CONTRA $\mathcal{I}_{wgp}$-CONTINUITY

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Abstract. In this paper, the concepts of $\mathcal{I}_{wgp}$-closed sets and $\mathcal{I}_{wgp}$-open sets are investigated and further they are used to define and study a new class of functions called contra $\mathcal{I}_{wgp}$-continuous functions in ideal spaces. We discuss the relationships of such class with some other related functions.

1. Introduction and preliminaries

Throughout this paper, by a space X, we always mean a topological space (X,τ) with no separation properties assumed. Let H be a subset of X. We denote the interior, the closure and the complement of a subset H by int(H), cl(H) and X\H or H^c, respectively. The set of all open sets containing a point x∈X is denoted by $\sum(x)$ [6].

Definition 1.1. [11] *A subset H of a space X is said to be preopen if H⊆int(cl(H)). The complement of a preopen set is called preclosed.*

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Definition 1.2. [9] *A space X is said to be regular if for each closed set F of X and each $x \notin F$, there exist disjoint open sets P and Q such that $x \in P$ and $F \subseteq Q$.*

Definition 1.3. [13] *A space X is called locally indiscrete if every open set is closed.*

Definition 1.4. [18] *A space X is called Urysohn if for every pair of points $x,y \in X$, $x \neq y$ there exist $U \in \sum(x)$, $V \in \sum(y)$ such that $cl(U) \cap cl(V) = \emptyset$.*

The collection of all clopen subsets of X will be denoted by $CO(X)$. We set $CO(X,x) = \{V \in CO(X) | x \in V\}$ for $x \in$X [12].

Definition 1.5. [14] *A space X is said to be*

(1) *Ultra Hausdorff if for each pair of distinct points x and y in X there exist $U \in CO(X, x)$ and $V \in CO(X, y)$ such that $U \cap V = \emptyset$.*
(2) *Ultra normal if each pair of non-empty disjoint closed sets can be separated by disjoint clopen sets.*

Definition 1.6. [6] *Let $f : (X,\tau) \to (Y,\sigma)$ be any function. Then the subset $G(f) = \{(x, f(x)) : x \in X\}$ of the product space $(X \times Y, \tau \times \sigma)$ is called the graph of f.*

An ideal $\mathcal{I}$ on a space X is a non-empty collection of subsets of X which satisfies (i) P$\in\mathcal{I}$ and Q$\subseteq$P$\Rightarrow$Q$\in\mathcal{I}$ and (ii) P$\in\mathcal{I}$ and Q$\in\mathcal{I}\Rightarrow$P$\cup$Q$\in\mathcal{I}$. Given a space X with an ideal $\mathcal{I}$ on X and if $\wp$(X) is the set of all subsets of X, a set operator $(.)^* : \wp(X) \to \wp(X)$, called a local function [10] of H with respect to τ and $\mathcal{I}$ is defined as follows: for H$\subseteq$X, $H^*(\mathcal{I},\tau) = \{x \in X | U \cap H \notin \mathcal{I}$ for every $U \in \tau(x)\}$ where $\tau(x) = \{U \in \tau | x \in U\}$. We will make use of the basic facts about the local functions [[8], Theorem 2.3] without mentioning it explicitly. A Kuratowski closure operator $cl^*(.)$ for a topology $\tau^*(\mathcal{I},\tau)$, called the $\star$-topology, finer than τ, is defined by $cl^*(H) = H \cup H^*(\mathcal{I},\tau)$ [17]. When there is no chance for confusion, we will simply write H^* for $H^*(\mathcal{I},\tau)$ and τ^* for $\tau^*(\mathcal{I},\tau)$. If $\mathcal{I}$ is an ideal on X, then (X,τ,$\mathcal{I}$) is called an ideal space. $\mathcal{N}$ is the ideal of all nowhere dense subsets in (X,τ). A subset H of an ideal space (X,τ,$\mathcal{I}$) is called $\mathcal{I}_g$-closed [4] if $H^* \subseteq U$ whenever H$\subseteq$U and U is open.

Definition 1.7. *A function $f : (X,\tau,\mathcal{I}) \to (Y,\sigma)$ is called $\mathcal{I}_g$-continuous* [7] *if the inverse image of every closed set in Y is $\mathcal{I}_g$-closed in X.*

Let us say that $w \subseteq \wp(X)$ is a weak structure (briefly WS) on X iff $\emptyset \in w$. Clearly each generalized topology and each minimal structure is a WS [2].

Each member of w is said to be w-open and the complement of a w-open set is called w-closed.

Let w be a weak structure on X and H⊆X. We define (as in the general case) i_w(H) is the union of all w-open subsets contained in H and c_w(H) is the intersection of all w-closed sets containing H [2].

Remark 1.8. [1] *If w is a WS on X, then $i_w(\emptyset)=\emptyset$ and $c_w(X)=X$.*

Theorem 1.9. [2] *If w is a WS on X and A,B $\in w$ then*

(1) $i_w(A)\subseteq A\subseteq c_w(A)$,

(2) $A\subseteq B\Rightarrow i_w(A)\subseteq i_w(B)$ *and* $c_w(A)\subseteq c_w(B)$,

(3) $i_w(i_w(A))=i_w(A)$ *and* $c_w(c_w(A))=c_w(A)$,

(4) $i_w(X-A)=X-c_w(A)$ *and* $c_w(X-A)=X-i_w(A)$.

