$$16\pi p = e^{-(ar^3+br^2+c)}\left[\left\{\frac{20b+6ar(5+lr^2)+4lr^2(b-l)+2mr(b-2l)+m(3ar^2-m)}{4}\right\}+\frac{m}{2r}-\frac{1}{r^2}\right]+\frac{1}{r^2} \tag{3.4}$$

$$2E = e^{-(ar^3+br^2+c)} \left[\left\{\frac{4lr^2(l-b)-6ar(1+lr^2)+2mr(2l-b)+m(m-3ar^2)-4b}{4}\right\} - \frac{m}{2r} - \frac{1}{r^2}\right] + \frac{1}{r^2} \tag{3.5}$$

$$4\pi\sigma = \left[\frac{\partial F^{41}}{\partial r} + \frac{2}{r}F^{41} + \left\{\frac{3ar^2+2r(b+l)+m}{2}\right\}F^{41}\right] e^{lr^2+mr+\frac{n}{2}} \tag{3.6}$$

Now matching the solution with Reissner–Nordstorm metric at the boundary $r = r_\Delta$ we have -

$$e^{-\lambda} = e^{ar_\Delta{}^3+br_\Delta{}^2+c} = \left(1 - \frac{2M}{r_\Delta} + \frac{Q_\Delta{}^2}{r_\Delta{}^2}\right) \tag{3.7}$$

$$e^{\nu} = e^{lr_\Delta{}^2+mr_\Delta+n} = \left(1 - \frac{2M}{r_\Delta} + \frac{Q_\Delta{}^2}{r_\Delta{}^2}\right) \tag{3.8}$$

Differentiate eq (i) and (ii) w.r.t r we get

$$-(3ar_\Delta{}^2 + 2br_\Delta)e^{-(ar_\Delta{}^3+br_\Delta{}^2+c)} = 2\left(\frac{M}{r_\Delta{}^2} - \frac{Q_\Delta{}^2}{r_\Delta{}^3}\right) \tag{3.9}$$

$$(2lr_\Delta + m)e^{(lr_\Delta{}^2+mr_\Delta+n)} = 2\left(\frac{M}{r_\Delta{}^2} - \frac{Q_\Delta{}^2}{r_\Delta{}^3}\right) \tag{3.10}$$

In particular, we take,

$$a = m = 0 \text{ and } b = 1$$

Then we get

$$16\pi p = e^{-(r^2+c)}\left[4l + lr^2(l-r) - 1 + \frac{1}{r^2}\right] - \frac{1}{r^2} \tag{3.11}$$

$$16\pi p \ = e^{-(r^2+c)}\left[5 + lr^2(1-l) - \frac{1}{r^2}\right] + \frac{1}{r^2} \tag{3.12}$$

$$2E \ = e^{-(r^2+c)}\left[\{lr^2(l-1) - 4\} - \frac{1}{r^2}\right] + \frac{1}{r^2} \tag{3.13}$$

$$4\pi\sigma \ = \left[\frac{\partial F^{41}}{\partial r} + \frac{2}{r}F^{41} + r(1+l)F^{41}\right] e^{\frac{lr^2+n}{2}} \tag{3.14}$$

At $r \ = \ c \ = \ 0$ these results give –

$$16\pi p_0 = (4l-1) \tag{3.15}$$

$$16\pi\rho_0 = 5 \tag{3.16}$$

$$E_0 \ = \ \left(-\frac{4}{2}\right) \ = -2 \tag{3.17}$$

For p_0 and ρ_0 to be positive we must have

$$4l - 1 \ > \ 0 \ \Rightarrow 4l \ > \ 1 \tag{3.18}$$

$$\Rightarrow l \ > 1/4$$

$$(l \ is \ + ve \ quantity)$$

Further for

$$\rho_0 \ \ \geq \ 3p_0 \ \Rightarrow \ 5 \ \geq \ 3\,[4l - 1]$$

$$\Rightarrow \ 2 \ \geq \ 3\,l \tag{3.19}$$

From condition [3.18] and [3.19] we have $3\,l \ \leq \ 2k \ \leq \ 8$

$$3\,lk \ \leq \ \ 2k \ \leq \ \ 8\,lk, \quad \text{where } K\epsilon\, N \tag{3.20}$$

