Use Of graph Theory In Transportation Problems and Different Networks

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Abstract:

One of the important issues in everyday life is optimization problem and reduction of the cost of distribution and transportation of goods. In modern day automobile traffic the problem of traffic congestion calls for the design of efficient control strategies The purpose of this paper is to examine the problem and its solution to the modelling induced by graph theory. Researchers are always in line with this objective by providing search tools trying various approaches to minimize such cost. This provides the motivation for this paper in transportation problem and different networks. Here we are considering the different networks i.e roads, railway , and a Air networks. In this it is argued that in order to have efficient and systematic solutions to a traffic control problem at an intersection, graph theoretic models of the problem are quite appropriate for its exploitation. Connectivity of compatibility graph of a traffic intersection can be used to study the most efficient route or the traffic control system to direct the traffic flow to its maximum capacity using the minimum number of edges or the minimum number of vertices.

When trying to estimate the maximum number of people who can fly from a city Ci to a city Cj through different routes and airports, one can use a networks N as a model. Let us assume the source s, as the origin Ci of the trip and the sink t, as the destination Cj, and the remaining internal nodes vi are the intermediate airports.

Keyword:- Graph Theory, Direct graph, Graph networks, Simple graphs .Multi graph, , Distance Balance, Null Graph, Compatibility Graph, Edge Connectivity.

INTRODUCTION

During the last decades, graph theory has attracted the attention of many researchers. Graph theory has provided very nice atmosphere for research of provable technique in discrete mathematics for researches. Many application in the computing, industrial, natural and social science are studied by graph theory.

Traffic theory is a physical phenomenon that aims at understanding and improving automobile traffic, and the problem associated with it such as traffic congestion. The traffic control problem is to minimize the waiting time of the public transportation while maintaining the individual traffic flow optimally. It worth mentioning that all graph are usually classified when we encounter to special graph in modelling of phenomena in real life. The graph theory networks have always been important in transportation and telecommunication. They have become more important for all business today, especially because of the internet. The internet has connected virtually everything today. It has connected everybody, everything, everywhere into a network. Of course the internet has also changed how existing networks behave. Graph theoretic paper part 1 and part 2 discuss of certain transportation problem and railway networks.

One of the main uses of traffic theory is the development of traffic models which can be used for estimation, prediction, and control related tasks for the automobile traffic process. In this paper graph theoretic model of a traffic control problem at an intersection is used for its solution. The resulting compatibility graph of the intersection and its connectivity is used to study the most efficient route or the traffic control system to direct the traffic flow to its maximum capacity using the minimum number of edges or the minimum number of vertices.

Use of Graph Theory in Transportation Networks: In solving problems in transportation networks Graph theory in mathematics is a fundamental tool. The term graph in mathematics has two different meaning. One is the graph of a function or the graph of a relation. The second usually related to "graph theory" is a collection of "vertices" or "nodal" and "links" or "edges" for purpose of this paper we are concerned with the latter type graph theory has been closely tied to its applications and its use first can be credited to transport ant followed by its application to other fields. In transportation graph theory is most commonly used to study problems.

Graphs are used to model situation in which a commodity is transported from one location to another. A common example is the water supply, where the pipelines are edge, vertices represent water users, pipe joins and so on. Highway systems can be thought of as transporting cars. In many examples it is natural to interpret some or all edges as directed. A common feature of transportation system is the existence of a capacity associated with each edge……..the maximum number of cars that can use a road in an hour.The maximum amount of water that can pass through a pipe and so on.

II INTELLIGENT TRANSPORTATION SYSTEM

The term Intelligent Transportation System (ITS) refers to information and communication technology applied to transport infrastructure and vehicles, that improves transport outcomes such as transport safety, transport productivity, transport reliability, informed traveller choice, environmental performance etc. ITS mainly comes from the problems caused by traffic congestion and synergy of new information technology for simulation, real time control and communication networks. Traffic congestion has been increased world wide as a result of increased motorization, urbanization, population growth and changes in population density. Congestion reduces efficiency of transportation infrastructure and increases travel time, air pollution and fuel consumption.

Intelligent Transport Systems vary in technologies applied, from basic management system such as car navigation; traffic signal control systems; container management system; variable message sign; automatic number plate recognition or speed cameras to monitor applications; such as security CCTV systems; and to more advanced applications that integrate live data and feedback from a number of other sources, such as parking guidance and information systems; weather information etc.

