A Comparative Study between Various Approaches On MCDM Models

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Abstract: We often face decision making problems in real life which are usually complex and ill structured to be considered through the examination of multi-criteria that will lead to an optimum decision. This paper presents a study on Dyslexia and deals with the problem of selecting suitable indoor game treatment. Here, a comparative study is done between fuzzy soft set, weighted sum model (WSM), fuzzy matrix in decision making and newly proposed method.

Keywords: Dyslexia, Fuzzy soft set, Weighted sum model (WSM), Fuzzy matrix in decision making.

1. INTRODUCTION

Disease is often constructed as a medical condition associated with specific symptoms and signs. Dyslexia is a developmental disorder which was not clear to researchers until the 20th century. In 1896 it became categorized as a learning disability and it has been debated for many years (Snowing et al, 2003). According to many different forms and ways Dyslexia has been defined. According to the US National Institutes of Health, it is a learning disability that will make child to struggle in reading, writing, pronouncing and sometimes in speaking. In the 1980's, the National Institute of Child Health and Human Development (NICHD) says if a child struggles with reading could not be explained by low intelligence, poor eye sight, inadequate educational opportunities, poor hearing, and then that child must be affected by the disease dyslexia. The facts of life for all children aregrowing, developing and learning. Each and every day children were facing many new concepts and various challenges. We can't imagine for a child who faces not only new challenges in life, but also to face such challenges while living with a learning disability.In this paper we dealt with one of the suitable indoor game treatments for dyslexia. Here, we have done a comparative study among fuzzy soft set, weighted sum model, fuzzy matrix in decision making, and newly proposed method. Molodtsovproposed fuzzy soft set in 1999[4]and its properties were studied in the work of Maji et al. (2001)[10].It is useful for solving the day to day problems in easiest way and this method helps to take decision making in a critical situation. Shimura discussed about Fuzzy sets concept in rank ordering objects in 1973 [6].In decision theory, the weighted sum model (WSM) is the simplest multi-criteria decision making (MCDM) method for evaluating a number of alternatives in terms of a number of decision criteria.

This paper is organized as follows: Following the introduction, Section 2 gives an overview of basic definitions on fuzzy algebra with two binary operations. In Section 3, we give a brief study on Dyslexia.Section 4 contains problem procedure for fuzzy soft set, WSM, fuzzy matrix in decision making. In Section 5, we introduced a new methodology for multi-criteria decision making (MCDM) and Section 6 deals with the application. In Section 7, Comparison between different approaches is given. And finally, Section 8 has conclusion.

2. FUZZY ALGEBRA WITH TWO BINARY OPERATIONS

The basic definitions involved in this study are taken from Meenakshi A.R.,(2008)[13].

Definition 2.1

A fuzzy algebra is a mathematical system with two binary operations +,'.' Defined on a set F satisfying the following properties:

1) **Idempotence:** $a + a = a; a \,.\, a = a$
2) **Commutativity:** $a + b = b + a; a \,.\, b = b \,.\, a$
3) **Associativity:** $a + (b + c) = (a + b) + c; a \,.\, (b \,.\, c) = (a \,.\, b) \,.\, c$
4) **Absorption:** $a + (a \,.\, b) = a; a \,.\, (a + b) = a$
5) **Distributivity:** $a \,.\, (b + c) = (a \,.\, b) + (a \,.\, c); a + (b \,.\, c) = (a + b) \,.\, (a + c)$
6) **Universal bounds:** $a + 0 = a; a + 1 = 1; a \,.\, 0 = 0; a \,.\, 1 = a$

Definition 2.2

Let $A = (a_{ij}) \in \mathfrak{F}_{mn}$ and $B = (b_{ij}) \in \mathfrak{F}_{mn}$. Then the matrix $A + B = (\sup\{a_{ij}, b_{ij}\}) \in \mathfrak{F}_{mn}$ is called the sum of A and B.

Definition 2.3

Let $A = (a_{ij}) \in \mathfrak{F}_{mn}$ and $B = (b_{ij}) \in \mathfrak{F}_{mn}$. Then the matrix $A \odot B = (\inf\{a_{ij}, b_{ij}\}) \in \mathfrak{F}_{mn}$ is called the multiplication of A and B.

