New Classes of Generalized Semi*-I_ω-Open Sets

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Abstract -The notions of α - I $_{\omega}$ - open sets and β - I $_{\omega}$ - open sets were studied. In this paper, we introduce and investigate the new notion called semi* - I $_{\omega}$ - open sets and I_ω-R-closed sets which is weaker than α - I $_{\omega}$ - open sets and stronger than β - I $_{\omega}$ - open sets. Also, we discussed some of the properties and characterizations.

Keywords: **semi*-I_ω-open, I_ω-R-closed sets.**

1. INTRODUCTION

A non empty collection of subsets of X in a topological spaces (X, τ) is said to be an ideal I if it satisfies (i) A ∈ I and B ⊂ A implies B ∈ I and (ii) A ∈ I and B ∈ I implies A U B ∈ I. .A topological space (X, τ) with an ideal I is called an ideal topological spaces or simply ideal space. Levine [11], Velicko [16] introduced the notions of generalized closed (briefly g – closed) and studied their basic properties. The notion of Ig – closed sets was first introduced by Dontchev [13] in 1999. Navaneetha Krishnan and Joseph [14] further investigated and characterized Ig – closed sets. Julian Dontchev and Maximillian Ganster [15] , Yuksel, Acikgoz and Noiri [12] introduced and studied the notions of δ – generalized closed (briefly δ g – closed) and δ – I closed sets respectively. The purpose of this paper is to introduce and study the concepts of new class of generalized classes of semi*-I_ω-open sets and I_ω-R-closed sets.

2. PRELIMINARIES

Throughout this paper, R (resp. N, Q, Q*, Z) denotes the set of all real numbers (resp. the set of all natural numbers, the set of all rational numbers, the set of all irrational numbers, the set of all integers). By space (X, τ) , we always mean a topological space (X, τ) with no separation properties assumed. If A ⊂ X, *cl(A)* and *int(A)* denotes the interior and closure of A in (X, τ). τ_u denotes the usual topology on R

Definition 2.1. [5] A space (X, τ) is called submaximal if every dense subset is open.

Definition 2.2. [6] A subset H of a space (X, τ) is said to be semi-ω-open if H ⊂ cl(int_ω (H)).

Definition 2.3. [6] A subset H of a space (X, τ) is said to be

1. semi*-ω-open if H ⊂ cl_ω (int(H)).
2. semi*-ω-closed if int_ω (cl(H)) ⊂ H.

Definition 2.4. [7] A subset H of an ideal topological space (X, τ, I) is said to be ∗-closed if H^* ⊂ H or cl^*(H) = H. The complement of an ∗-closed set is called ∗-open.

Definition 2.5. [8] A subset H of an ideal topological space (X,τ, I) is called ∗-dense if cl^*(H) = X.

Definition 2.6. [9] A subset H of an ideal topological space (X,τ, I) is called ∗-codense if X\H is ∗-dense.

Definition 2.7. [9] An ideal topological space (X, τ, I) is called I-submaximal if every ∗-dense subset of X is open.

Definition 2.8. [8] A subset H of an ideal topological space (X, τ, I) is called ∗-dense if cl^*(H) = X.

Definition 2.9. [9] A subset H of an ideal topological space (X, τ, I) is called $*$-condense if $X\backslash H$ is $*$-dense.
Proposition 3. [10] In R with usual topology τ_u and I = F, the ideal of all finite subsets of R, for the subset H = Q, $H^*(I) = Q^*(I = F) = R$.
Definition 3.1. [9] An ideal topological space (X, τ, I) is called I-submaximal if every $*$-dense subset of X is open.

3. Semi*-I_ω-open sets

In this section we introduce and study a new class of sets known as semi*-I_ω-open sets.

Definition 3.1. A subset H of an ideal topological space (X, τ, I) is said to be

(1) semi*-I_ω-open if $H \subset cl_\omega(int^*(H))$.

(2) semi*-I_ω -closed if $int_\omega(cl^*(H)) \subset H$.

The complement of a semi*-I_ω-open set is called semi*-I_ω -closed.

