Effects of Viscous Dissipation and Transverse Magnetic Field on a Flow of Heat Transfer over a Non Linear Stretching Sheet under Convective Boundary Condition

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Abstract— The effect of slip condition on MHD free convective flow of non Newtonian fluid past a non-linearly stretching sheet with viscous dissipation is analyzed. The governing partial differential equations are converted into nonlinear, ordinary, and coupled differential equations and are solved by using Keller-Box method. The effects of important parameters such as magnetic parameter, slip parameter, Prandtl number, Eckert number and the local Biot number are described through graphs. The numerical results are compared with the published data and are found to be in good agreement."

Keywords— **Casson fluid, Viscous Dissipation, Slip Flow, MHD.**

I. INTRODUCTION

The interest in flows of non-Newtonian fluids with heat transfer has grown substantially in recent years because such flows are extensively used in engineering and industrial applications (nuclear fuel slurries, oil recovery, food processing, paper production, glass blowing, plastic sheet formation, and extrusion of polymeric fluids and melts). Specifically, the boundary layer flows of non-Newtonian fluids are of special importance. A few studies in this direction can be found in [1] for relevant issues related to the behavior and modelling of non-Newtonian fluids. There are many non-Newtonian fluid models, e.g., the power-law model, Ellis model, Carreau model, and Bingham model [1]. Being used to describe a shear-thickening fluid, the power-law viscosity model yields a zero effective value of the "apparent viscosity" for small shear rates. Almost all fluids possess a finite, though very small, viscosity even if the velocity shear is very small in the outer region of the flow (i.e., the region outside the boundary layer). Hence, a viscosity model that gives a constant, albeit small, value for the apparent viscosity for the shear rate tending toward zero would better conform to the behavior of a real fluid. However, little interest has been paid to such models for the fluid viscosity with these properties, though they are widely used in chemical engineering. The Casson fluid model [2] is a prominent model for many fluids such as blood, chocolate, honey, etc. Casson fluid behaves as solid when the shear stress is less than the yield stress and it starts to deform when the shear stress becomes greater than the yield stress. Mehta et al. [3] investigated the repercussion of Casson fluid under yield stress through a homogeneous porous medium bounded by a circular tube. Sandeep et al. [4] reported heat and mass transfer behavior of Casson fluid over an exponential permeable stretching surface. Detailed study on Casson nanofluid under various aspects could be found in [5–7].