3. CASE STUDY

Dyslexia is a specific learning disability in reading. People get dyslexia from trauma or stroke after birth. This will cause the brain cells to die.He/she who affected with dyslexia will have trouble in reading accurately and fluently. Those maystruggle to read comprehension, spell and also to write.Comparing to girls it is found more in boys. It doesn't just affect learning. This disease can impact everyday activities and skills, as well. These include social interaction, memory and dealing with stress.

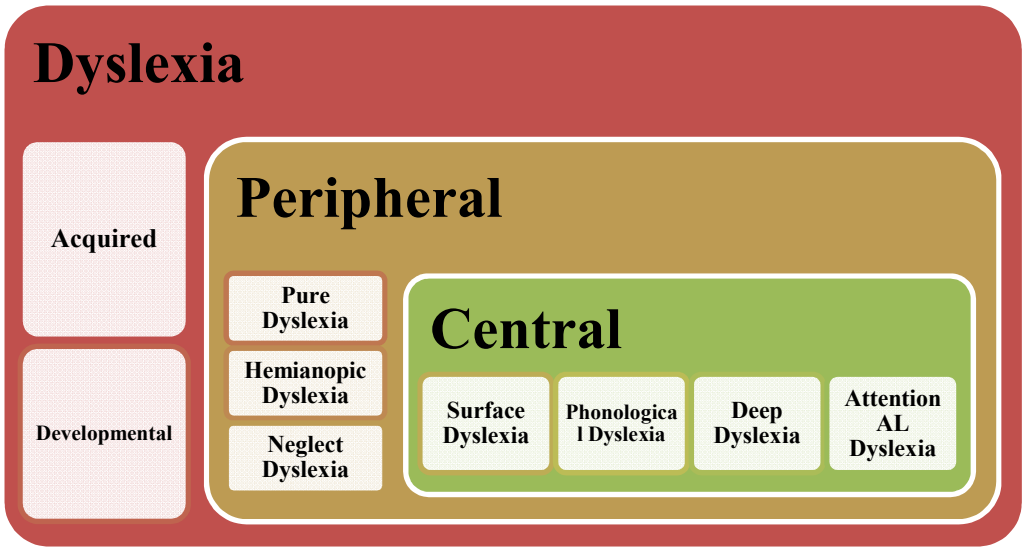

Figure 1 Classification of Dyslexia

- **Central Dyslexia**

Surface Dyslexia: Regular pronunciations are read easily. Example: Ball, but those irregular pronunciations are difficult. Example: ISLAND.

Phonological Dyslexia: He/she can easily read familiar words, but they feel very difficult to read unfamiliar words.

Deep Dyslexia: It is combination of Semnatic Dyslexia **(**he/she can't correlate the meaning of the world, which they spell or read) and Phonological Dyslexia(he/she can read the word, but says unrelated meaning instead of the actual meaning).

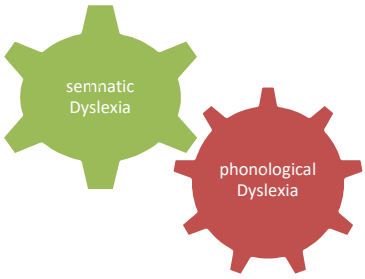

Figure 2

Attention AL Dyslexia: He/she can't read blended words, but can read the same word when split into halves.

- **Peripheral Dyslexia**

Pure Dyslexia: He/she unable recognizes the written letters in a sequence.
Hemianopic Dyslexia: It will damage the visual cortex. Hence, visual field will loss and that he/she will beslow reader.
Neglect Dyslexia: He/she will leave some part of word or beginning of word is left or wrongly spelled.

Other features

- Learning disability.
- No sex predilection; but frequently in male.
- When diagnosed in childhood, then the prognosis outcome is good.
- He/she need social support.
- Poor memory capacity.
- Depression.

Signs of dyslexia will differ by age to age. Here are some examples of signs of dyslexia are given in figure 3.