Example 3.2 In R with usual topology τ_u and ideal $I = \{\phi\}$, $H = R\backslash\{0\}$ is not semi*-I_ω-closed, since $int_\omega(cl^*(H)) = int_\omega(cl(H)) = int_\omega(R) = R \not\subset H$.
Example 3.3. In R with usual topology τ_u and ideal I = P(R), $H = R\backslash\{0\}$ is semi*-I_ω-closed since $int_\omega(cl^*(H)) = int_\omega(H) = H \subset H$.
Proposition 3.4. For a subset of an ideal topological space (X, τ, I), every semi*-ω-open set is semi*-I_ω--open.
Proof. If H is semi*-ω-open, then $H \subset cl_\omega(int(H)) \subset cl\omega(int^*(H))$. Therefore H is semi*-$I_\omega$- open.
Remark 3.5. The converse of Proposition 3.4. is not true.
Example 3.6. In R with usual topology τ_u and ideal I = P(R), H = Q is semi*-I_ω-closed, since $int_\omega(cl^*(H)) = int_\omega(H) = H$. But H = Q is not semi*-I_ω closed, since $int_\omega(cl(H)) = int_\omega(R) = R \not\subset H$.
Proposition 3.7. A subset H of an ideal topological space (X, τ, I) is semi*-I_ω-open if and only if $cl_\omega(H) = cl_\omega(int^*(H))$.
Proof. If H is semi*-I_ω-open set, then $H \subset cl_\omega(int^*(H))$ and $cl_\omega(H) \subset cl_\omega(int^*(H))$. But $cl_\omega(int^*(H)) \subset cl_\omega(H)$. Hence $cl_\omega(H) = cl_\omega(int^*(H))$. Conversely, $H \subset cl_\omega(H) = cl_\omega(int^*(H))$ by assumption. Therefore H is semi*-I_ω-open.
Definition 3.8. A subset H of an ideal topological space (X,τ,I) is said to be a t-I_ω*-set if $int_\omega(cl^*(H)) = int_\omega(H)$.
Example 3.9. In R with usual topology τ_u and ideal $I = \{\phi\}$,

1. H=(0,1] is a t-I_ω*-set, since $int_\omega(H) = (0,1)$ and $int_\omega(cl^*(H)) = int_\omega(cl(H)) = int_\omega([0,1]) = (0, 1)$.
2. $H = Q^*$ is not a t-I_ω* -set, since $int_\omega(H) = H$ and $int_\omega(cl^*(H)) = int_\omega(cl(H)) = int_\omega(R) = R$.

Proposition 3.10. In an ideal topological space (X, τ, I),every $*$-closed set is a t-I_ω* -set.
Proof. Let H be a $*$-closed set. Then $H = cl^*(H)$ and $int_\omega(cl^*(H)) = int_\omega(H)$ which proves that H is a t-I_ω* -set.
Remark 3.11. The converse of Proposition 3.10 is not true.
Example 3.12. In R with usual topology τ_u and ideal $I = \{\phi\}$, H = (0, 1] is t-I_ω* -set by (1) of *Example 3.9.*
But H = (0; 1] is not $*$-closed, since $cl^\omega(H) = cl(H) = [0, 1] \neq H$.

Proposition 3.13. In an ideal topological space (X, τ, I), every t-I_ω* set is t-I_ω* -set.

Proof. If H is a t-I_ω* -set, then $int_\omega(cl^*(H)) = int(H) \subset int_\omega(H) \subset int_\omega(cl^*(H))$.

Thus $int_\omega(cl^*(H)) = int_\omega(H)$ and hence H is a t-I_ω* -set.

Remark 3.14. The converse of Proposition 3.13. is not true.

Example 3.15. In R with usual topology τ_u and ideal I = P(R), $H = (0, 1) \cap Q^*$ is a t-I_ω* -set since $int_\omega(cl^*(H)) = int_\omega(H)$. But H is not a t-$I_\omega$* set since $int_\omega(cl^*(H)) = int_\omega(H) = H \neq \phi = int(H)$.

Theorem 3.16. A subset H of an ideal topological space (X, τ, I) is semi*-I_ω-closed if and only if H is a t-I_ω* -set.

Proof. H is a semi*-I_ω-closed in X , X\ H is semi*-I_ω-open $\Rightarrow cl_\omega(X\setminus H) = cl_\omega(int^*(X\setminus H))$ by Proposition 3.13.$\Rightarrow$ $X\setminus int_\omega(H) = X \setminus int_\omega(cl^*(H))$, $int_\omega(H) = int_\omega(cl^*(H))$, H is a t-I_ω* -set.

Proposition 3.17. If A and B are t-I_ω* -sets of an ideal topological space (X, τ, I), then $A\cap B$ is a t-I_ω* -set.

Proof. Let A and B be t-I_ω* -sets. Then $int_\omega(A \cap B) \subset int_\omega(cl^*(A \cap B)) = int_\omega(cl^*(A) \cap cl^*(B)) = int_\omega(cl^*(A)) \cap int_\omega(cl^*(B)) = int_\omega(A) \cap int_\omega(B) = int_\omega(A \cap B)$.

Thus $int_\omega(A \cap B) = int_\omega(cl^*(A \cap B))$ and hence $A \cap B$ is a t-I_ω* -set.

Definition 3.18. A subset H of an ideal topological space (X, τ, I) is said to be semi-I_ω-regular if H is semi-I_ω-open and a t-I_ω* -set.