MHD is the field of science which addresses the dynamics of electrically conducting fluid. The key fact behind the MHD is that the applied magnetic field induces the current, the consequences of this phenomenon produces Lorentz force which affects fluid motion significantly. The dynamics of electrically conducting fluids is mathematically formulated with well-known Navier–Stokes equations along with Maxwell equations. The nature contains variety of MHD fluids like plasmas, salt water, electrolysis etc. Currently, MHD is a topic of intense research due to its use in many industrial processes like magnetic materials processing, glass manufacturing, magneto hydrodynamics electrical power generation. Furthermore, it has applications in astrophysics and geophysics as well, e.g. it is used in solar structure, geothermal energy extraction, radio propagation etc. Whereas, MHD pumps and MHD flow meters are some products of engineering which utilized the magneto hydrodynamics phenomenon. So, magneto hydrodynamics is recently experienced a period of great enlargement and differentiation of subject matter. Alfven [8] discovered electro magneto-hydrodynamic waves in his work. Liao [9] discussed the magneto hydrodynamics power law fluid flow over continuously stretching sheet. Solution was computed by utilizing HAM. In this analysis, he described effects of magneto hydrodynamics on skin friction coefficient by considering variations in power-law index. He suggested that effects of MHD are prominent for shear thinning than shear thickening fluids. Power law fluid flow in the presence of MHD over stretching sheet was studied by Cortell [10]. In this problem, he calculated the numerical solution by way of shooting method and analyzed the model for variations in power law index and Hartmann number. He concluded that Hartmann number is a source of decrease in both Newtonian and non-Newtonian fluids velocity. Ishak et al. [11] investigated the incompressible flow of Newtonian fluid over stretching cylinder under the influence of magneto hydrodynamics. The solution was calculated numerically by applying Keller-Box method. They illustrated that MHD causes decline in velocity profile while enhances the skin friction coefficient. Nadeem et al. [12] examined the flow of MHD Casson fluid over a porous stretching sheet in three dimensions. Numerical solution of the problem was found by applying Runge–Kutta Fehlberg numerical scheme. They proved that Hartmann number reduces both horizontal and vertical velocity profiles, but it increases skin friction coefficients. Akbar et al. [13] investigated the Jeffrey fluid in small intestine under the impact of magnetic field and found exact solution. Ellahi et al. [14] discussed the blood flow of two-dimensional Prandtl fluid through taperd arties. The solution was calculated analytically and variations are found against physical parameters. Gangadhar [15] studied the effects of soret and dufour on MHD heat and mass transfer towards a vertical plate with convective condition. Rushi Kumar and Gangadhar [16] observed that an increase in wall suction increases the boundary layer thickness and decreases the skin friction. Gangadhar and Bhaskar Reddy [17] conclude that an increase in wall suction enhances the boundary layer thickness and reduce the skin friction together with the heat and mass transfer rate at the moving plate surface. Venkata Subba Rao et al. [18] investigated the unsteady boundary layer flow of Casson fluid due to transverse magnetic field with thermal radiation. Gangadhar et al. [19] studied the MHD micropolar nanofluid over a permeable stretching/shrinking sheet with Newtonian heating.

An analysis of heat transfer with viscous dissipation effects was firstly considered by Brinkman [20]. He investigated the effect of viscous heating in Newtonian fluids. Lin et al. [21] discussed the effect of viscous dissipation on thermal entrance heat transfer region in the laminar pipe flow with convective boundary conditions. The flow of incompressible viscous fluid over the porous stretching surfaces was analyzed by Anjali Devi and Ganga [22]. The influence of MHD and viscous dissipation were discussed in this problem. The solution was computed by hyper geometric functions for special case when plate is stretched linearly and concluded that influence of Eckert number is insignificant on temperature. Singh [23] formulated the problem on heat and mass transfer of MHD boundary layer flow of incompressible viscous fluid past the moving vertical porous plate. He discussed heat transfer with viscous dissipation and temperature dependent variable viscosity and solved the problem numerically.

Mabood et al. [24] analyzed the impact of viscous dissipation on MHD stagnation point flow of water based nanofluid over a flat plate. The Runge–Kutta Fehlberg integration scheme was utilized to compute solution of modeled differential equations. Gangadhar [25] investigated the radiation, heat generation viscous dissipation and magneto hydrodynamic effects on the laminar boundary layer about a flat-plate in a uniform stream of fluid (Blasius flow), and about a moving plate in a quiescent ambient fluid (Sakiadis flow) both under a convective surface boundary condition.

So far as we are aware, no attempt has ever been made to study the impact of magnetic field, viscous dissipation on free convective flow along a non - linearly stretching sheet with slip condition. In this paper, the governing partial differential equations of the flow and temperature fields are reduced to ordinary differential equations, which are then, solved numerically using Keller – Box method.