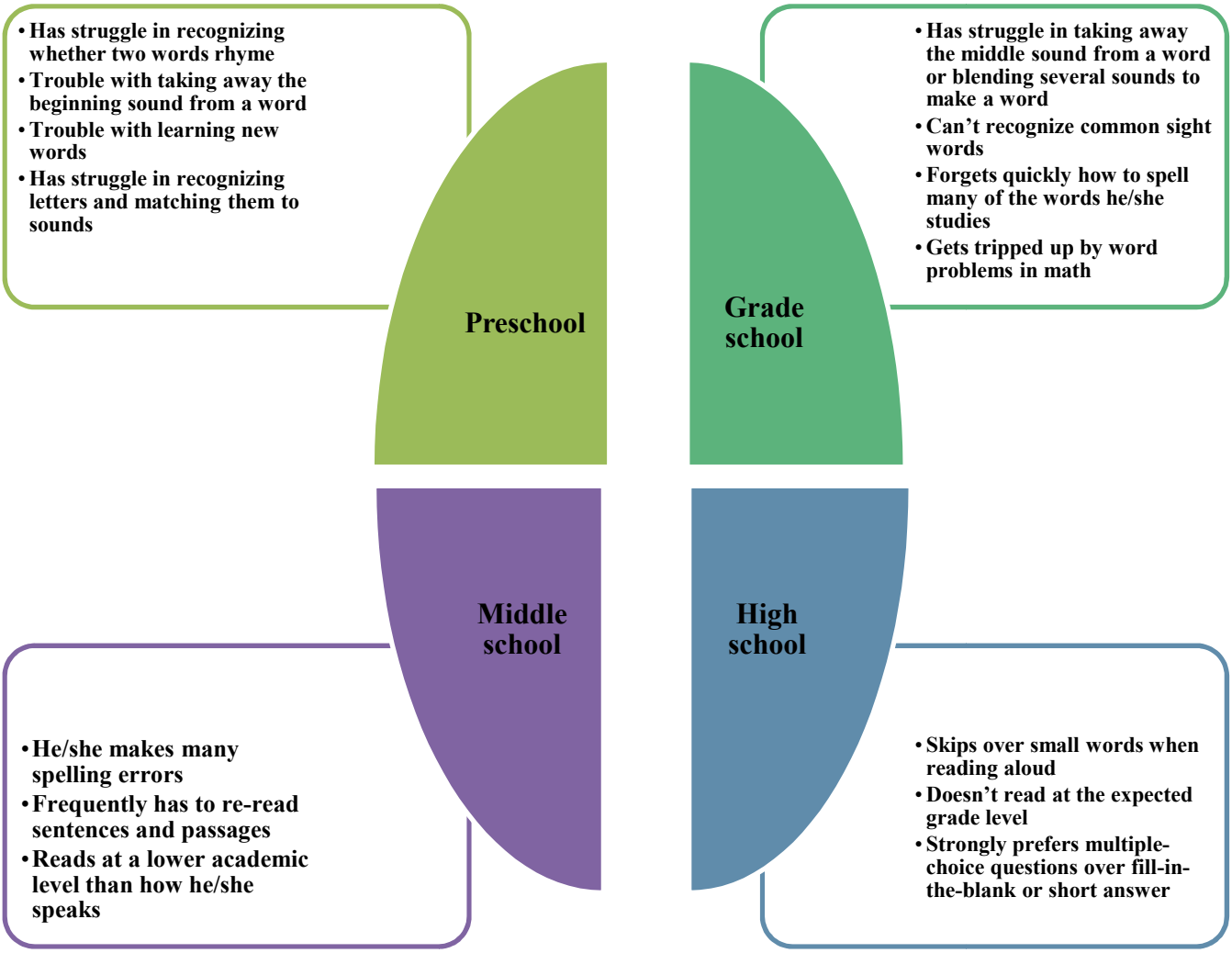

Figure 3: Signs of Dyslexia for different ages

For dyslexia there is no curative treatment, but we can reduce the symptoms. One of the way is to reduce the symptoms of dyslexia is playing indoor games. Indoor games can be one of the best ways to keep mind healthy and active. It will creates happiness and reduce stress, an opportunity to spend time together, memory formation and cognitive skills, improves immune system, coordination and so on. Playing indoor games is one of an excellent treatment for dyslexia. Every indoor game has different rules and assists in different ways in the development of kid's mind. Indoor games will help to increase the mental power of the child and considered to be the best mind exercise.In this paper dealt with one of the suitable indoor game treatments for dyslexia.The criteria and alternatives to defineone of the suitable indoor game treatments for dyslexia are created using a simple questionnaire. The questionnaire answered by the expertshelped in defining the following problem.The alternatives are Video games (g_1), Table tennis (g_2), Chess (g_3), Snooker (g_4), and Bowling (g_5). The criteria are Co-ordination (x_1), IQ (x_2), Brain exercise (x_3), Muscle toning (x_4), Fun (x_5), Memory power (x_6), Multitasking skill (x_7), Reading skill (x_8).