Definition 1.10. [1] *Let w be a WS on a space X. Then H⊆X is said to be wg-closed if cl(H)⊆U whenever H⊆U and U is w-open in X.*

The complement of a wg-closed set is called a wg-open set.

Remark 1.11. [1] *For a WS w on a space X, every w-closed set is gw-closed but not conversely.*

Let w be a WS on X and H⊆X. Then H$\in\pi(w)$ if H⊆$\mathrm{i}_w(i_w(H))$ [2].

Definition 1.12. [16] *Let w be a WS on a space X, then H⊆X is called a wgp-closed set if cl(H)⊆U whenever H⊆U$\in\pi(w)$.*

The complement of wgp-closed set is a wgp-open set.

Remark 1.13. [16] *For a WS w on a space X, every w-closed set is wgp-closed but not conversely.*

Proposition 1.14. [16] *If H$\in\tau$ then H$\in\pi(w)$.*

2. Properties of Contra $\mathcal{I}_{wgp}$-continuity

Definition 2.1. *Let w be a WS on a space X. Then X is said to be wgp-normal if each pair of non-empty disjoint closed sets can be separated by disjoint wgp-open sets.*

Example 2.2. (1) *Let* X=$\{a,\ b,\ c\}$, τ=$\{\emptyset,\ X,\ \{c\},\ \{a,\ b\}\}$ *and* w= $\{\emptyset,\ X,\ \{a\},\ \{a,\ b\}\}$. *Then wgp-open sets are* $\emptyset$, X, $\{a\}$, $\{c\}$, $\{a,\ b\}$, $\{a,\ c\}$. *Clearly* X *is wgp-normal.*

(2) *Let* X=$\{a,\ b,\ c\}$, τ=$\{\emptyset,\ X,\ \{a\},\ \{a,\ b\},\ \{a,\ c\}\}$ *and* w=$\{\emptyset,\ X,\ \{a,\ b\},\ \{b,\ c\},\ \{a,\ c\}\}$. *Then wgp-open sets are* $\emptyset$, X, $\{a\}$, $\{a,\ b\}$, $\{a,\ c\}$. *Clearly* X *is not wgp-normal.*

Definition 2.3. *Let* w *be a WS on a space X. A function* f : $(X,\tau,\mathcal{I}) \rightarrow (Y,\sigma)$ *is said to be*

(1) *contra wgp-continuous if for each open set* V *in* (Y,σ), $f^{-1}(V)$ *is wgp-closed in* (X,τ).

(2) *contra* w*-continuous* [15] *if for each open set* V *in* (Y,σ), $f^{-1}(V)$ *is* w*-closed in* (X,τ).

(3) *contra continuous* [3] *if for each closed set* V *in* (Y,σ), $f^{-1}(V)$ *is open in* (X,τ).

(4) *contra* $\mathcal{I}_g$*-continuous* [14] *if for each open set* V *in* (Y,σ), $f^{-1}(V)$ *is* $\mathcal{I}_g$*-closed in* $(X,\tau,\mathcal{I})$.

Proposition 2.4. *Every contra* w*-continuous function is contra wgp-continuous.*

Proof. Let w be a WS on a space X. Let $f : (X,\tau) \rightarrow (Y,\sigma)$ be a contra w-continuous function and let V be any open set in Y. Then, $f^{-1}(V)$ is w-closed in X. Since every w-closed set is wgp-closed, $f^{-1}(V)$ is wgp-closed in X. Therefore f is contra wgp-continuous.

However, converse need not be true as seen from the following Example.

Example 2.5. *Let* X=Y=$\{a,\ b,\ c\}$, τ=σ=$\{\emptyset,\ \{c\},\ \{a,\ b\},\ X=Y\}$ *and* w=$\{\emptyset,\ X,\ \{a\},\ \{a,\ b\}\}$. *Then* w *is a WS on a space X. Also the identity function* $f : (X,\tau) \rightarrow (Y,\sigma)$ *is contra wgp-continuous but not contra* w*-continuous.*

Definition 2.6. *Let w be a WS on a space X. A graph $G(f)$ of a function $f:(X,\tau)\to(Y,\sigma)$ is said to be contra wgp-closed in $(X\times Y)$ if for each $(x,y)\in(X\times Y)\backslash G(f)$, there exist an $P\in wGPO(X)$ containing x and a closed set Q of (Y,σ) containing y such that $f(P)\cap Q=\emptyset$ where $wGPO(X)$ denotes the family of all wgp-open sets of X.*

Example 2.7. *Let $X=Y=\{a,\ b,\ c\}$, $\tau=\sigma=\{\emptyset,\ \{a\},\ \{b\},\ \{c\},\ \{a,\ b\},\ \{a,\ c\},\ \{b,\ c\}$ $X=Y\}$ and $w=\{\emptyset,\ \{a\},\ \{b,\ c\},\ X\}$. Let $f:(X,\tau)\to(Y,\sigma)$ be an identity function. Then w is a WS on a space X and $G(f)$ is contra wgp-closed in $X\times Y$.*

Definition 2.8. *Let w be a WS on an ideal space $(X,\tau,\mathcal{I})$. A subset $H\subseteq X$ is said to be $\mathcal{I}_{wgp}$-closed if $H^{\star}\subseteq U$ whenever $H\subseteq U\in\pi(w)$.*

The complement of an $\mathcal{I}_{wgp}$-closed set is called $\mathcal{I}_{wgp}$-open.