Reissner – Nordstrom metric at boundary $r \ = \ r_\Delta$ we get

$$e^{-({r_\Delta}^2+c)} = \left(1 - \frac{2M}{r_\Delta} + \frac{{Q_\Delta}^2}{{r_\Delta}^2}\right) \tag{3.21}$$

$$e^{(l{r_\Delta}^2+n)} = \left(1 - \frac{2M}{r_\Delta} + \frac{{Q_\Delta}^2}{{r_\Delta}^2}\right) \tag{3.22}$$

$$-r_\Delta\; e^{-({r_\Delta}^2+c)} = \left(\frac{M}{{r_\Delta}^2} - \frac{{Q_\Delta}^2}{{r_\Delta}^3}\right) \tag{3.23}$$

$$lr_\Delta\; e^{(l{r_\Delta}^2+n)} = \left(\frac{M}{{r_\Delta}^2} - \frac{{Q_\Delta}^2}{{r_\Delta}^3}\right) \tag{3.24}$$

4. DISSCUSSION

In the present paper we have found some solutions of Einstein-Maxwell Field equations for some charged fluid sphere. The explicit solutions of the scale factor are found via the assumption of metric potentials as $\lambda = ar^3 + br^2 + c\,, \nu = lr^2 + mr + n$. Here the metric is regular and can be matched to the Reissner-Nordstrom metric and pressure is finite. In the limit of vanishing charge, the solutions reduce to the interior solutions of an uncharged sphere. Here we get at $r = c = 0$ these results give –

$$16\pi p_0 = (4l - 1) \tag{4.1}$$

$$16\pi \rho_0 = 5 \tag{4.2}$$

$$E_0 = \left(-\frac{4}{2}\right) = -2 \tag{4.3}$$

For p_0 and ρ_0 to be positive we must have

$$4l - 1 > 0 \Rightarrow 4l > 1 \tag{4.4}$$

$$\Rightarrow l > 1/4$$

(l is +ve quantity)

Further for

$$\rho_0 \geq 3p_0 \Rightarrow 5 \geq 3\,[4l - 1]$$

$$\Rightarrow 2 \geq 3\,l \tag{4.5}$$

From condition [3.18] and [3.19] we have $3\,l \leq 2k \leq 8\,l,$

$$3\,lk \leq 2k \leq 8\,lk, \quad \text{where } K \epsilon N \tag{4.6}$$

- Reissner–Nordstrom metric at boundary $r = r_\Delta$ also calculated.
- Further for realistic model $p > 0, \rho > 0$, which will impose further restrictions on our solutions.

5. APPLICATIONS

General relativity is very important to human society. How wonderful it would be if evolutionary research could prompt novel insights and practical applications for some of human kind's for most challenges today. Such a natural science survey broadly addressing all known ordered system across all of cosmic time [36,37] and potential identifying a complexity metric of white significant from quarks to Quasars and from Microbes to minds does seemingly of a humanity sum guidance at a time of accelerating global troubles on planet earth. Some important application are given below-

* Helpful to study for stellar body and its behaviour.
* It is useful to climate application and evolution.
* Rising energy use on earth.
* The renewable sources especially solar heat energy.
* Implication for global warming.
* Machine application for cosmic evolution.
* Humans advancing and machines arising the human and machine interface.
* Implication for smart machine.
* Economic application of cosmic evolution.
* Implication of world economics etc.

In General Relativity, the test particles which are point like and neutral fall along geodesies of the Riemannian space-time V_4 of General Relativity. This behaviour can be derived from the energy-momentum law of General Relativity or from the field equations. In Einstein-Cartan theory a typical massive test mass carries spin and therefore falls neither along a straight line nor along a shortest line (geodesic). The long distance behaviour of Einstein-Cartan theory is the same as in General Relativity but the short distance behaviour is distinctly different. Adamovicz shown that in the case of the linear approximation Einstein-Cartan theory and the general relativity gives the same metrics of space-time.

➢ *Acknowledgment*

We are grateful to Prof. R.B.S. Yadav, P.G. Department of Mathematics, Magadh University, Bodh Gaya and anonymous reviewers for helping us to improve the paper.