4. FUZZY SOFT SET, WEIGHTED SUM MODEL (WSM) AND FUZZY MATRIX IN DECISION MAKING

4.1 Problem procedure for fuzzy soft set

1. Input the performance evaluation of the indoor games by experts as matrices.
2. Find the average of the corresponding entries of all the matrices.
3. Multiply the weightage of the selection criteria of the person to the corresponding entries of each row to get the comprehensive decision matrix(in this problem we have given the weightage for each criteria as 0.07, 0.09, 0.2, 0.1, 0.05, 0.18, 0.19, 0.12).
4. Formulate the comparison table.
5. Find the row-sum and column-sum of the comparison table.
6. Obtain the score for each indoor games and the indoor game with maximum score is recommended as one of the suitable indoor game treatment for dyslexia.

4.2 Problem procedure forWeighted Sum Model (WSM)

1. Input the performance evaluation of the indoor games by experts as matrices.
2. Find the average of the corresponding entries of all the matrices.
3. Assume that all the criteria are benefit criteria.
4. Then, the total importance of alternative G_i, denoted as $G_i^{\text{WSM-score}}$, is defined as follows:

$$G_i^{\text{WSM-score}} = \sum_{j=1}^{n} w_j \quad a_{ij}, \text{ for I = 1, 2, 3,…,m.}$$

Where, w_j is the relative weight of importance of the criterion X_j and a_{ij} is the performance value of alternative G_i when it is evaluated in terms of criterion X_j.

5. For the maximization case, the best alternative is the one that yields the maximum total performance value.

4.3 Problem procedure forfuzzy matrix in decision making

1. Input the performance evaluation of the indoor games by experts as matrices.
2. Find the average of the corresponding entries of all the matrices.
3. Multiply the weightage of the selection criteria of the person to the corresponding entries of each row to get the comprehensive decision matrix.
4. Then, formulate the pair wise function.
5. Using relativity function we find the comparison matrix and here, the suitable indoor game treatment for dyslexia is the one that yields the minimum value.

5.New Approach Using Multi-Criteria Decision Making Method For Selecting Suitable Indoor Game Treatment For Dyslexia

1. Input the performance evaluation of the indoor games by experts as matrices.
2. Find the supremum (sup)/infimum (inf) of the corresponding entries of the matrices. If the goals need not to be supremum or infimum, then find average of the corresponding entries of the matrices say R. then, find R^T.
3. Then, apply the condition if$a_{ij} \geq 0.5$ = 1 and if$a_{ij} < 0.5$= 0 and find row-sum.Then, the indoor game with maximum row-sum is recommended as one of the suitable indoor game treatment for dyslexia.

Mathematical modelling of the problem

Suppose there are m alternatives (indoor games) G = $\{g_1, g_2, \dots, g_q\}$ andan experts have taken some selection criteria as X = $\{x_1, x_2, \dots, x_r\}$ for preference evaluation of the indoor games. The performance evaluation is expressed as fuzzy soft set, where F: X→P(G), for each decision makers.

From an expert 1 (D_1)

$$(D_1, X) = \begin{array}{c} \\ u_1 \\ u_2 \\ u_3 \\ \vdots \\ u_q \end{array} \begin{array}{c} \begin{array}{ccccc} x_1 & x_2 & x_3 & \dots & x_r \end{array} \\ \begin{bmatrix} a_{11}^{(1)} & a_{12}^{(1)} & a_{13}^{(1)} & \cdots & a_{1r}^{(1)} \\ a_{21}^{(1)} & a_{22}^{(1)} & a_{23}^{(1)} & \cdots & a_{2r}^{(1)} \\ a_{31}^{(1)} & a_{32}^{(1)} & a_{33}^{(1)} & \cdots & a_{3r}^{(1)} \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ a_{q1}^{(1)} & a_{q2}^{(1)} & a_{q3}^{(1)} & \cdots & a_{qr}^{(1)} \end{bmatrix} \end{array}$$