Example 3.19. In R with usual topology τ_u and ideal $I = \{\phi\}$,

1. H = (0,1] is a t-I_ω* -set by (1) of Example 3.9. Also $cl^*(int_\omega(H)) = cl^*((0,1)) = cl((0,1)) = [0, 1] \supset H$. Thus H is semi-$I_\omega$-open. Hence (0, 1] is semi-I_ω-regular.
2. $H = Q^*$ is not a t-I_ω* -set by (2) of Example 3.9. Hence Q^* is not semi-I_ω-regular.

Remark 3.20. In an ideal topological space (X, τ, I),

1. Every semi-I_ω-regular set is semi-I_ω-open.
2. Every semi-I_ω-regular set is t-I_ω* -set.

The converses of (1) and (2) of Remark 3.20. are not true in general as illustrated in the following Example.

Example 3.21. In R with usual topology τ_u and ideal $I = \{\Phi\}$, the subset $H = Q^*$ is semi-I_ω-open by Example 3.9.

But $H = Q^*$ is not semi-I_ω-regular by (2) of Example 3.9.

Theorem 3.22. A subset of an ideal topological space (X, τ, I) is semi-I_ω-regular if and only if it is both β -I_ω-open and semi*-I_ω-closed.

Proof. If H is semi-I_ω-regular, then H is both semi-I_ω-open and a t-I_ω* -set. Since H is semi-I_ω-open, H is β -I_ω-open . Also H is a t-I_ω* -set by assumption. Hence H is semi*-I_ω-closed. Conversely, let H be semi*-I_ω-closed and β-I_ω-open. Since H is semi*-I_ω closed, by Theorem 3.16, H is a t-I_ω* -set. Since H is β -I_ω- open, $H \subset cl^*(int_\omega(cl^*(H))) = cl^*(int_\omega(H))$. Therefore H is semi-$I_\omega$-open. Since H is both semi-I_ω-open and a t-I_ω* -set, H is semi-I_ω-regular.

Remark 3.23. The following Example shows that the concepts of β -I_ω-openness and semi*-I_ω-closedness are independent.

Example 3.24.(i) In R with the topology $\tau = \{\phi, R, Q^*\}$ and ideal I = P(R), H = Q is semi*-I_ω -closed, since $int_\omega(cl^*(H)) = int_\omega(H) = \phi \subset H$. But H = Q is not β -I_ω-open, since $cl^*(int_\omega(cl^*(H))) = cl^*(int_\omega(H)) = cl^*(\Phi) = H$

(ii) In R with usual topology τ_u and ideal $I = \{\Phi\}$, H = Q is β -I_ω-open, since $cl^*(int_\omega(cl^*(H))) = cl^*(int_\omega(cl(H))) = cl^*(int_\omega(R)) = R$ H. But H = Q is not semi*-I_ω -closed, since $int_\omega(cl^*(H)) = int_\omega(cl(H)) = int_\omega(R) = R \not\subset H$.

4. *I_ω-R-closed sets*

In this section we introduce and study a new class of sets known as I_ω-R-closed sets.

Definition 4.1. A subset H of an ideal topological space (X, τ, I) is called I_ω-R-closed if $H = cl^*(int_\omega(H))$.

Example 4.2. In R with the topology $\tau = \{\Phi, R; Q^*\}$ and ideal I = P(R),

1. H = Q is not I_ω-R-closed, since $cl^*(int_\omega(H)) = cl^*(\phi) = \phi \neq H$.
2. $H = Q^*$ is I_ω-R-closed for $cl^*(int_\omega(H)) = cl^*(H) = H$.

Theorem 4.3. For a subset H of an ideal topological space (X, τ, I), the following properties are equivalent.

1. H (≠, ϕ) is I_ω-R-closed.
2. There exists a non-empty ω-open set G such that $G \subset H = cl^*(G)$.
3. There exists a non-empty ω-open set G such that $H = G \cup ((cl^*(G) - G)$.

Proof. (1) ⇒ (2): Suppose H(≠, ϕ) is an I_ω-R-closed set. Then $H = cl^*(int_\omega(H))$.

Let $G = int_\omega(H)$. G is the required ω-open set such that G $H = cl^*(G)$.

(2) ⇒ (3): Since $H = cl^*(G) = G \cup ((cl^*(G) - G)$ where G is a nonempty ω-open set, (3)follows.

(3) ⇒ (1): $H = G \cup ((cl^*(G) - G))$ implies that $H = cl^*(G) = cl^*(int_\omega(G)) \subset cl^*(int_\omega(H))$, since G is ω-open and $G \subset H$. Also $cl^*(int_\omega(H)) \subset cl^*(H) = cl^*(G) = H$. Therefore $H = cl^*(int_\omega(H))$ which implies that H is I_ω-R-closed.