II. MATHEMATICAL FORMULATION

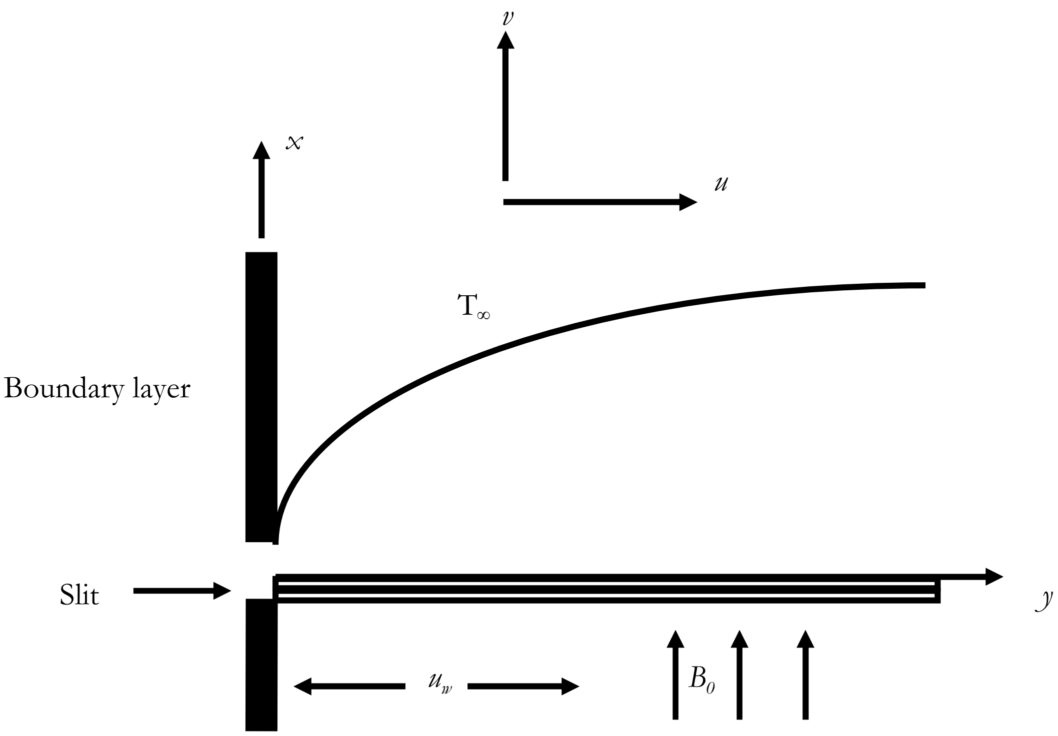

Figure 1: Schematic diagram of the problem

Consider steady two dimensional and incompressible free conventional flow of Casson fluid over a nonlinearly stretching sheet saturated in a porous medium under the influence of magnetic field. Further, the slip and Newtonian heating effects are also taken into account. The sheet is stretched in a nonlinear way along the x-axis with the velocity of $u_w(x) = cx^n$ and y-direction is taken for fluid flow with the origin fixed (see Fig.1).

Here c is constant and n $(n \geq 0)$ is the nonlinear stretching sheet parameter, n=1 represents the linear sheet case and $n \neq 1$ is for nonlinear case. A non-uniform transverse magnetic field of strength B_0 is applied normal to the sheet. The ambient fluid temperature is denoted by T_∞. The induced magnetic field is neglected due to the small magnetic Reynolds number. Under these assumptions the rheological equation for incompressible flow of Casson fluid is given by (see Bhattacharya 2013; Sharada and Shankar, 2015)

$$\tau_{ij} = \begin{pmatrix} 2(\mu_\beta + p_y/\sqrt{2\pi})e_{ij}, & \pi > \pi_c \\ 2(\mu_\beta + p_y/\sqrt{2\pi_c})e_{ij}, & \pi < \pi_c \end{pmatrix}$$

Here $\pi = e_{ij}e_{ij}$ and e_{ij} is the (i,j) – th component of the deformation rate, π is the product of the component of deformation rate with itself, π_c is the critical value of this product based on the non-Newtonian model, μ_B is the plastic dynamic viscosity of the non-Newtonian fluid and p_y is the yield stress of the fluid. The governing equations of continuity, momentum and energy are given by