From an expert 2 $(D_2$

$$(D_2, X) = \begin{array}{c} \\ u_1 \\ u_2 \\ u_3 \\ \vdots \\ u_q \end{array} \begin{array}{c} \begin{array}{ccccc} x_1 & x_2 & x_3 & \dots & x_r \end{array} \\ \begin{bmatrix} a_{11}^{(2)} & a_{12}^{(2)} & a_{13}^{(2)} & \cdots & a_{1r}^{(2)} \\ a_{21}^{(2)} & a_{22}^{(2)} & a_{23}^{(2)} & \cdots & a_{2r}^{(2)} \\ a_{31}^{(2)} & a_{32}^{(2)} & a_{33}^{(2)} & \cdots & a_{3r}^{(2)} \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ a_{q1}^{(2)} & a_{q2}^{(2)} & a_{q3}^{(2)} & \cdots & a_{qr}^{(2)} \end{bmatrix} \end{array}$$ And so on.

According to the goal taking the sup/inf/avg of all the above matrices we get the evaluation matrix as

$$\text{R}= \begin{array}{c} \\ u_1 \\ u_2 \\ u_3 \\ \vdots \\ u_q \end{array} \begin{array}{c} \begin{array}{ccccc} x_1 & x_2 & x_3 & \dots & x_r \end{array} \\ \begin{bmatrix} \overline{a}_{11} & \overline{a}_{12} & \overline{a}_{13} & \dots \overline{a}_{1r} \\ \overline{a}_{21} & \overline{a}_{22} & \overline{a}_{23} & \dots \overline{a}_{2r} \\ \overline{a}_{31} & \overline{a}_{32} & \overline{a}_{33} & \dots \overline{a}_{3r} \\ \vdots & \vdots & \vdots & \dots \vdots \\ \overline{a}_{q1} & \overline{a}_{q2} & \overline{a}_{q3} & \dots \overline{a}_{qr} \end{bmatrix} \end{array}$$

Where$\overline{a}_{ij} = \sup\left(a_{ij}^k\right)/ \inf\left(a_{ij}^k\right)/$ avg $(a_{ij}^k) i = 1,2, \dots, q \text{ and } j = 1,2, \dots r$.

Now, find R^T. If $a_{ij} \geq 0.5$ then, give value as 1 and if $a_{ij} < 0.5$ then, give value as 0 and find row-sum. Let r_i^s denoted row-sum of indoor games and is calculated by using the formula

$$r_i^s = \sum_{j=1}^{r} a_{ij}$$

Then, the indoor game with maximum row-sum is recommended as the suitable indoor game treatment for dyslexia.

6. Application for Selecting Suitable Indoor Game Treatment for Dyslexia

The model proposed in this paper is intended to select asuitable indoor game treatment for dyslexia in fuzzy space.

Notation	Indoor games
g_1	Video games
g_2	Table tennis
g_3	Chess
g_4	Snooker
g_5	Bowling

Table 1: Notations for the Alternatives

Anexperts considers 5indoor games given in Table.1, and that satisfies the criteria$x_1 = Co-ordination,\ \ x_2 = IQ,\ \ x_3 = Brain\ exercise,\ \ x_4 = Muscle\ toning,\ \ x_5 = fun$ $x_6 = Memory\ power,\ \ x_7 = Multitasking\ skill,\ \ x_8 = Reading\ skill$with conditions given in Table.2

Notation	Criteria	Conditions
$\mathbf{x_1}$	Co − ordination	Avg
$\mathbf{x_2}$	IQ	Avg
$\mathbf{x_3}$	Brain exercise	Sup
$\mathbf{x_4}$	Muscle toning	Sup
$\mathbf{x_5}$	Fun	Inf
$\mathbf{x_6}$	Memory power	Sup
$\mathbf{x_7}$	Multitasking skill	Sup
$\mathbf{x_8}$	Reading skill	Sup

Table 2: Weightage for Criteria

An experts provided the information about the suitable indoor game treatment for dyslexiaconsidering$x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8$. The information provided by an expertsforms the fuzzy soft sets$(D_1,\ \ X), (D_2,\ \ X)$ and$(D_3,\ \ X)$over G.