Theorem 4.4. For each β -I_ω open subset H of an ideal topological space (X, τ, I), $cl^*(H)$ is I_ω-R-closed.

Proof. Suppose H is β -I_ω - open. Then $H \subset cl^*(int_\omega(cl^*(H)))$ and so $cl^*(H) \subset Cl^*(int_\omega(cl^*(H))) \subset cl^*(H)$ which implies that $cl^*(H) = cl^*(int_\omega(cl^*(H)))$. Therefore $cl^*(H)$ is I_ω-R-closed.

Theorem 4.5. For a subset H of an ideal topological space (X, τ, I), the following properties are equivalent.

1. H is I_ω-R-closed.
2. H is semi-I_ω-open and ∗-closed.
3. H is β -I_ω-open and ∗-closed.

Proof. (1) ⇒ (2): If H is I_ω-R-closed, then $H = cl^*(int_\omega(H))$. Since $H \subset cl^*(int_\omega(H))$, H is semi-$I_\omega$-open. Also, $H = cl^*(H)$ and thus H is ∗-closed.

(2) ⇒ (3): It follows from the fact that every semi-Iω-open set is a β - Iω -open.

(3) ⇒ (1): Suppose H is β - Iω -open and ∗-closed. Then $H \subset cl(int_\omega(cl^*(H)))$ and $= cl^*(H)$. Now $cl^*(int_\omega(H)) \subset cl^*(H) = H$. Also, $H \subset cl^*(int_\omega(H))$. Therefore $= cl^*(int_\omega(H))$ which implies that H is I_ω-R-closed.

Remark 4.6 .The following Examples show that

1. The concepts of semi-I_ω-openness and ∗-closedness are independent.
2. The concepts of β -I_ω-openness and∗closedness are independent.

Example 4.7. In R with usual topology τ_u and ideal I = {ϕ }

1. $H = Q^*$ is semi-I_ω-open by Example 3.5.21. But H is not ∗-closed for $cl^*(H) = cl(H) = R \neq H$.
2. N = the set of natural numbers is not semi-$I_!$-open by Example 3.23. But N is ∗-closed for $cl^*(N) = cl(N) = N$.

Example 4.8. In R with usual topology τ_u and ideal I = { ϕ },

1. The subset $H = Q^*$ is semi-I_ω-open by Example 3..21. and hence β-I_ω-open But $H = Q^*$ is not ∗-closed for $cl^*(H) = cl(H) = R \neq H$.

2. N = the set of all natural numbers is ∗-closed by (2) of Example 4.7. But N is not β- I_ω-open for $cl^*(int_\omega(cl^*(N))) = cl^*(int_\omega(cl(N))) = cl^*(int_\omega(N)) = cl^*(\phi) = \phi \subset N$.

Definition 4.9 An ideal topological space (X, τ, I) is called $I_!$-submaximal if every *-dense subset of X is ω-open.

Proposition 4.10 1. Every submaximal space is I-submaximal.

2. Every I-submaximal space is Iω-submaximal.

Proof. (1) If (X, τ) is submaximal and H is *-dense in the ideal topological space (X, τ, I), then $cl^*(H) = X$. But $X = cl^*(H) \subset cl(H)$ implies cl(H) = X. Thus H is dense in X and by assumption H is open in X. This shows that (X, τ, I) is I-submaximal.

(2) Proof follows directly since any open set is ω -open.

The converses of (1) and (2) in Proposition 4.10 are not true in general as illustrated below.

Example 4.11. In (R, τ, I) where = τ_u, the usual topology and ideal I = P(R), if H is any *-dense subset, then $cl^*(H)$= R and so H = R which is open. Thus (R, τ_u, I) is I-submaximal. But in (R, τ_u), Q is dense in R since cl(Q) = R. But Q is not open. This shows that (R, τ_u) is not submaximal.

2. In the ideal topological space (N, τ, I), where N is the set of all natural numbers, τ= { ϕ, N }and ideal I = {ϕ }, if H is any *-dense subset in N, then H ⊂ N. Since N is countable, H is ω-open. Thus (N, τ, I) is I_ω-submaximal. The subset A = {1} is *****-dense in N for $cl^*(A)$ = cl(A) = N. But A = {1} is not open in N. This shows that (N, τ, I) is not I-submaximal.

Theorem 4.12 For an ideal topological space (X, τ, I), the following are equivalent.

1. X is I_ω-submaximal,
2. Every *-codense subset of X is ω-closed.

Proof. X is I_ω -submaximal , every *****-dense subset of X is ω-open , every *-co-dense subset of X is ω-closed since a subset A is *-dense in X if and only if X - A is *-codense in X.

CONCLUSION

We investigated the properties and characterizations of semi*-I_ω-open sets, and I_ω-R-closed sets in ideal topological spaces.

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