The traffic flow predictions will be delivered to the drivers via different channels such as roadside billboards, radio stations, internet, and on vehicle GPS (Global Positioning Systems) systems. One of the components of an ITS is the live traffic data collection. To collect accurate traffic data sensors have to be placed on the roads and streets to measure the flow of traffic. Some of the constituent technologies implemented in ITS are namely, Wireless Communication, Computational technologies, Sensing technologies, Video Vehicle Detection etc. Sensing technologies, which is the present interest, mean briefly the following:

The technological advances in telecommunication and information technology, coupled with microchip, RFID (Radio Frequency Identification), and inexpensive intelligent beacon sensing technologies, have enhanced the technical capabilities that will facilitate safety of the traffic participants for intelligent transportation system globally. Sensing systems for ITS are vehicle- and infrastructure based networked systems i.e. Intelligent vehicle technologies. Infrastructure sensors are such as in road reflector devices that are already installed or embedded in the road or surrounding the road e.g. on building, posts and signs, as required and may be manually disseminated during preventive road construction maintenance or by sensor injection machinery for rapid development.

III COMPATIBILITY GRAPH AND ITS CONNECTIVITY

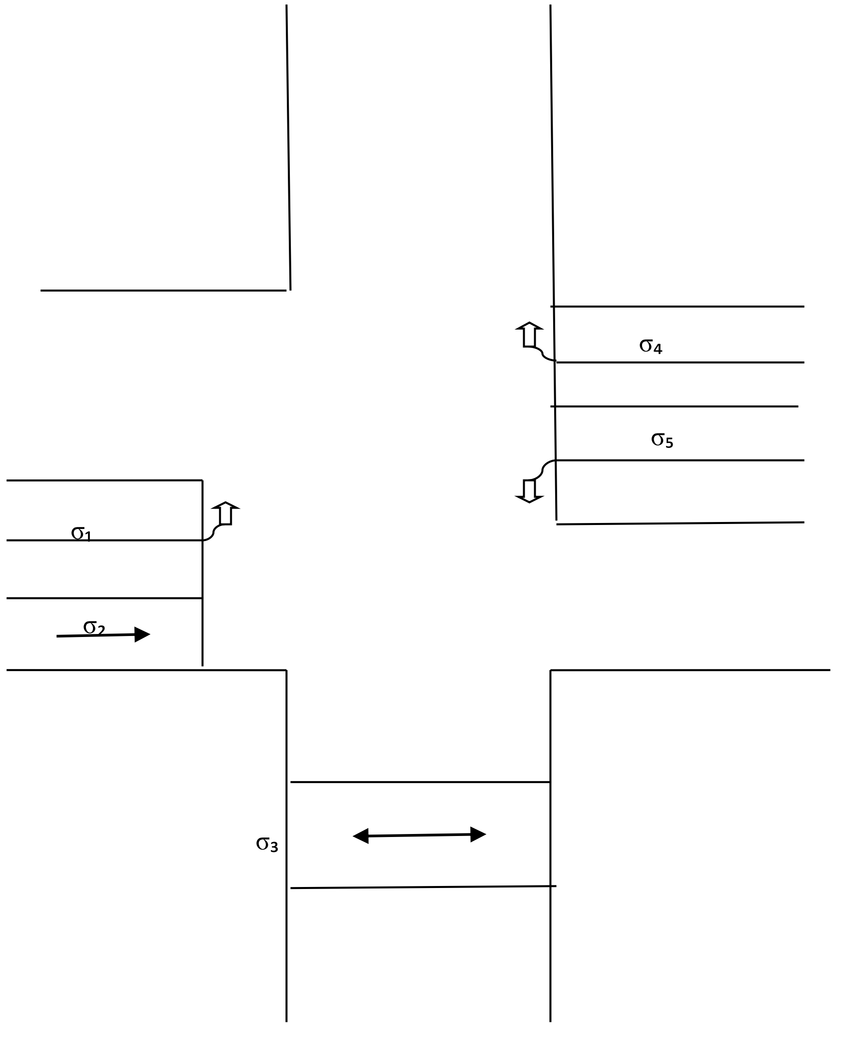
Vehicles approaching an intersection prepare themselves to perform a certain manoeuvre i.e. to drive through, turn left, or turn right at an intersection. The vehicles that perform this manoeuvre represent a flow component. Such an arrival flow component is called a traffic stream. For solving the control problem it is necessary to know relations between traffic streams at an intersection. Traffic streams mean briefly the following :

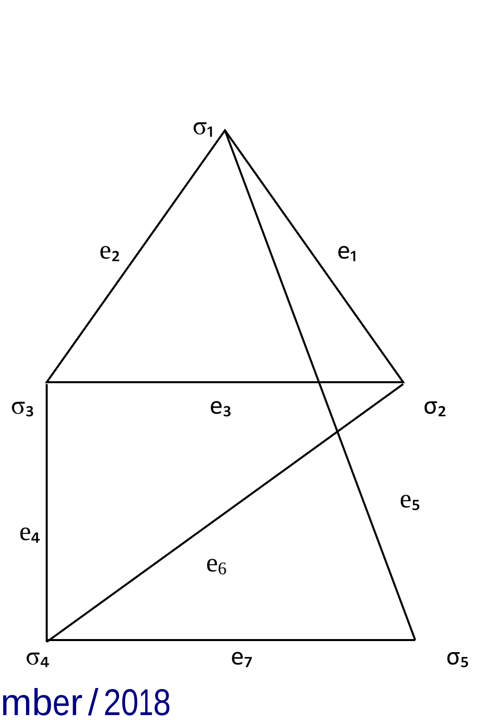
Traffic streams σ_i on an intersection are elements of the set of traffic stream, σ i.e