$$(D_1,\ \ X) = \begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \\ x_8 \end{array} \begin{array}{c} \begin{array}{ccccc} g_1 & g_2 & g_3 & g_4 & g_5 \end{array} \\ \begin{bmatrix} 0.4 & 0.65 & 0.6 & 0.45 & 0.5 \\ 0.3 & 0.43 & 0.9 & 0.5 & 0.6 \\ 0.55 & 0.5 & 0.8 & 0.6 & 0.7 \\ 0.4 & 0.8 & 0.7 & 0.73 & 0.75 \\ 0.8 & 0.63 & 0.6 & 0.75 & 0.7 \\ 0.7 & 0.55 & 0.9 & 0.6 & 0.57 \\ 0.4 & 0.75 & 0.8 & 0.5 & 0.52 \\ 0.65 & 0.3 & 0.7 & 0.42 & 0.31 \end{bmatrix} \end{array}$$

$$(D_2,\ \ X) = \begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \\ x_8 \end{array} \begin{array}{c} \begin{array}{ccccc} g_1 & g_2 & g_3 & g_4 & g_5 \end{array} \\ \begin{bmatrix} 0.5 & 0.65 & 0.7 & 0.45 & 0.6 \\ 0.2 & 0.4 & 0.9 & 0.45 & 0.5 \\ 0.6 & 0.55 & 0.9 & 0.62 & 0.7 \\ 0.3 & 0.9 & 0.8 & 0.7 & 0.75 \\ 0.8 & 0.62 & 0.6 & 0.77 & 0.7 \\ 0.71 & 0.4 & 0.9 & 0.55 & 0.6 \\ 0.45 & 0.7 & 0.75 & 0.5 & 0.6 \\ 0.6 & 0.2 & 0.72 & 0.4 & 0.3 \end{bmatrix} \end{array}$$

$$(D_3,\ \ X) = \begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \\ x_8 \end{array} \begin{array}{c} \begin{array}{ccccc} g_1 & g_2 & g_3 & g_4 & g_5 \end{array} \\ \begin{bmatrix} 0.52 & 0.7 & 0.65 & 0.5 & 0.55 \\ 0.2 & 0.47 & 0.92 & 0.51 & 0.57 \\ 0.5 & 0.6 & 0.9 & 0.65 & 0.77 \\ 0.25 & 0.82 & 0.8 & 0.7 & 0.72 \\ 0.8 & 0.65 & 0.6 & 0.77 & 0.75 \\ 0.7 & 0.5 & 0.9 & 0.6 & 0.67 \\ 0.4 & 0.73 & 0.7 & 0.55 & 0.67 \\ 0.6 & 0.21 & 0.7 & 0.47 & 0.31 \end{bmatrix} \end{array}$$

Now setting the goal of the above three fuzzy soft sets we get the performance evaluation matrix.

$$\begin{array}{ccccc} g_1 & g_2 & g_3 & g_4 & g_5 \end{array}$$

$$R = \begin{matrix} x_1(Avg) \\ x_2(Avg) \\ x_3(Sup) \\ x_4(Sup) \\ x_5(inf) \\ x_6(Sup) \\ x_7(Sup) \\ x_8(Sup) \end{matrix} \begin{bmatrix} 0.473 & 0.67 & 0.65 & 0.47 & 0.55 \\ 0.23 & 0.43 & 0.91 & 0.49 & 0.56 \\ 0.6 & 0.6 & 0.9 & 0.65 & 0.77 \\ 0.4 & 0.9 & 0.8 & 0.73 & 0.75 \\ 0.8 & 0.62 & 0.6 & 0.75 & 0.7 \\ 0.71 & 0.55 & 0.9 & 0.6 & 0.67 \\ 0.45 & 0.75 & 0.9 & 0.6 & 0.67 \\ 0.65 & 0.3 & 0.72 & 0.47 & 0.31 \end{bmatrix}$$

$$R^T = \begin{matrix} & x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & x_7 & x_8 \\ g_1 \\ g_2 \\ g_3 \\ g_4 \\ g_5 \end{matrix} \begin{bmatrix} 0.473 & 0.23 & 0.6 & 0.4 & 0.8 & 0.71 & 0.45 & 0.65 \\ 0.67 & 0.43 & 0.6 & 0.9 & 0.62 & 0.55 & 0.75 & 0.3 \\ 0.65 & 0.91 & 0.9 & 0.8 & 0.6 & 0.9 & 0.9 & 0.72 \\ 0.47 & 0.49 & 0.65 & 0.73 & 0.75 & 0.6 & 0.6 & 0.47 \\ 0.55 & 0.56 & 0.77 & 0.75 & 0.7 & 0.67 & 0.67 & 0.31 \end{bmatrix}$$