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 0\ , \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\left(1+\frac{1}{\beta}\right)\frac{\partial^2 u}{\partial y^2} - \frac{\sigma B^2 x}{\rho}u \tag{2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} + \frac{\upsilon}{c_p}.\left(\frac{\partial u}{\partial y}\right)^2 \tag{3}$$

In the above equations *u* and *v* denote the velocity component in *x* and *y* directions respectively, ρ is the fluid density, ν is kinematic viscosity, β is the Casson fluid parameter, σ is the electrically conductivity of the fluid, $B(x) = B_0 x^{\frac{n-1}{2}}$ is the magnetic field with constant magnetic strength B_0, *T* is the fluid temperature, *a* is the thermal diffusivity of the Casson fluid.

The corresponding boundary conditions for the problem can be written as follows:

$$u = cx^n + N_1\nu\left(1+\frac{1}{\beta}\right)\frac{\partial u}{\partial y},\ \ v = 0,\ \ T = T_w\ \ at\ y = 0, \tag{4}$$

$$u \to 0,\ T \to T_\infty\ \ as\ \ \ y \to \infty \tag{5}$$

Here $N_1(x) = Nx^{\frac{n-1}{2}}$ denotes velocity of slip factor depends on *x*.

We introduce the following similarity transformations:

$$\psi = \sqrt{\frac{2\nu c x^{n+1}}{n+1}}f(\eta),\ \ \ \eta = \sqrt{\frac{(n+1)cx^{n-1}}{2\nu}}y,\ \ \ \ \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty} \tag{6}$$

where the stream function ψ is defined by the following relations

$$u = \frac{\partial\psi}{\partial y},\ \ v = -\frac{\partial\psi}{\partial x}. \tag{7}$$

The above expression also satisfies the continuity equation (1). From equations (2) – (7), one arrives at the following non-dimensional systems:

$$\left(1+\frac{1}{\beta}\right)f''' + ff'' - \frac{2n}{n+1}f'^2 - Mf' = 0 \tag{8}$$

$$\theta''+\Pr f\theta'+Ec\Pr f''^2=0 \tag{9}$$

The corresponding transformed boundary conditions are:

$$f(0)=0,\ f'(0)=1+\lambda\left(1+\frac{1}{\beta}\right)f''(0),\ \theta(0)=1\ , \tag{10}$$

$$f'(\infty)=0,\ \theta(\infty)=0 \tag{11}$$

Where prime denote derivatives with respect to η and the parameters are defined as: magnetic parameter, $M=\dfrac{2\sigma B_0^2}{\rho c(n+1)}$, Prandtl number $\Pr=\dfrac{\nu}{\alpha}$, slip parameter $\lambda=N\sqrt{\dfrac{(n+1)c\nu}{2}}$ and Eckert number $Ec=\dfrac{u_w^2}{c_p(T_w-T_\infty)}$

The parameters with physical interest are the skin friction coefficient Cf_x and Nusselt number Nu_x and are defined as follows

$$Cf_x=\frac{\tau w}{\rho u_w^2},\quad Nu_x=\frac{xq_w}{\alpha(T_w-T_\infty)}, \tag{12}$$

where τ_w and q_w are the wall skin friction and heat flux defined by :

$$\tau_w=\mu_B\left(1+\frac{1}{\beta}\right)\left[\frac{\partial u}{\partial y}\right]_{y=0}\ ,\qquad q_w=-\alpha\left(\frac{\partial T}{\partial y}\right)_{y=0}. \tag{13}$$

Using equation (6) in equation (13), we get the following non-dimensional skin friction coefficient and local Nusselt number

$$(\mathrm{Re})^{1/2}Cf_x\sqrt{\frac{2}{n+1}}=\left(1+\frac{1}{\beta}\right)f''(0)\ ,\quad (\mathrm{Re})^{-1/2}Nu_x\sqrt{\frac{2}{n+1}}=-\theta'(0). \tag{14}$$

III. SOLUTION OF THE PROBLEM

As Equations (8)-(9) are nonlinear, it is not possible to obtain the closed form solutions. as a result, the equations through the boundary conditions (10) & (11) are solved numerically by means of a finite-difference scheme recognized as the Keller-box method. The major steps in the Keller-box method to obtain the numerical solutions are the following:

i). Decrease the specified ODEs to a system of first order equations.
ii). write down the condensed ODEs to finite differences.
iii). Linearized the algebraic equations by using Newton's method and write down them in vector form.
iv). Solve the linear system through the block tridiagonal elimination technique.