$\sigma = (\sigma_1, \sigma_2, \sigma_3, \dots, \sigma_i)$

where $\sigma_1, \sigma_2, \sigma_3, \dots, \sigma_i$ are the individual traffic streams of an intersection.

To study the traffic control problem at an arbitrary intersection, it has to be modelled mathematically by using a simple graph for the traffic collection data problem. The set of edges of the underlying graph will represent the communication link between the set of nodes i.e. traffic streams at an intersection. In the graph representing the traffic control problem, the traffic streams which can move simultaneously at an intersection without any conflict will be joined by an edge and the streams which cannot move together will not be connected by an edge.


σ4
σ5
σ1
σ2
σ3


σ1
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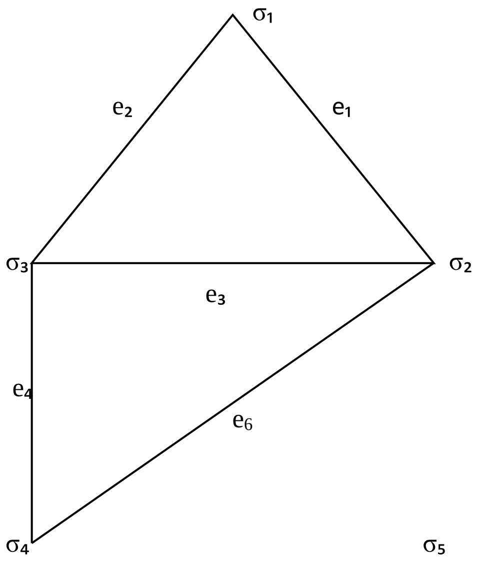
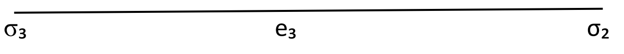




Some theorems:

a) The edge connectivity of a graph *G* cannot exceed the degree of the vertex with the smallest degree in *G*.

Proof :

The vertex v_i be the vertex with the smallest degree in G.

Let $d(v_i)$ be the degree of $v_{i.}$

Vertex v_i can be separated from G by removing the $d(v_i)$ edges incident on vertex v_i.

Hence the theorem.

b) The vertex connectivity of any graph *G* can never exceed the edge connectivity of *G*.

Proof :

Let α denote the edge connectivity.

Therefore,∃ a cutset S in G with α edges.

Let S partition of the vertices of G into subsets V_1 and V_2.

By removing atmost α vertices V_1 or V_2 which the edges in S are incident.

We can effect the removal of S from G.

Hence the theorem.

c) The maximum vertex connectivity one can achieve with a graph *G* of *n* vertices and *e* edges $(e \geq n - 1)$ is the integral part of the number $2e/n$; i.e., $[2e/n]$

Proof :

Every edge in G contributes two degree.

The total (2e degree S) is divided among n-vertices.

Therefore,there must be atleast one vertex in G whose degree is equal to or less than the number 2e/n.

The vertex connectivity of G cannot exceed this number.

First construct a n-vertex regular graph of degree [2e/n] and then add the remaining e-(n/2).

[2e/n] edges arbitary.

Vertex connectivity $\leq$ Edge connectivity $\leq$ 2e/n

and maximum vertex connectivity possible= [2e/n].

The traffic sensors can be placed on each edge in a cut-set of *G* determined by its edge connectivity as well as on each vertex of *G* determined by its vertex connectivity. These sensors will provide complete traffic information for the control system. Thus optimal locations for the traffic sensors can be obtained by using edge connectivity and vertex connectivity of the compatibility graph *G*.

One way street problem: Robin's Theorem, the first problem we consider has to do with movement of traffic. If traffic where to move more rapidly and with fewer delays in our cities, this would alleviate wasted energy and air pollution. It has sometimes been argued that making certain streets one-way would move traffic more efficiently. We consider the one-way and, if so, how to do it. Of course, it is always possible to make certain streets in a city one-way simply put up a one-way street sign. What is desired is to do in such a way that it is still possible to get from any place to any other place. Let us begin with that it is still possible to get from any place to any other place. Let us begin with the simplified problem where every street is currently two-way and it is desired to make every street one-way in the future. We can formulate this problem graph theoretically by taking the street corners as the vertices of a graph. And drawing an edge between two street corners if only if these corners are currently joined by a two-way street. We wish to place a direction on each edge of this graph we speak of orienting each edge-so that in the resulting digraph. It is possible to go from any place to any other place.