The family of all $\mathcal{I}_{wgp}$-open sets of $(X,\tau,\mathcal{I})$ is denoted by $\mathcal{I}wGPO(X)$.

Definition 2.9. *Let w be a WS on an ideal space $(X,\tau,\mathcal{I})$. Then $(X,\tau,\mathcal{I})$ is said to be $\mathcal{I}_{wgp}$-normal if each pair of non-empty disjoint closed sets can be separated by disjoint $\mathcal{I}_{wgp}$-open sets.*

Example 2.10.

(1) *Let $X=\{a,\ b,\ c\}$, $\tau=\{\emptyset,\ X,\ \{a\},\ \{b,\ c\}\}$, $w=\{\emptyset,\ X,\ \{b,\ c\}\}$ and $\mathcal{I}=\{\emptyset\}$. Then $\mathcal{I}_{wgp}$-open sets are $\emptyset$, X, $\{a\}$, $\{b,\ c\}$. Clearly $(X,\tau,\mathcal{I})$ is $\mathcal{I}_{wgp}$-normal.*

(2) *Let $X=\{a,\ b,\ c\}$, $\tau=\{\emptyset,\ X,\ \{a\},\ \{a,\ b\},\ \{a,\ c\}\}$, $w=\{\emptyset,\ X,\ \{a\},\ \{a,\ c\},\ \{a,\ b\}\}$ and $\mathcal{I}=\{\emptyset\}$. Then $\mathcal{I}_{wgp}$-open sets are $\emptyset$, X, $\{a\}$, $\{a,\ b\}$, $\{a,\ c\}$. Clearly $(X,\tau,\mathcal{I})$ is not $\mathcal{I}_{wgp}$-normal.*

Definition 2.11. *Let w be a WS on an ideal space $(X,\tau,\mathcal{I})$. A function $f:(X,\tau,\mathcal{I})\to(Y,\sigma)$ is said to be $\mathcal{I}_{wgp}$-continuous if $f^{-1}(V)$ is $\mathcal{I}_{wgp}$-closed in $(X,\tau,\mathcal{I})$ for each closed set V in (Y,σ).*

Definition 2.12. *Let w be a WS on an ideal space $(X,\tau,\mathcal{I})$. A function $f:(X,\tau,\mathcal{I})\to(Y,\sigma)$ is said to be contra $\mathcal{I}_{wgp}$-continuous if $f^{-1}(V)$ is $\mathcal{I}_{wgp}$-closed in $(X,\tau,\mathcal{I})$ for each open set V in (Y,σ).*

Proposition 2.13. *Let w be a WS on an ideal space $(X, \tau, \mathcal{I})$. If $\tau \subseteq w$ then every $\mathcal{I}_{wgp}$-closed set is $\mathcal{I}_g$-closed.*

Proof. The result follows immediately from the given condition.

However, converse need not be true as seen from the following Example.

Example 2.14. *Let X=$\{a, b, c\}$, τ=$\{\emptyset, \{a\}, X\}$, w=$\{\emptyset, \{a\}, \{b\}, X\}$ and $\mathcal{I}$=$\{\emptyset, \{c\}\}$. Then $\tau \subseteq w$. Also $\{b\}$ is an $\mathcal{I}_g$-closed set but not $\mathcal{I}_{wgp}$-closed.*

Proposition 2.15. *For a WS w on an ideal space $(X, \tau, \mathcal{I})$, every wgp-closed set is $\mathcal{I}_{wgp}$-closed.*

Proof. The proof follows immediately from the fact that H$^\star \subseteq$cl(H).

However, converse need not be true as seen from the following Example.

Example 2.16. *Let X=$\{a, b, c, d\}$, τ=$\{\emptyset, \{b\}, \{b, c, d\}, X\}$, w=$\{\emptyset, \{a, b, c\}, X\}$ and $\mathcal{I}$=$\{\emptyset, \{c\}\}$. Then $\{c\}$ is an $\mathcal{I}_{wgp}$-closed set but not wgp-closed.*

Proposition 2.17. *Every contra wgp-continuous function is contra $\mathcal{I}_{wgp}$-continuous.*

Proof. Let w be a WS on an ideal space $(X, \tau, \mathcal{I})$. Let $f : (X, \tau, \mathcal{I}) \to (Y, \sigma)$ be a contra wgp-continuous function and let V be any open set in Y. Then, $f^{-1}(V)$ is wgp-closed in X. Since every wgp-closed set is $\mathcal{I}_{wgp}$-closed, $f^{-1}(V)$ is $\mathcal{I}_{wgp}$-closed in X. Therefore f is contra $\mathcal{I}_{wgp}$-continuous.