One of the factors so as to be affecting the correctness of the method is the suitability of the preliminary guesses. The accurateness of the method depends on the alternative of the preliminary guesses. The choices of the primary guesses depend on the convergence criteria and the boundary conditions (10) & (11). The subsequent primary guesses are chosen

$$f_0(\eta) = \frac{1}{1+\lambda(1+1/\beta)}\left(1-e^{-\eta}\right),\ p_0(\eta) = e^{-\eta}, \theta_0(\eta) = e^{-\eta}$$

In this study, a consistent grid of size $\Delta\eta$ = 0.006 is found to be convince the convergence and the solutions are obtained through an error of tolerance 10^{-5} in all cases. In our study, this gives regarding six decimal places perfect to the majority of the agreed quantities.

IV. RESULTS AND DISCUSSIONS

p Numerical computation was carried out for several non-dimensional parameters, namely magnetic parameter *M*, slip parameter λ, Casson parameter β, non linear stretching parameter *n*, Eckert number *Ec*, and the Prandtl number *Pr*. Figures 2-9 have plotted to demonstrate the effect of parameters on the flow field and heat transfer characteristics.

Figure 2 is plotted to study dimensionless velocity for different values of Casson fluid parameter β. It is observed that velocity decreases with an increase in β. Physically, with increase in β the fluid become more viscous and in result the fluid velocity reduces. Also, the momentum boundary layer thickness decreases as β increases. Further, as $\beta \to \infty$ the present phenomenon reduces to Newtonian. Figure 3 demonstrates the variations of β on temperature profile. It is found that increasing values of β enhances the fluid temperature in the boundary layer.

Figure 4 plotted for the influence of magnetic field parameter M on non – dimensional velocity profiles. It is evident that the figure that a raise in magnetic field parameter decelerates the velocity profile. This is due to fact that an enhance in the magnetic field parameter develops the reverse force to the flow, is called Lorentz flow. This force has propensity to reduce the velocity boundary layer. Figure 5 exhibit the effects of magnetic field parameter on dimensionless temperature profile. It is noticed that fluid temperature increases with increasing values of *M*.

Figure 6 shows the variation of λ on velocity profile. It is found that velocity is a decreasing function of λ. The reduction in momentum boundary layer thickness is also observed. It is because the momentum provided by stretching sheet is partly transmitted to the Casson fluid under the velocity-slip boundary condition. The opposite nature of temperature is noticed for the effect of λ as shown in Figure 7. It can be seen from figure 8 that the non- dimensional temperature increases with an increase in *Ec*. From figure 9, it is observed that temperature profiles decreases with higher values of *Pr*. Since the Prandtl number represents the ratio of momentum diffusivity to thermal diffusivity as an increase in the Prandtl number decreases the thermal boundary layer of the cylinder. Therefore the figures show that an increase in Pandtl number *Pr*, reduces in the temperature profile at a given point of flow parameter.

In order to regulate the method used in the present study and to make a decision the accuracy of the present analysis and to compare with the results available (Cortell [26]) relating to the local skin-friction coefficient and found in an agreement (see table 1). From the Table 2 that skin friction coefficient increases with increase in magnetic and non -linearly stretching parameters. The skin friction coefficient decreases with higher values of Casson fluid and slip parameters this is observed from Table 2. It can be seen directly forwardly from Table 3 that the magnitude of local Nusselt number increases with increasing values of the Prandtl number. Further, the magnitude of Nusselt number is a decreasing function of β, *M*, n, λ and *Ec*.