Chinese Postman's Problem: In 1962, A Chinese mathematician called Kuan Mei-ko was interested in a postman delivering mail to a number of streets. Such that the total distance walked by the postman was as short possible. How could the postman ensure that the distance walked was minimum.

Following example: - A postman has to start at A, walk along all 13 streets and return to A. The numbers on each edge represent the Length, in meters, of each street. The problem is to find a train that uses all the edges of a graph with minimum Length .We will return to solving this actual problem later, but initially we will look at drawing various graphs. The Chinese postman is traversable graphs given below.

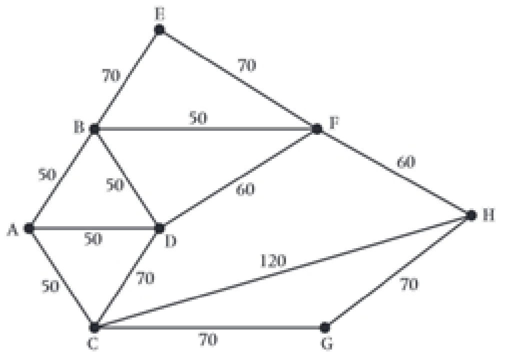
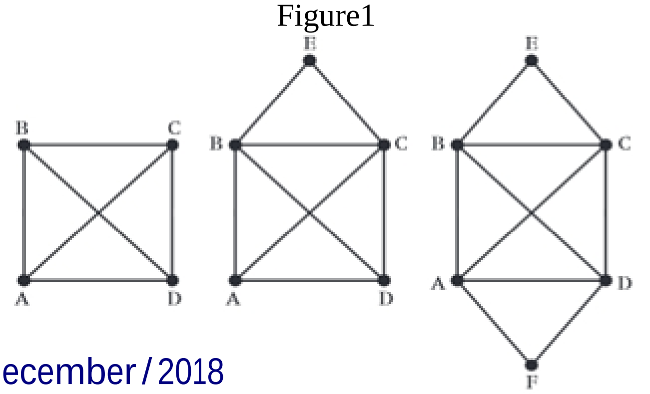


Figure1



Graph 1 Graph 2 Graph 3

Figure 2

From these Graph we find;

- It is impossible to draw graph 1 without either taking the pen off the paper or re- tracing an edge.
- We can draw graph 2, but only by starting at either A or D-in each case the path will end at the other vertex of DorA.
- Graph 3 can be drawn regardless of the starting position and you will always return to the start vertex.

In order to establish the differences, we must consider the order of the vertices for each graph. The following When the order of all the vertices is even the graph is

Vertex	Order
A	3
B	3
C	3
D	3

Graph 01

Vertex	Order
A	3
B	4
C	4
D	3
E	2

Graph 02

Vertex	Order
A	4
B	4
C	4
D	4
E	2
F	2

Graph 03

When the order of all the vertices is even the graph is Travers able. When there are two odd vertices we can draw the graph but the start and end vertices are different. When there are four odd vertices the graph can‟t be drawn without repeating an edge

Chinese postman algorithm : An algorithm for finding an optimal Chinese postman route is.
Step 1 :- List all odd vertices.
Step 2 :- List all possible paring of odd vertices.
Step 3 :- For each pairing find the edges that connect the vertices with the minimum weight.
Step 4 :- Find the pairing such that the sum of the weights is minimized.
Step 5 :- On the original graph add the edges that have been found in step 4.
Step 6 :- The length of an optimal Chinese postman route is the sum of all the edges added to the total found in step 4.
Step 7 :- A route corresponding to this minimum weight can then be easily found.

Now we apply the algorithm to the original problem in fig. 01.as;
Step 01 the odd vertices are A and H.
Step 02 there is only one way of pairing these odd vertices namely AH.
Step 03 the shortest way of joining A to H is using the path AB, BF, FH a total length of 160.
Step 04 these edges on to the original network in this fig

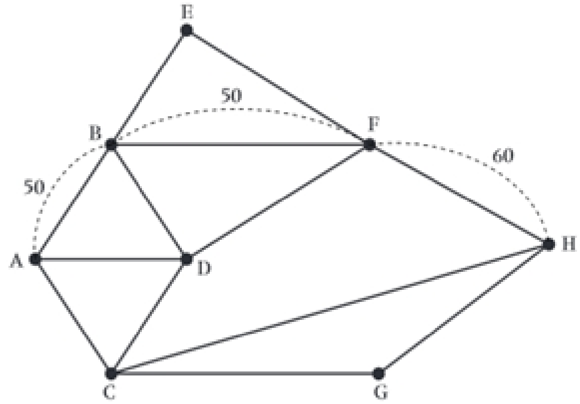


Figure 3

Step 05 the length of the optimal Chinese postman route is the sum of all the edges in the original network. Which is 840 mtr Plus the answer found in step 4, which is 160 mtr., Hence the length of the optimal Chinese postman route is 100 mtr.
Step 06 one possible route corresponding to this length is ADCGHCABDFBEFHFBA, but many other possible routes of the sum minimum length can be found.