However, converse need not be true as seen from the following Example.

Example 2.18. *Let X=Y=$\{a, b, c\}$, τ=σ=$\{\emptyset, \{a\}, X$=$Y\}$, $\mathcal{I}$=$\{\emptyset, \{a\}\}$ and w=$\{\emptyset, X, \{a\}, \{c\}\}$. Then the identity function $f : (X, \tau, \mathcal{I}) \to (Y, \sigma)$ is contra $\mathcal{I}_{wgp}$-continuous but not contra wgp-continuous.*

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Remark 2.19. *The following two examples show that the concepts of $\mathcal{I}_{wgp}$-continuity and contra $\mathcal{I}_{wgp}$-continuity are independent of each other.*

Example 2.20. *Let X=Y=$\{a, b, c\}$, τ=σ=$\{\emptyset, \{a\}, X$=$Y\}$, $\mathcal{I}$=$\{\emptyset, \{a\}\}$ and w=$\{\emptyset, \{a\}, \{c\}, X\}$. Let $f : (X, \tau, \mathcal{I}) \to (Y, \sigma)$ be defined by f(a)=b, f(b)=a and f(c)=c. Since the inverse image of every open set of Y is $\mathcal{I}_{wgp}$-closed in X, f is contra $\mathcal{I}_{wgp}$-continuous. For the closed set $\{b, c\}$ of Y, $f^{-1}(\{b, c\})$=$\{a, c\}$ is not $\mathcal{I}_{wgp}$-closed in $(X, \tau, \mathcal{I})$. Therefore f is not $\mathcal{I}_{wgp}$-continuous.*

Example 2.21. *Let X=Y=$\{a, b, c\}$, τ=σ=$\{\emptyset, \{a\}, \{b\}, \{a, b\}, X$=$Y\}$, $\mathcal{I}$=$\{\emptyset, \{a, c\}\}$ and w=$\{\emptyset, \{b\}, \{a, c\}, X\}$. Let $f : (X, \tau, \mathcal{I}) \to (Y, \sigma)$ be defined by f(a)=a, f(b)=b and f(c)=c. Since the inverse image of every closed set of Y is $\mathcal{I}_{wgp}$-closed in X, f is $\mathcal{I}_{wgp}$-continuous. For the open set $\{b\}$ of (Y, σ), $f^{-1}(\{b\})$=$\{b\}$ is not $\mathcal{I}_{wgp}$-closed in $(X, \tau, \mathcal{I})$. Therefore f is not contra $\mathcal{I}_{wgp}$-continuous.*

Proposition 2.22. *If $\tau \subseteq w$, then every contra $\mathcal{I}_{wgp}$-continuous function is contra $\mathcal{I}_g$-continuous.*

Proof. The proof follows immediately from Propositioin 2.13.

However, converse need not be true as seen from the following Example.

Example 2.23. *Let X=Y=$\{a, b, c\}$, τ=$\{\emptyset, \{a\}, X\}$, σ=$\{\emptyset, \{b\}, \{a, c\}, Y\}$, w=$\{\emptyset, X, \{a\}, \{a, c\}\}$ and $\mathcal{I}$=$\{\emptyset, \{c\}\}$. Then the identity function $f : (X, \tau, \mathcal{I}) \to (Y, \sigma)$ is contra $\mathcal{I}_g$-continuous but not contra $\mathcal{I}_{wgp}$-continuous.*

Theorem 2.24. *Let w be a WS on an ideal space $(X, \tau, \mathcal{I})$. Let $f : (X, \tau, \mathcal{I}) \to (Y, \sigma)$ be a function. Then the following are equivalent:*

(1) f is contra $\mathcal{I}_{wgp}$-continuous.
(2) The inverse image of each closed set in Y is $\mathcal{I}_{wgp}$-open in X.
(3) For each point x in X and each closed set Q in Y with $f(x) \in$Q, there is an $\mathcal{I}_{wgp}$-open set P in X containing x such that f(P)$\subseteq$Q.

Proof. (1) $\Rightarrow$ (2) Let G be a closed set in Y. Then $Y-G$ is open in Y. By definition of contra $\mathcal{I}_{wgp}$-continuity, $f^{-1}(Y-G)$ is $\mathcal{I}_{wgp}$-closed in X. But $f^{-1}(Y-G) = X-f^{-1}(G)$. This implies $f^{-1}(G)$ is $\mathcal{I}_{wgp}$-open in X.

(2) $\Rightarrow$ (3) Let $x\in$X and Q be any closed set in Y with $f(x)\in$Q. By (2), $f^{-1}(Q)$ is $\mathcal{I}_{wgp}$-open in X. Set $P = f^{-1}(Q)$. Then there is an $\mathcal{I}_{wgp}$-open set P in X containing x such that f(P)$\subseteq$Q.

(3) $\Rightarrow$ (1) Let $x\in$X and Q be any closed set in Y with $f(x)\in$Q. Then $Y-Q$ is open in Y with $f(x)\in$Q. By (3), there is an $\mathcal{I}_{wgp}$-open set P in X containing x such that f(P)$\subseteq$Q. This implies $P = f^{-1}(Q)$. Therefore, $X-P = X-f^{-1}(Q) = f^{-1}(Y-Q)$ which is $\mathcal{I}_{wgp}$-closed in X.