References :

[1] *Alder, R.J., Bazin, M. and Schiffer, M.M. (1974) Introduction to general relativity (Mc-Graw Hill, N.Y.) Chapter 14 of 2nd ed.*
[2] *Bonnor, W.B. (1960) : Z. Physik, 160, [59].*
[3] *Junevicous, G.J.G. (1976) : J. Phys., A9, [2069].*
[4] *Krori, K.D. and Barua, J. (1975) : J. Phys. A. Math, 8;[508].*
[5] *Kyle, C.F. and Martein, A.W. (1967) : I I Nuovo Cim, 50; [583-604].*
[6] *Nduka, A. (1977) : Acta, Phys. Polon, B8, [75].*
[7] *Singh, T. and Yadav, R.B.S. (1978) : Acta. Phys. Polon, B9, [475].*
[8] *Z. Shi-Chang (1982) : G.R.G., 15, [4].*
[9] *Wilson, S. J. (1969) : Can. J. Phys., 47, [2401]*
[10] *Xingxiang, W. (1987) : G.R.G., 19 [729].*
[11] *Yadav, R.B.S., et. al. (2005) Proc. of Math. Soc. BHU, Vol. No. 21 [107-112].*
[12] *Yadav, R.B.S. et al. (2007) ANUSANDHAN, Vol. no. 09,[91-93].*
[13] *Maurya, S.K., Gupta, Y.K., Ray, S.: arXiv:1502.01915 [gr- qc] (2015)*
[14] *Pant, N., Maurya, S.K.: Appl. Math. Comput. 218, 8260, (2012)*
[15] *Pant, D.N., Sah, A.: J. Math. Phys. 20, 2537 (1979)*

[16] *Papapetrou, A.: Proc. R. Ir. Acad., Sci. 51, 191 (1947)*
[17] *Patel, L.K., Kopper: Aust. J. Phys. 40, 441 (1987)*
[18] *Patel, L.K., Mehta, N.P.: Aust. J. Phys. 48, 635 (1995)*
[19] *Paul, B.C., Chattopadhyay, P.K., Karmakar, S., Tikekar, R.: Mod. Phys, Lett. A 26, 575 (2011)*
[20] *Rao, J.K., Trivedi, M.M.: Pram¯ana 51, 663 (1998)*
[21] *Ratanpal,B.S.,Thomas,V.O., Pandya,D.M*
[22] *Venkateshwarlu, R., Satish, J. et.al. (2013): IJAER, LRS Bianchi-III (Trespace time. Journal), Vol. 4,No-3*
[23] *Wagoner, R. V. (1970): Phys. Rev., D11, 3209.*
[24] *Walker, A. G. (1935): Quart. J. Math. Oxford Ser., 6, 81.*
[25] *Walker, A. G. (1936): Proc. Lond. Moth. Sco., 42, 90.*
[26] *Wang, X. X. (2003): Chin Phys. Rev. phys. lett., 20, 1205.*
[27] *Weyssenhoff, J., Rabbe, A. (1947): Acta. Phys. Pol., 9, 7.*
[28] *Yadav, A.K. et.al. (2011): Bianchi type V String Cosmological models in general relativity Pramana J. of Phys., 76, 681-90.*
[29] *Yadav, A.K., Pradhan, A. and Singh A.K. (2012) : Astrophys. Space Sci., 337, 379.*
[30] *Yadav, A.K. and Yadav, L. (2010): International Journal of theoretical physics [LRS and Bianchi type-III an isotropic dark energy model with constant deacceleration parametre., Vol. -50,No.-1,Pg. No.- 218-227.*
[31] *Yadav, A. K., Saha, B. (2012): Astrophys. and Space Science. [LRS, Bianchi Type-I an isotropic cosmological model with dominance of dark energy]. Vol.- 337, No.- 2, Pg. No. - 759-765.*
[32] *Yilmaz, I. (2006): G.R.G., 38, 1397.*

[33] *Zeldovich, Ya. B., and Novikou, I. D. (1971): Relativistic Astrophys, Vol. - I, The Univ. of Chicago Press., P. 459.*
[34] *Zeldovich, Ya. B., et.al. (1975): Sov. Phys. JETP., 40, 1.*
[35] *Zeldovich, Ya. B., (1981): Sov. Phys. Usp., 24, 216.*
[36] *H., Moor, J.H., Søraker, J.H. and Steinhart, E., Eds., Singularity Hypotheses, Springer, Berlin, 413-438.* *http://dx.doi.org/10.1007/978-3-642-32560-1_20*
[37] *Intergovernmental Panel on Climate Change (IPCC) (2013) Working Group I: The Physical Science Basis of Climate Change,5th Assessment.* *https://www.ipcc.ch/report/ar5/wg1*