Use of Graph Theory in Railway Networks

One of the most important users of graphs with respect to applications in railway signaling systems is the derivation of paths.

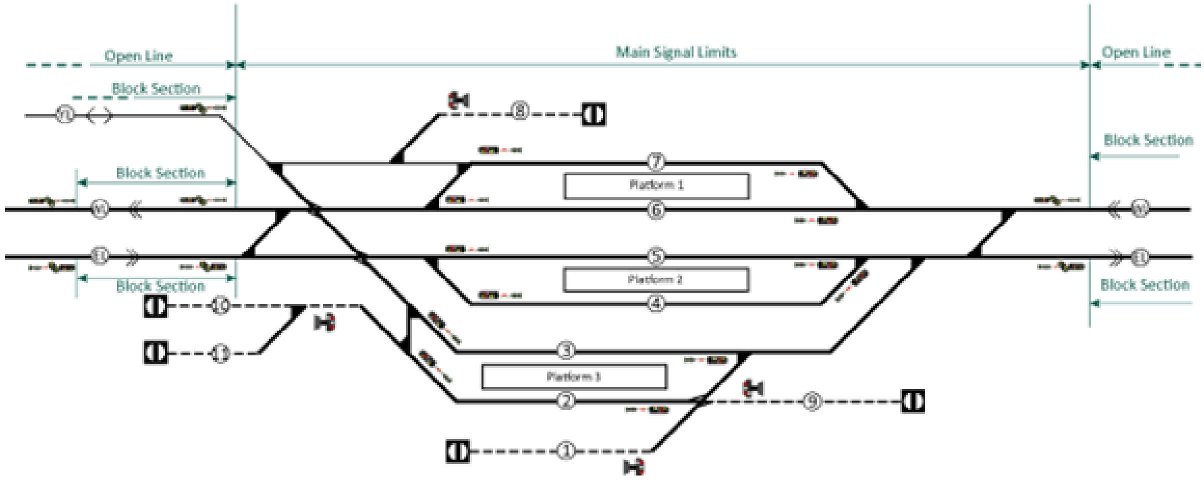


Figure 4

A railway network in special graphs called double vertex graphs. A user can edit the networks topology graphically. Every element of the graph hold various attributes. An edge for example holds a track sections length, gradient, maximum speed for different train categories and much more. A user can create and manage objects for edges and vertices and also signals, switches, stations and router shows an example for a station.

Output data There is evaluation in the form of diagrams of train movement , track occupation and line profiles. Every station produces output about all the trains. That used it, including arrival, stopping and departure times. The user can view output data in either a diagram or and excel table and ASC II table.