V. CONCLUSIONS

In the present work, the MHD free convective boundary layer flow and heat transfer over a non-linearly stretching sheet with viscous dissipation under slip condition have been investigated. The present results are in good concurrence with those reported in open literature for some special cases. From the study, the following remarks can be summarized."

1. Velocity profile decreases with the increase in β, λ and *M*.
2. Temperature profile increases with the increase in β, λ, *M* and *Ec*.
3. Temperature profile decreases with the increase in *Pr*.
4. Local skin friction coefficient decreases with the increase in β and λ.
5. Local skin friction coefficient increases with the increase in *M* and *n*.
6. Local Nusselt number decreases with the increase in β, M, n, λ and *Ec*.
7. Local skin friction coefficient increases with the increase in *Pr*.

GRAPHS:

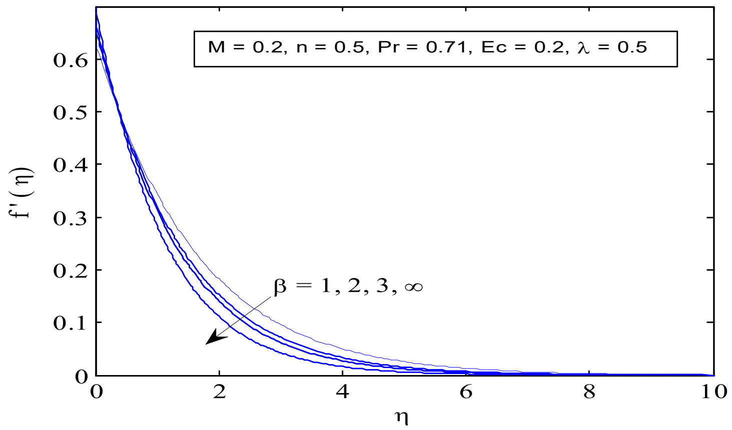

Fig. 2 Velocity profile for different values of β.

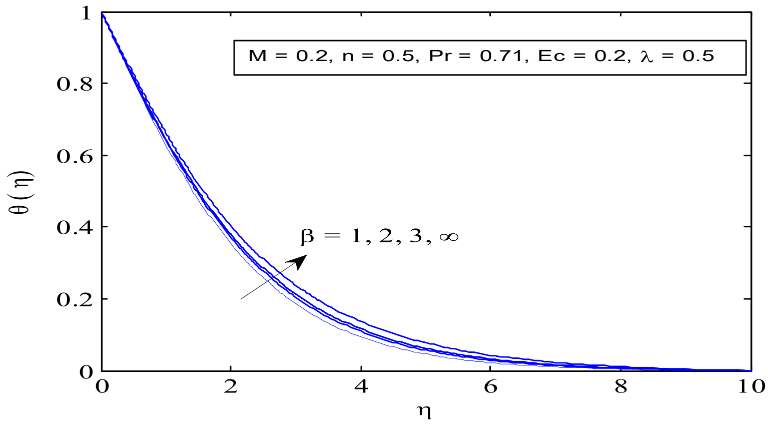

Fig. 3 Temperature profile for different values of β

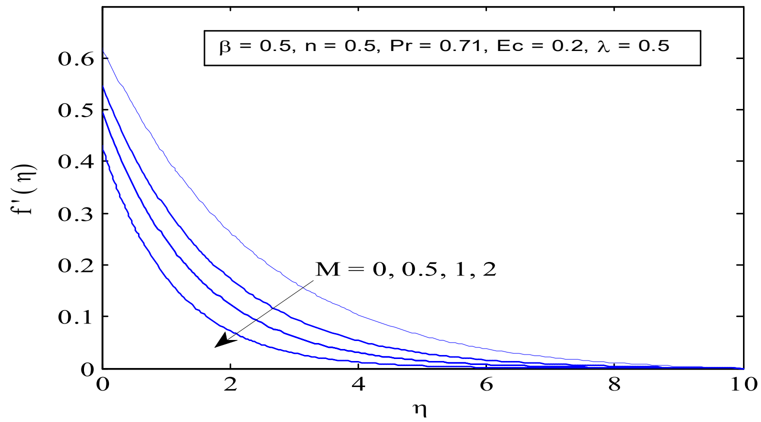

Fig. 4 Velocity profile for different values of M.