Theorem 2.25. *Let w be a WS on an ideal space $(X,\tau,\mathcal{I})$ and let $f:(X,\tau,\mathcal{I})\to(Y,\sigma)$ and $g:(Y,\sigma)\to(Z,\mu)$. Then the following properties hold:*

(1) If f is contra $\mathcal{I}_{wgp}$-continuous and g is continuous then $g\circ f$ is contra $\mathcal{I}_{wgp}$-continuous.

(2) If f is contra $\mathcal{I}_{wgp}$-continuous and g is contra continuous then $g\circ f$ is $\mathcal{I}_{wgp}$-continuous.

(3) If f is $\mathcal{I}_{wgp}$-continuous and g is contra continuous then $g\circ f$ is contra $\mathcal{I}_{wgp}$-continuous.

Proof. (1) Let V be any closed set in Z. Since g is continuous, $g^{-1}(V)$ is closed in Y. Since f is contra $\mathcal{I}_{wgp}$-continuous, $(g\circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is $\mathcal{I}_{wgp}$-open in X. Therefore $g\circ f$ is contra $\mathcal{I}_{wgp}$-continuous.

(2) Let V be any closed set in Z. Since g is contra continuous, $g^{-1}(V)$ is open in Y. Since f is contra $\mathcal{I}_{wgp}$-continuous, $(g\circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is $\mathcal{I}_{wgp}$-closed in X. Therefore $g\circ f$ is $\mathcal{I}_{wgp}$-continuous.

(3) Let V be any closed set in Z. Since g is contra continuous, $g^{-1}(V)$ is open in Y. Since f is $\mathcal{I}_{wgp}$-continuous, $(g\circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is $\mathcal{I}_{wgp}$-open in X. Therefore $g\circ f$ is contra $\mathcal{I}_{wgp}$-continuous.

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Theorem 2.26. *Let w be a WS on an ideal space $(X,\tau,\mathcal{I})$. If a function $f:(X,\tau,\mathcal{I})\to(Y,\sigma)$ is contra $\mathcal{I}_{wgp}$-continuous and Y is regular, then f is $\mathcal{I}_{wgp}$-continuous.*

Proof. Let x be an arbitrary point of X and Q be an open set of Y containing $f(x)$. Since Y is regular, there exists R$\in\tau$ such that $f(x)\in$R$\subseteq cl$(R)$\subseteq$Q. Since f is contra $\mathcal{I}_{wgp}$-continuous, by Theorem 2.24, there exists an $\mathcal{I}_{wgp}$-open set P containing x such that $f(P)\subseteq cl$(R). Thus $f(P)\subseteq cl$(R)$\subseteq$Q. Hence f is $\mathcal{I}_{wgp}$-continuous.

Definition 2.27. *Let w be a WS on an ideal space $(X,\tau,\mathcal{I})$. Then $(X,\tau,\mathcal{I})$ is said to be an $\mathcal{I}_{wgp}$-space if every $\mathcal{I}_{wgp}$-open set of X is open in X.*

Example 2.28.
(1) *Let X=$\{a,\ b,\ c\}$, τ=$\{\emptyset,\ \{a\},\ \{b\},\ \{a,\ b\},\ X\}$, w=$\{\emptyset,\ \{a\},\ \{b\},\ \{a,\ b\},\ X\}$ and $\mathcal{I}$=$\{\emptyset,\ \{a,\ c\}\}$. Then $\mathcal{I}_{wgp}$-open sets are $\{a\}$, $\{b\}$, $\{a,\ b\}$, $\emptyset$, X. Then $(X,\tau,\mathcal{I})$ is $\mathcal{I}_{wgp}$-space.*
(2) *Let X= $\{a,\ b,\ c\}$, τ=$\{\emptyset,\ \{a\},\ X\}$, w= $\{\emptyset,\ \{a\},\ \{a,\ c\},\ X\}$ and $\mathcal{I}$=$\{\emptyset,\ \{c\}\}$. Then $\mathcal{I}_{wgp}$-open sets are $\{a\}$, $\{a,\ b\}$, $\emptyset$, X. Then $(X,\tau,\mathcal{I})$ is not $\mathcal{I}_{wgp}$-space.*

Theorem 2.29. *Let w be a WS on an $\mathcal{I}_{wgp}$-space X. If a function $f:(X,\tau,\mathcal{I})\to(Y,\sigma)$ is contra $\mathcal{I}_{wgp}$-continuous then f is contra continuous.*

Proof. Let V be any closed set in Y. Since f is contra $\mathcal{I}_{wgp}$-continuous, $f^{-1}(V)$ is $\mathcal{I}_{wgp}$-open in X. Since X is an $\mathcal{I}_{wgp}$-space, $f^{-1}(V)$ is open in X. Therefore f is contra continuous.