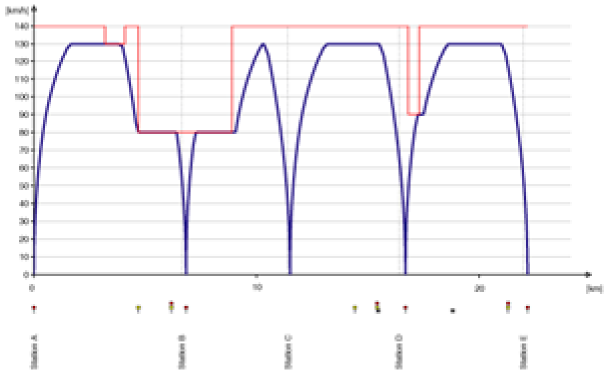
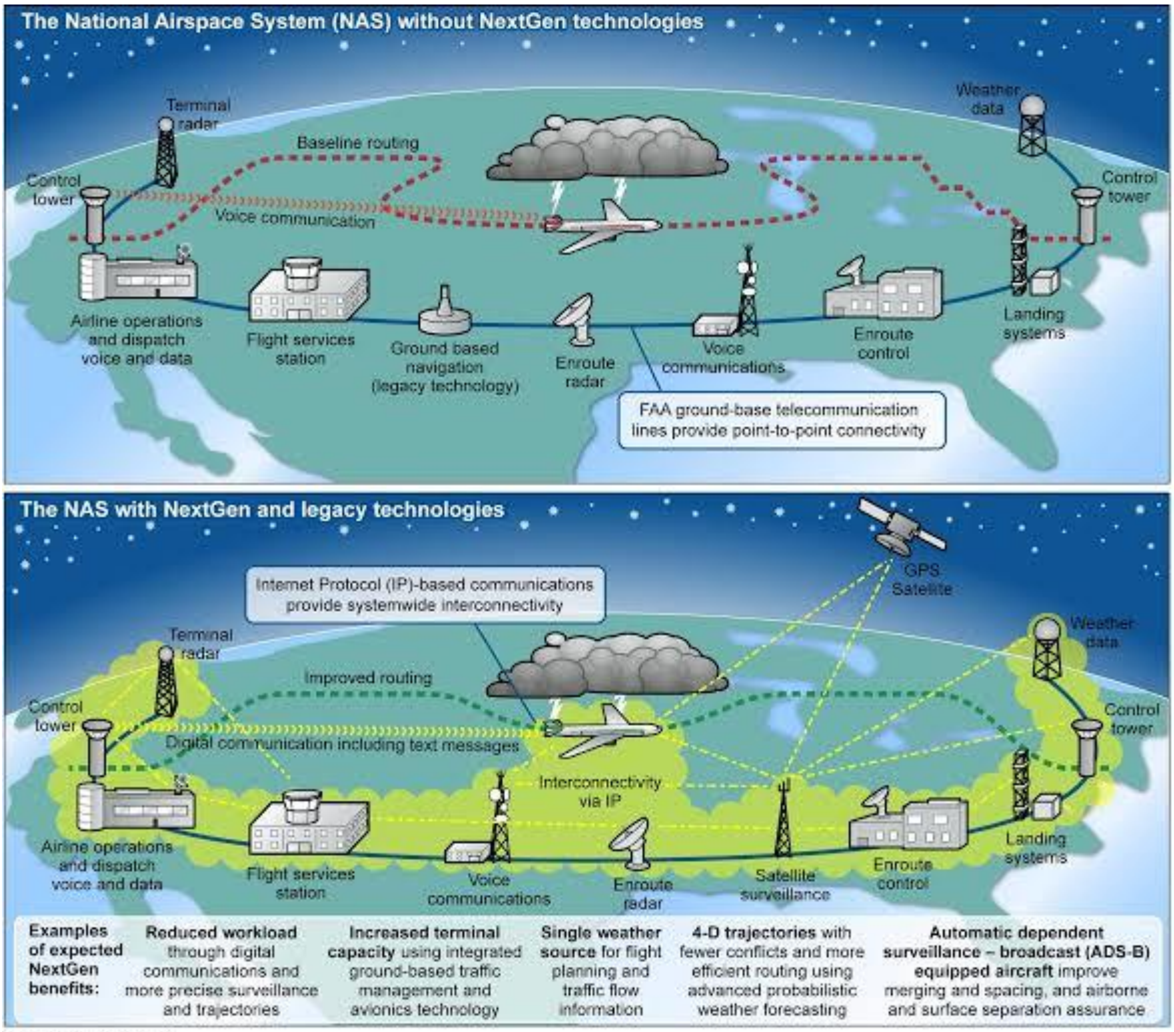


Figure 5

Maritime Traffic :-

Let ui, i = 1, 2………, m and vj j = 1, 2…….., n are different seaports and some products are ready for shipment at ui to vj. let si be the quantity available at ai and dj the quantity demanded at vj. How should the products be shipped?

Here also, network serves as a model. That is ui, i = 1, 2………, m and vj j = 1, 2…….., n are treated as nodes and shipping routes can be represented by arcs of the form (ui, vj) with a capacity equal to the shipping capacity between the two seaports. Two new nose s and t are introduced as a source and sink, respectively such that join s to each ui by an arc with capacity c(s, ui) = si and join each node vi to t by an arc with capacity c(vj, t) = dj. A maximum flow for this transportation network yields the quantity of products to ship along each route in order to satisfy all demands, if this is possible.



Source: GAO. | GAO-15-370

Figure 6

Graph Theory use Air Traffic Control Network :- Air traffic control is an essential element of the communication structure which supports air transportation.Two basic for air traffic control (ATC) are safely and efficiency of air traffic movement. ATC organizes the air space to achieve the objective of a safe, expeditious and orderly flow of air traffic. The increasing range of aircraft technology means more attention to the allotment of air space. The problem is future compounded by the fact that busy airports sustain excessive lending and departure rates and airports themselves are invariably situated within busy terminal areas and in close proximity to other airports. Future more, these airports are often sited near the junction of air routers serving other destinations.