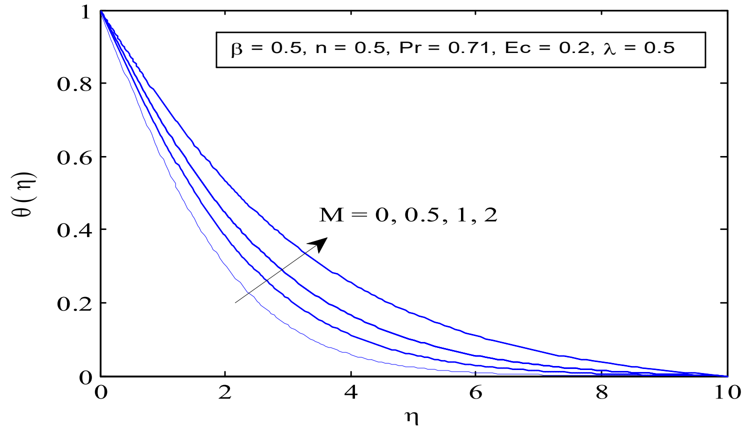

Fig. 5 Temperature profile for different values of M.

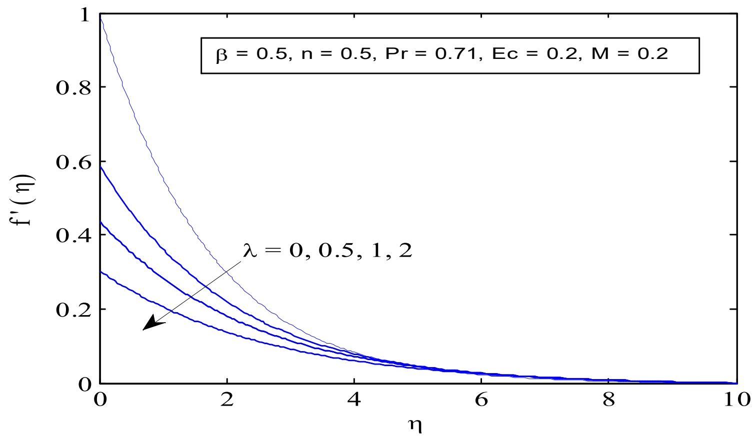

Fig. 6 Velocity profile for different values of λ.

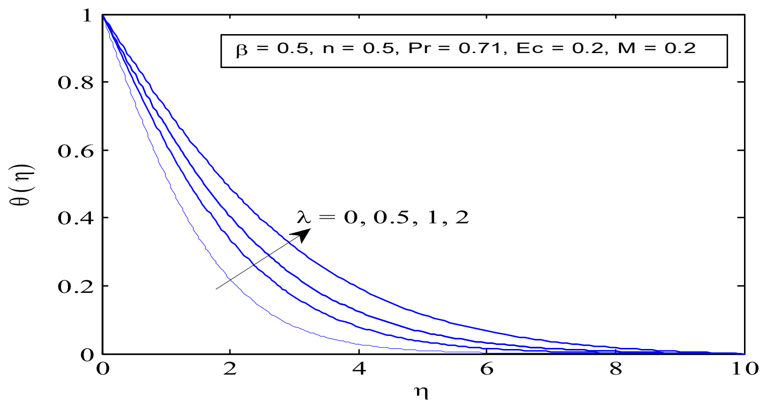

Fig. 7 Temperature profile for different values of λ.