Definition 2.30. *Let w be a WS on an ideal space $(X,\tau,\mathcal{I})$. Then $(X,\tau,\mathcal{I})$ is said to be $\mathcal{I}_{wgp}$-T_2 space if for each pair of distinct points x and y in $(X,\tau,\mathcal{I})$, there exist an $\mathcal{I}_{wgp}$-open set P containing x and an $\mathcal{I}_{wgp}$-open set Q containing y such that $P\cap Q$=$\emptyset$.*

Example 2.31.
(1) *Let X=$\{a,\ b,\ c\}$, τ=$\{\emptyset,\ \{a\},\ \{b\},\ \{c\},\ \{a,\ b\},\ \{a,\ c\},\ \{b,\ c\},\ X\}$, w=$\{\emptyset,\ \{a,\ b\},\ \{a,\ c\},\ \{b,\ c\},\ X\}$ and $\mathcal{I}$=$\{\emptyset,\ \{a\}\}$. Then $(X,\tau,\mathcal{I})$ is $\mathcal{I}_{wgp}$-T_2 space.*
(2) *Let X=$\{a,\ b,\ c\}$, τ=$\{\emptyset,\ \{a\},\ \{b\},\ \{c\},\ \{a,\ b\},\ \{a,\ c\},\ \{b,\ c\},\ X\}$, w=$\{\emptyset,\ \{a,\ b\},\ \{a,\ c\},\ \{b,\ c\},\ X\}$ and $\mathcal{I}$=$\{\emptyset,\ \{a\}\}$. Then $(X,\tau,\mathcal{I})$ is not $\mathcal{I}_{wgp}$-T_2 space.*

Theorem 2.32. *If w is a WS on an ideal space $(X, \tau, \mathcal{I})$ and for each pair of distinct points x_1, x_2 in X, there exists a function f from $(X, \tau, \mathcal{I})$ into a Urysohn space Y such that $f(x_1) \neq f(x_2)$ and f is contra $\mathcal{I}_{wgp}$-continuous at x_1 and x_2, then X is $\mathcal{I}_{wgp}$-T_2.*

Proof. Let x_1 and x_2 be any two distinct points in X. Then by hypothesis, there is a function $f : (X, \tau, \mathcal{I}) \to (Y, \sigma)$, such that $f(x_1) \neq f(x_2)$. Let $y_i = f(x_i)$ for $i = 1, 2$. Then $y_1 \neq y_2$. Since Y is Urysohn, there exist open neighbourhoods Q_{y_1} and Q_{y_2} of y_1 and y_2 respectively in Y such that $cl(Q_{y_1}) \cap cl(Q_{y_2}) = \emptyset$. Since f is contra $\mathcal{I}_{wgp}$-continuous, there exists an $\mathcal{I}_{wgp}$-open set P_{x_i} of x_i in X such that $f(P_{x_i}) \subseteq \mathrm{cl}(Q_{y_i})$ for $i = 1, 2$. Hence we get $P_{x_1} \cap P_{x_2} = \emptyset$ because $cl(Q_{y_1}) \cap cl(Q_{y_2}) = \emptyset$. Thus X is $\mathcal{I}_{wgp}$-T_2.

Corollary 2.33. *Let w be a WS on an ideal space $(X, \tau, \mathcal{I})$. If f is a contra $\mathcal{I}_{wgp}$-continuous injection of $(X, \tau, \mathcal{I})$ into a Urysohn space (Y, σ), then $(X, \tau, \mathcal{I})$ is $\mathcal{I}_{wgp}$-T_2.*

Proof. Let x_1 and x_2 be any pair of distinct points in X. Since f is contra $\mathcal{I}_{wgp}$-continuous and injective, we have $f(x_1) \neq f(x_2)$. Therefore by Theorem 2.32, X is $\mathcal{I}_{wgp}$-T_2.

Corollary 2.34. *Let w be a WS on an ideal space $(X, \tau, \mathcal{I})$. If f is a contra $\mathcal{I}_{wgp}$-continuous injection of $(X, \tau, \mathcal{I})$ into a Ultra Hausdorff space (Y, σ), then $(X, \tau, \mathcal{I})$ is $\mathcal{I}_{wgp}$-T_2.*

Proof. Let x_1 and x_2 be any two distinct points in X. Then since f is injective and Y is Ultra Hausdorff, $f(x_1) \neq f(x_2)$ and there exist two clopen sets V_1 and V_2 in Y such that $f(x_1) \in V_1, f(x_2) \in V_2$ and $V_1 \cap V_2 = \emptyset$. Then $x_i \in f^{-1}(V_i) \in \mathcal{I}w\mathrm{GPO(X)}$ for $i = 1, 2$ and $f^{-1}(V_1) \cap f^{-1}(V_2) = \emptyset$. Thus X is $\mathcal{I}_{wgp}$-T_2.