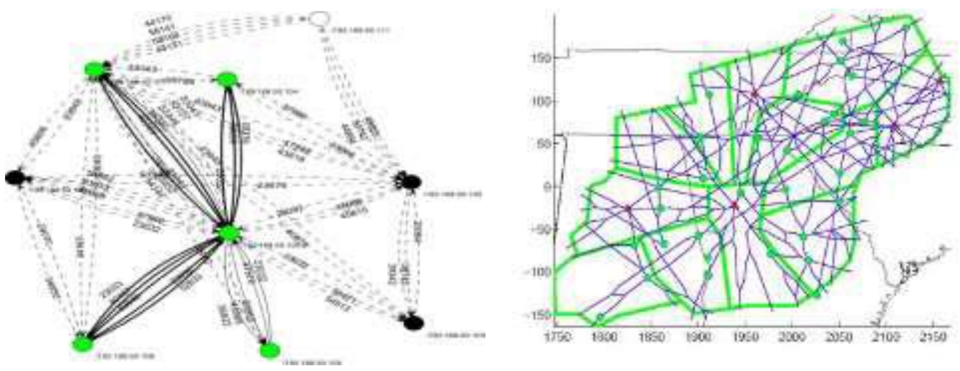


Figure 7

The tern air traffic control is defined as service provided for the purpose of

- ❖ Preventing collision between aircraft on the air
- ❖ Assist in preventing collision between air craft moving on the apron or the maneuvering area.
- ❖ Expedite and maintain an orderly flow of air traffic and
- ❖ Providing information useful for safe and effective conduct of flights

To mange air traffic system there are three basic types of manned facilities, namely, air route traffic control centre, the airport traffic control tower and the flight service station.

Air route traffic control centers (ARTCC) - The ARTCC is to control air traffic network within the assigned area . That is the area which is outside the confines of air spaces designated for the provision of air traffic services by approach control and aerodrome control. Each centre has control of a definite geographic area and is concerned primarily with the control of aircraft operating under IFR. For ease of operation of work on area control unit is divided into sectors. These sectors are usually longitudinal in dimension having specific boundary which are delineated by en route reporting points. In some cases sectors are also divided vertically. Permitting a separate sector responsibility for the air routes within the upper air space. The sectors are required to work in close liaison, one with another, their manning and method of operation of being primarily determined by the nature of technical equipment provided to carry out the tasks. Aircraft must not be permitted to penetrate the airspace of another sector or ARTCC unless prior coordination has taken place. It can be observed that an aircraft flight plan is transferred between sectors within an ARTCC and between ARTCC's when crossing the ARTCC boundary. At the boundary points marking the limits of ARTCC, the aircraft is released to and adjacent centre or to terminal control or an approach control facility.

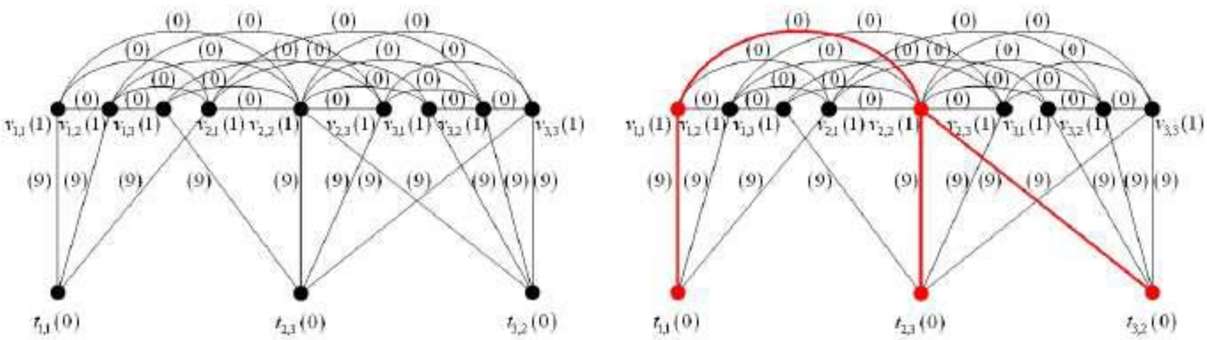


Figure 8

Terminal Approach Control : These purpose is to protect the flight path of aircraft leaving the airways system to land at the airport in the terminal or alternatively the flight path of aircraft departing the terminal for and en route airway. When these are several airports in and urban area. One facility may control traffic to all these airports. An approach control of busy airport can handle as many as fine stacks of arriving aircraft which have been transferred to it by ARTCC.