Then, apply the condition if $a_{ij} \geq 0.5 = 1$ and if $a_{ij} < 0.5 = 0$ and find row-sum.

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	Row-Sum
g_1	0	0	1	0	1	1	0	1	4
g_2	1	0	1	1	1	1	1	0	6
g_3	1	1	1	1	1	1	1	1	8
g_4	0	0	1	1	1	1	1	0	5
g_5	1	1	1	1	1	1	1	0	7

Table 3

From the table 3 the indoor game with maximum row-sum is

g_3 i.e., Chess. Hence, Chess is recommended as one of the suitable indoor game treatment for dyslexia.

7. Comparison between different approaches

Indoor games	Fuzzy soft set		WSM		Fuzzy matrix		Proposed method	
	Final value	Rank	Final value	Rank	Final value	Rank	Final value	Rank
g_1	-10	5	0.5158	5	1	5	4	5
g_2	-5	3	0.565	3	0.8	4	6	3
g_3	20	1	0.7981	1	0.143	1	8	1
g_4	-7	4	0.5648	4	0.6	3	5	4
g_5	2	2	0.6039	2	0.33	2	7	2

Table 4: Comparative result

By table 4, we have seen that g_3 (chess) is the best way to reduce symptoms of dyslexia followed by g_5 (bowling).

8. CONCLUSION

While comparing to fuzzy soft set, weighted sum model (WSM), fuzzy matrix, we can conclude that the concept of new approach in multi-criteria decision making problems using by soft set and fuzzy algebra method is very much interesting and has simple calculation for solving the day to day problems.

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REFERENCES

1) *Zadeh, L. A. (1965), "Fuzzy sets, Information and Control", 8(3): 338–353.*
2) *Klir, G.J., and Yan, B. (1995), "Fuzzy Sets and Fuzzy Logic Theory and Applications", Prentice-Hall International, Inc. London.*
3) *Hwang, C.L., and Yoon, K., (1981), "Multiple Attribute Decision Making: Methods and Applications", New York: Springer-Verlag.*
4) *Molodtsov, D.,(1999), "Soft set theory – First Results", Computer and Mathematics with Applications, 37 19-31.*
5) *Zadeh,L.A., (1970),"Decision Making in a fuzzy environment", Management Science, 17, B144-B164.*
6) *Shimura, M., (1973), "Fuzzy sets concept in rank ordering objects", J. Math. Anal. Appl., 43, 717-733.*
7) *Kahraman, C. (2008), "Multi-Criteria Decision Making methods and fuzzy sets".*
8) *Molodtsov, D. (2004), "The Theory of Soft Sets, URSS Publishers", Moscow (in Russian).*
9) *Molodtsov, D. (1999), "Soft set theory-first results", Computers & Mathematics with Applications 37 (4–5) 19–31.*
10) *Maji, P.K.,Biswas,R., Roy,A.R., (2001), "Fuzzy soft sets", The Journal of Fuzzy Mathematics 9 (3) 589–602.*
11) *Maji,P.K., Roy,A.R., Biswas,R., (2002), "An application of soft sets in a decision making problem", Computers & Mathematics with Applications 44 (8–9) 1077–1083.*
12) *Stanujkic, D.,Zavadskas, E.K., (2015), "A modified weighted sum method based on the decision-maker's preferred levels of performances," Studies in Informatics and Control, vol. 24, no. 4, pp. 461–469.*
13) *Virginraj, A.,Hemavathi, S., (2016), "Solving the decision making problem using decagonal fuzzy number and fuzzy matrix", International Journal of Mathematics And its Applications,Volume 4, Issue 4, 155–162.*
14) *Meenakshi, A.R., (2008), "FUZZY MATRIX: Theory and Applications", MJP publishers.*