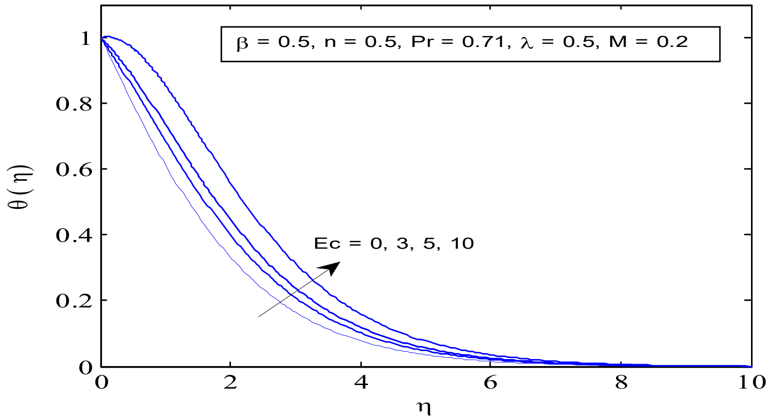

Fig. 8 Temperature profile for different values of Ec.

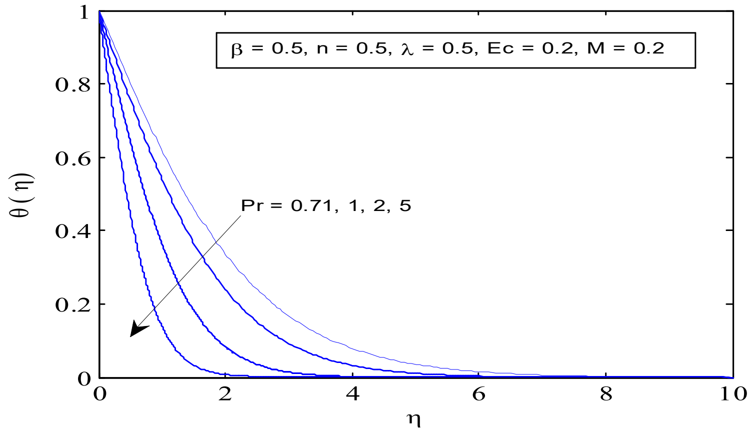

Fig. 9 Temperature profile for different values of Pr.

TABLES:

Table: 1

comparison of $-f''(0)$ for various values of n with M = λ =0 and $\beta = 10^8$

n	Cortell [26]	Present Results
0	0.627547	0.6276
0.2	0.766758	0.7668
0.5	0.889477	0.8896
1	1.0	1.0000
3	1.148588	1.1486
10	1.234875	1.2349
100	1.276768	1.2768

Table: 2

Skin friction coefficient for different values of β, M & λ.

β	M	n	λ	$-\left(1+\frac{1}{\beta}\right)f''(0)$
0.5	0.2	0.5	0.5	0.8269
1				0.7419
1.5				0.7046
2				0.6833
	0.4			0.8844
	0.6			0.9326
	0.8			0.9739
		1		0.8634
		1.5		0.8826
		2		0.8944
			1	0.5614
			1.5	0.4289
			2	0.3484

Table: 3

Local Nusselt number for different values of $\beta, M, Ec, \Pr$ & λ.

β	M	n	λ	Pr	Ec	$-\theta'(0)$
0.5	0.2	0.5	0.5	0.71	0.2	0.4006
1						0.3874
1.5						0.3787
2						0.3728
	0.4					0.3755
	0.6					0.3537
	0.8					0.3346
		1				0.3879
		1.5				0.3811
		2				0.3769
			1			0.3458
			1.5			0.3105
			2			0.2852
				2		0.7384
				3		0.9246
				5		1.2180
					2	0.3173
					3	0.2710
					5	0.1784

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