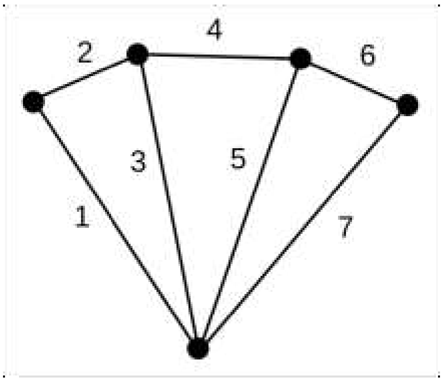
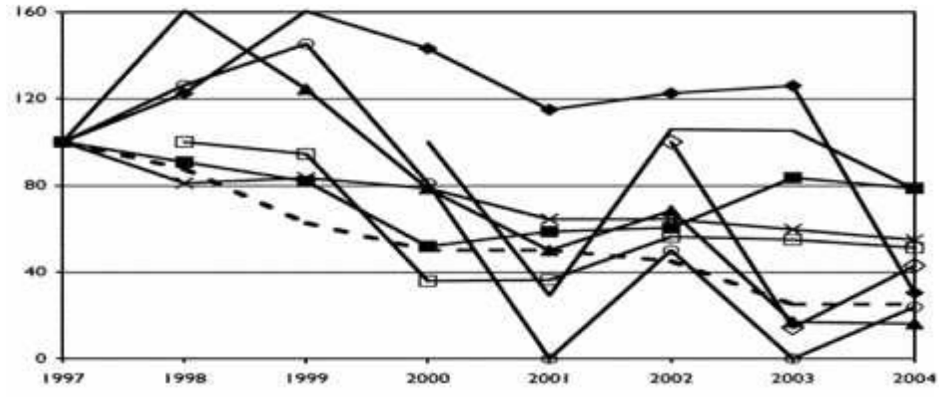


Figure 9

Air Traffic Control Tower : The modern airport, control room sits on the top of a concrete stalk or on top of a brick building placed at permissible height within the clearance angles of the airport runways. Seeing by eye, what is actually happening within the immediate environment of the airport and on its surface is what this part of the ATC service is all about. Usually at busy airports these would be two controllers. The air controllers and the ground controller. Air controller is responsible for aircraft which are flying in the vicinity of airport traffic zone and for aircraft taking off and landing. Ground movement on the airport surface. It is essential for him to see, as much as possible of the airport surface including its taxi ways and exit point form the runway in use.



The Traveling Salesman Problem :

The traveling salesman problem consists of a salesman and set of cities.The salesman has to visit each one of the cities starting from a certain one(e.g. the hometown) and returning to the same city.The challenge of the problem is that the traveling sales man wants to minimize the total length of the trip.

Solution :

The traveling salesman problem can be described as follows:

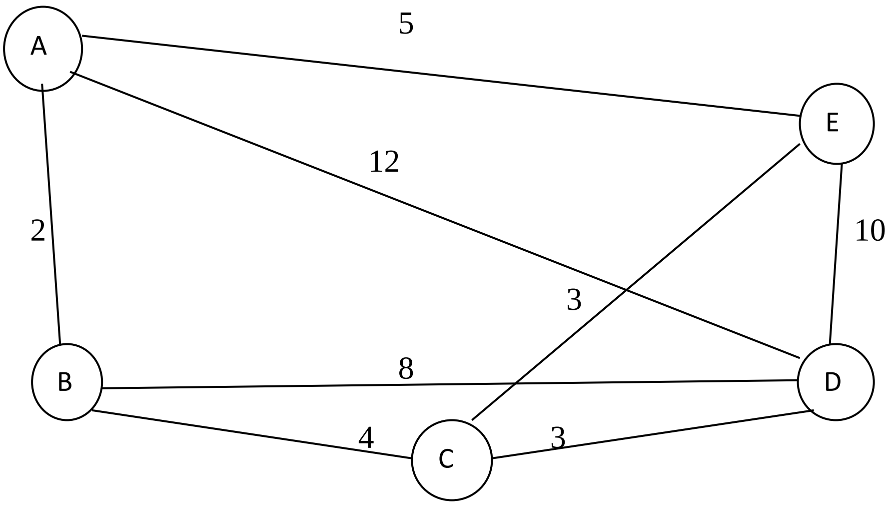
Traveling Salesman Problem={(G,f,t): G=(V,E) a complete graph,

f is a function VxV→Z,

t€Z,

G is a graph that contains a traveling salesman tour with cost that doesnot exceed t}

Consider the following set of cities:



The problem lies in finding a minimal part passing from all vertices once.

For example,The path Path1 {A,B,C,D,E,A} and the path Path2 {A,B,C,E,D,A} pass all the vertices but path1 has total length of 24 and path2 has a total length of 31.

CONCLUSION

In this paper edge connectivity as well as vertex connectivity are used as graph theoretic tools to study traffic control problem at an intersection. As the edges of the edge connectivity represents the flow of traffic at an intersection, the waiting time of the traffic participants can be minimized by controlling the edges of the edge connectivity. This can be achieved by placing traffic sensors on each such edges of the edge connectivity of the transportation network which will provide complete traffic information of the network. Transportation Problems and Railway Networks are especially to project the idea of graph theory. Researcher may get some information related to graph theory and Transportation Problems and Railway Networks field and can get some ideas related to their field of research. This paper is designed to benefit the students of computer science to gain depth knowledge of transportation problems and railway networks.

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