Theorem 2.35. *Let w be a WS on an ideal space $(X, \tau, \mathcal{I})$. If $f : (X, \tau, \mathcal{I}) \to (Y, \sigma)$ is a contra $\mathcal{I}_{wgp}$-continuous, closed injection and Y is Ultra normal, then $(X, \tau, \mathcal{I})$ is $\mathcal{I}_{wgp}$-normal.*

Proof. Let G_1 and G_2 be disjoint closed subsets of X. Since f is closed and injective, $f(G_1)$ and $f(G_2)$ are disjoint closed subsets of Y. Since Y is Ultra normal, $f(G_1)$ and $f(G_2)$ are separated by disjoint clopen sets Q_1 and Q_2 respectively. Hence $G_i \subseteq f^{-1}(Q_i), f^{-1}(Q_i) \in \mathcal{I}w$GPO(X) for $i = 1, 2$ and $f^{-1}(Q_1) \cap f^{-1}(Q_2) = \emptyset$. Thus X is $\mathcal{I}_{wgp}$-normal.

Definition 2.36. *Let w be a WS on an ideal space $(X, \tau, \mathcal{I})$. A graph $G(f)$ of a function $f : (X, \tau, \mathcal{I}) \to (Y, \sigma)$ is said to be contra $\mathcal{I}_{wgp}$-closed if for each $(x, y) \in (X \times Y)\backslash G(f)$, there exists $P \in \mathcal{I}w$GPO(X) containing x and a closed set Q of (Y, σ) containing y such that $f(P) \cap Q = \emptyset$.*

Example 2.37. *Let $X=Y=\{a, b, c\}$, $\tau=\sigma=\{\emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, X=Y\}$, $w=\{\emptyset, \{a, b\}, \{a, c\}, \{b, c\}, X\}$ and $\mathcal{I}=\{\emptyset, \{a\}\}$. Let $f : (X, \tau, \mathcal{I}) \to (Y, \sigma)$ be an identity function. Then $G(f)$ is contra $\mathcal{I}_{wgp}$-closed in $X \times Y$.*

Theorem 2.38. *Let w be a WS on an ideal space $(X, \tau, \mathcal{I})$. If $f : (X, \tau, \mathcal{I}) \to (Y, \sigma)$ is contra $\mathcal{I}_{wgp}$-continuous and (Y, σ) is Urysohn, then $G(f)$ is contra $\mathcal{I}_{wgp}$-closed in $X \times Y$.*

Proof. Let $(x, y) \in (X \times Y)\backslash G(f)$, then $f(x) \neq y$ and there exist open sets Q, R of Y such that f(x)∈Q, y∈R and cl(Q)∩cl(R)=∅. Since f is contra $\mathcal{I}_{wgp}$-continuous there exists P∈$\mathcal{I}w$GPO(X) containing x such that f(P)⊆cl(Q). Since cl(Q)∩cl(R)=∅, we have f(P)∩cl(R)=∅. This shows that $G(f)$ is contra $\mathcal{I}_{wgp}$-closed in $X \times Y$.

Remark 2.39. *The following Example shows that the condition Urysohn on the space (Y, σ) in Theorem 2.38 cannot be dropped.*

Example 2.40. *Let $X=Y=\{a, b, c\}$, $\tau=\sigma=\{\emptyset, \{a\}, X=Y\}$, $w=\{\emptyset, \{a, b\}, X\}$ and $\mathcal{I}=\{\emptyset, \{a\}\}$. Then Y is not a Urysohn space. Also the identity function $f : (X, \tau, \mathcal{I}) \to (Y, \sigma)$ is contra $\mathcal{I}_{wgp}$-continuous but not contra $\mathcal{I}_{wgp}$-closed.*

Corollary 2.41. *Let w be a WS on a space X. If $f:(X,\tau)\to(Y,\sigma)$ is contra wgp-continuous function and (Y,σ) is a Urysohn space, then $G(f)$ is contra wgp-closed in $X\times Y$.*

Proof. The proof follows from the Theorem 2.38 if $\mathcal{I}$=$\{\emptyset\}$.

Remark 2.42. *The following Example shows that the condition Urysohn on the space (Y,σ) in Corollary 2.41 cannot be dropped.*

Example 2.43. *Let X=Y=$\{a, b, c\}$, τ=σ=$\{\emptyset, \{c\}, \{a, b\}, X{=}Y\}$, and w=$\{\emptyset, \{a\}, \{a, b\}, X\}$. Then Y is not a Urysohn space. Also the identity function $f:(X,\tau)\to(Y,\sigma)$ is contra wgp-continuous but not contra wgp-closed.*

Definition 2.44. *Let w be a WS on an ideal space $(X,\tau,\mathcal{I})$. Then $(X,\tau,\mathcal{I})$ is said to be $\mathcal{I}_{wgp}$-connected if $(X,\tau,\mathcal{I})$ cannot be expressed as the union of two disjoint non-empty $\mathcal{I}_{wgp}$-open subsets of $(X,\tau,\mathcal{I})$.*

Example 2.45.

