On BIQUADRATIC EQUATION with FIVE UNKNOWNS $x^4 - y^4 = 2(z+w)p^2$

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Abstract: Biquadratic Equation is examined for getting non integral solutions. Two various kinds of different solutions to the equation analysed are found. Some fascinating results about the solutions are obtained.

NOTATIONS

obl_n = oblong number of order n
$t_{m,n}$ =polygonal number of rank 'n' with sides m
Keywords: Biquadratic , nasty number

METHOD OF ANALYSIS

The cubic Diophantine equation with four unknowns to be solved for getting non – zero integral solution is

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The cubic Diophantine equation with four unknowns to be solved for getting non – zero integral solution is

$x^4 - y^4 = 2(z+w)p^2$ --------- (1)

On substituting the linear transformations

$x = u + u, \quad y = u - v, \quad z = 2uv + 1$

$w = 2uv - 1$ ---- (2) in equation (1) ,

$(u+v)^4 - (u-v)^4 = 2((2uv+1) + (2uv-1))p^2$

$\Rightarrow ((u+v)^2)^2 - ((u-v)^2)^2 = 2(4uv)p^2$

$\Rightarrow ((u+v)^2 + (u-v)^2)((u+v)^2 - (u-v)^2)(2u^2 + 2v^2)(4uv)$

$\Rightarrow 18(u^2+v^2)uv = 2(4uv)p^2$

$\Rightarrow u^2 + v^2 = p^2$ --- (3)

We obtain four different patterns of integral solutions to (1) through solving (3) which are illustrated as follows :

Pattern 1:

$Let\, u = 2\,rs; \quad v = r^2 - s^2;$

The distinct integral solutions of (1) are expressed by,

$x = 2rs + r^2 - s^2$

$y = 2rs - r^2 + s^2$

$z = 4rs(r^2 - s^2) + 1$

$w = 4rs(r^2 - s^2) - 1$
$p = r^2 + s^2$

Properties :

i. $x(1,s) + y(1,s) \equiv 0 \pmod{4}$
ii. $x(1,s) - p(1,s) \equiv 0 (mod2)$
iii. $z(1,s) - w(1,s) \equiv 0(mod2)$
iv. $3y(1,s) + 3P(1,s)$ *is a nasty number*
v. $x(r,1) - p(r,1) \equiv 0\ (mod2)$
vi. $x(r,1) + p(r,1) = 4t_{3,r}$
vii. $2(x(r,1) + p(r,1)$ is a perfect square

Pattern 2:

Letting $u^2 + v^2 = p^2 * 1 and\ p^2 = a^2 + b^2$

Equation (3) becomes $u^2 + v^2 = p^2 * \frac{(3+4i)(3-4i)}{25}$

$$= (a^2 + b^2)^2 * \frac{(3+4i)(3-4i)}{25}$$

$$\Rightarrow (u + iv)(u - iv) = (a + ib)^2(a - ib)^2 * \frac{(3+4i)(3-4i)}{25}$$

$$\Rightarrow (u + iv) = (a + ib)^2 \frac{(3+4i)}{5}$$

$$\Rightarrow (u + iv) = \frac{1}{5}(a^2 - b^2 + 2aib)(3 + 4i)$$

$$= \frac{1}{5}(3a^2 - 3b^2 + 6aib - 4ia^2 - 4ib^2 - 8ab)$$

Equating real and imaginary parts,

$$u = \frac{1}{5}(3a^2 - 3b^2 - 8ab)$$

$$v = \frac{1}{5}(6ab - 4a^2 - 4b^2)$$

Taking $a = 5A \ ; b = 5B$

$$u = 3A^2 - 3B^2 - 40AB$$

$$v = -4A^2 - 4B^2 + 30AB$$

The distinct integral solutions of (1) are expressed by,

$x = -a^2 - 7b^2 - 10ab$
$y = 7a^2 + b^2 - 70ab$
$z = (6a^2 - 6b^2 - 80ab)(-4a^2 - 4b^2 + 30ab) + 1$
$w = (6a^2 - 6b^2 - 80ab)(-4a^2 - 4b^2 + 30ab) - 1$

Properties for pattern 2:

i. $x(a,1)+y(a,1)$ is a nasty number

ii. $x(a,1)-y(a,1)+z(a,1)-w(a,1)\equiv 0\ (mod2)$

iii. $x(1,b)+y(1,b)+80ab\ is\ a\ nasty\ number$

iv. $x(1,b)-y(1,b)\equiv 0\ \ (mod4)$

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