1. *Let X=$\{a, b, c\}$, τ=$\{\emptyset, \{a\}, \{a, c\}, \{a, b\}, X\}$, w=$\{\emptyset, \{a\}, \{a, b\}, \{a, c\}, X\}$ and $\mathcal{I}$=$\{\emptyset\}$. Then $\mathcal{I}_{wgp}$-open sets are $\{a\}$, $\{a, b\}$, $\{a, c\}$, $\emptyset$, X. Then $(X,\tau,\mathcal{I})$ is $\mathcal{I}_{wgp}$-connected.*
2. *Let X=$\{a, b, c\}$, τ=$\{\emptyset, \{a\}, \{b, c\}, X\}$, w=$\{\emptyset, \{a\}, \{a, b\}, X\}$ and $\mathcal{I}$=$\{\emptyset\}$. Then $\mathcal{I}_{wgp}$-open sets are $\emptyset$, $\{a\}$, $\{b, c\}$, X. Then $(X,\tau,\mathcal{I})$ is not $\mathcal{I}_{wgp}$-connected.*

Theorem 2.46. *Let w be a WS on an ideal space $(X,\tau,\mathcal{I})$. Then a contra $\mathcal{I}_{wgp}$-continuous image of a $\mathcal{I}_{wgp}$-connected space is connected.*

Proof. Let $f:(X,\tau,\mathcal{I})\to(Y,\sigma)$ be a contra $\mathcal{I}_{wgp}$-continuous function of an $\mathcal{I}_{wgp}$-connected space $(X,\tau,\mathcal{I})$ onto a space (Y,σ). If possible, let Y be disconnected. Let M and N form a disconnection of Y. Then M and N are clopen and Y=M∪N where M∩N=∅. Since f is contra $\mathcal{I}_{wgp}$-continuous, $X=f^{-1}(Y)=f^{-1}(M\cup N)=f^{-1}(M)\cup f^{-1}(N)$, where $f^{-1}(M)$ and $f^{-1}(N)$ are nonempty $\mathcal{I}_{wgp}$-open sets in X. Also $f^{-1}(M)\cap f^{-1}(N)=\emptyset$. Hence X is not $\mathcal{I}_{wgp}$-connected. This is a contradiction. Therefore Y is connected.

Definition 2.47. *Let w be a WS on a space (X, τ). Then (X, τ) is said to be wgp-connected if (X, τ) can not be expressed as the union of two disjoint non-empty wgp-open subsets of (X, τ).*

Example 2.48. (1) *Let $X=\{a, b, c\}$, $\tau=\{\emptyset, \{b\}, \{b, c\}, X\}$ and $w=\{\emptyset, \{b, c\}, \{a, b\}, X\}$. Then (X, τ) is wgp-connected.*

(2) *Let $X=\{a, b, c\}$, $\tau=\{\emptyset, \{b\}, \{a, c\}, X\}$ and $w=\{\emptyset, \{a\}, \{c\}, \{a, b\}, X\}$. Then (X, τ) is not wgp-connected.*

Corollary 2.49. *Let w be a WS on a space X. Then a contra wgp-continuous image of a wgp-connected space is connected.*

Proof. The proof follows from the Theorem 2.46 if $\mathcal{I}=\{\emptyset\}$.

Lemma 2.50. *For a WS w on an ideal space $(X, \tau, \mathcal{I})$, the following are equivalent.*

(1) X is $\mathcal{I}_{wgp}$-connected.
(2) The only subset of X which are both $\mathcal{I}_{wgp}$-open and $\mathcal{I}_{wgp}$-closed are the empty set $\emptyset$ and X.

Proof. (1) $\Rightarrow$ (2). Let G be an $\mathcal{I}_{wgp}$-open and $\mathcal{I}_{wgp}$-closed subset of X. Then $X-G$ is both $\mathcal{I}_{wgp}$-open and $\mathcal{I}_{wgp}$-closed. Since X is $\mathcal{I}_{wgp}$-connected, X can be expressed as union of two disjoint non-empty $\mathcal{I}_{wgp}$-open sets X and $X-G$, which implies $X-G$ is empty.

(2) $\Rightarrow$ (1). Suppose $X=P\cup Q$ where P and Q are disjoint non-empty $\mathcal{I}_{wgp}$-open subsets of X. Then P is both $\mathcal{I}_{wgp}$-open and $\mathcal{I}_{wgp}$-closed. By assumption either P=$\emptyset$ or X which contradicts the assumption that P and Q are disjoint nonempty $\mathcal{I}_{wgp}$-open subsets of X. Therefore X is $\mathcal{I}_{wgp}$-connected.

Definition 2.51. [5] *Let $f:(X,\tau)\to(Y,\sigma)$ be a function. Then f is called preclosed if $f(V)$ is preclosed in Y for each closed set V of X.*

Theorem 2.52. *Let w be a WS on an ideal space $(X, \tau, \mathcal{I})$. Let $f : (X, \tau, \mathcal{I}) \to (Y, \sigma)$ be a surjective preclosed contra $\mathcal{I}_{wgp}$-continuous function. If X is an $\mathcal{I}_{wgp}$-space, then Y is locally indiscrete.*

Proof. Suppose that Q is open in Y. Since f is contra $\mathcal{I}_{wgp}$-continuous, $f^{-1}(Q) = P$ is $\mathcal{I}_{wgp}$-closed in X. Since X is an $\mathcal{I}_{wgp}$-space, P is closed in X. Since f is preclosed, then Q is preclosed in Y. Now we have cl(Q)=cl(int(Q))⊆Q. This means that Q is closed and hence Y is locally indiscrete.

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