Existence and Uniqueness of Solutions for Special Random Impulsive Differential Equation

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Abstract— In this paper, we study the existence and uniqueness of the mild solution for special random impulsive differential equations of the form

$$x'(t) = Ax(t) + f\big(t, x(t), U\,x(t), V\,x(t)\big), t \neq \xi_k, t \geq \tau$$
$$x(\xi_k) = b_k(\tau_k)\, x(\xi_k^-),\ k = 1, 2, 3, \ldots,$$
$$x_{t_0} = x_0$$

In real separable Hilbert space X through fixed point technique, where A is the infinitesimal generator of a strongly continuous semi group of bounded linear operator T (t). An example is also provided to illustrate the theory.

Keywords— **Existence, Uniqueness, Fixed point theorem, Random impulses, Special random impulsive differential equation.**

I. INTRODUCTION

An abrupt change of state at certain moments of time is the one of the main features of evolution processes of many diverse fields like Physics, Population dynamics, Aeronautics, Economics, Telecommunications and Engineering etc., When compared to the total duration of the process acting in the form of impulses, the duration of these changes are negligible. Many real world phenomena processes the sudden state changes. Hence the study of impulsive differential equation is begun significant. This study opens a large space for natural frameworks for many real time mathematical phenomenons. At the same time, the uncertainties and complexities related to deterministic equations fails to describe the system precisely; many stochastic models were developed to overcome this challenge.

When the impulses exist at random points, then the solutions of the differential equations is a stochastic process. It is very different from deterministic impulsive differential equations and also it is different from stochastic differential equations. Thus the random impulsive equations give more realistic than deterministic impulsive equations. Impulsive differential equations with random coefficients offer a natural and rational approach. Wu and Meng [12] first introduced random impulsive ordinary differential equations and investigated the boundedness of solutions by Liapunovs direct method. There are few more publications in this field, Iwankievicz and Nielsen [6], investigated dynamic response of non-linear systems to poisson distributed random impulses. Sanz-Serna and Stuart [9] first brought disssipative differential equations with random impulses and used Markov chains to simulate such systems. Tatsuyuki et al [10] presented a mathematical model of random impulse to depict drift motion of granules in chara cells due to myosin-actin interaction.

In [4], the author studied the existence, uniqueness, continuous dependence, Ulam stabilities and exponential stability of random impulsive semi linear differential equations under sufficient condition by using the contraction mapping principle. In [2,11] the author has studied some properties of random type impulsive differential systems. Motivated by the above mentioned works, the main purpose of this paper is to study the random impulsive differential equations. We utilize the technique developed by [7, 8, 14].

This paper is organized as follows: Some preliminaries are presented in Section 2. In Section 3, we investigate the existence and uniqueness of solution of special random impulsive differential equation. Moreover, Lipschitz condition has to be relaxed on the impulsive terms in the deriving results. Finally in Section 4, we give an example to motivate our results.

I. PRELIMINARIES

Let X be a real separable Hilbert space and Ω a non empty set. Assume that τ_k is a random variable defined from Ω to $D_k \{= (0, d_k)\}$ for all k = 1, 2, 3, … where $0 < d_k < +\infty$. Consequently, assume that τ_i and τ_j are independent with each other as $i \neq j$ for i, j =1, 2,... Let $\tau \in \Re$ be a constant. For notational convenience, we denote $\Re_\tau = [\tau, T]$. We consider the differential equations with random impulsive of the form

$$x'(t) = Ax(t) + f(t, x(t), U x(t), V x(t)), t \neq \xi_k, t \geq \tau \qquad (2.1)$$

$$x(\xi_k) = b_k(\tau_k)\, x(\xi_k^-), \quad k = 1, 2, 3, ..., \qquad (2.2)$$

$$x_{t_0} = x_0 \qquad (2.3)$$

Where A is the infinitesimal generator of a strongly continuous semigroup of bounded linear operator T(t) in X; $f: \Re_\tau \times X \times X \times X \to X$, $b_k: D_k \to \Re$ for each k = 1, 2, 3, . . . , ; $\xi_0 = t_0 \in [\tau, T]$ and $\xi_k = \xi_{k-1} + \tau_k$ for k = 1, 2, 3, . . . , here $t_0 \in \Re_\tau$ is arbitrary real number. Obviously, $t_0 = \xi_0 < \xi_1 < \xi_2 < . . . < \xi_k < . . .$; $x(\xi_k^-) = \lim_{t \uparrow \xi_k} x(t)$ according to their path with the norm $\| x \| = \sup_{\tau \leq t \leq T} | x(s) |$ for each t satisfying $\tau \leq t \leq T$

$$Ux(t) = \int_{t_0}^{t} K(t, s)x(s)ds, \quad K \in C[D, \Re^+]$$

$$Vx(t) = \int_{t_0}^{T} H(t, s)x(s)ds, \quad H \in C[D_0, \Re^+]$$

Where $D = \{ (t, s) \in \Re^+ : t_0 \leq s \leq t \leq T \}$, $D_0 = \{ (t, s) \in \Re^+ : t_0 \leq t, s \leq T \}$ Let us denote $\{\mathscr{B}_t, t \geq 0\}$ be the simple counting process generated by $\{\xi_n\}$, that is, $\{\mathscr{B}_t \geq t\} = \{\xi_n \leq t\}$, and denote $\mathscr{F}_t$ the σ - algebra generated by $\{\mathscr{B}_t, t \geq 0\}$. Then $(\Omega, P, \{\mathscr{F}_t\})$ is a probability space. Let $L_2 = L_2(\Omega, \{\mathscr{F}_t\}, X)$ denote the Hilbert space of all $\{\mathscr{F}_t\}$ –measurable square integrable random variables with values in X.

Let $\mathscr{B}$ denote Banach space $\mathscr{B}([\tau, T], L_2)$, the family of all $\{\mathscr{F}_t\}$- measurable random variable ψ with norm

$$\| \psi \|^2 = \sup_{t_0 \in [\tau, T]} E \| \psi \|^2.$$

Definition 2.1. *Consider the inhomogeneous initial value problem where $f : [0, T] \to X$.*

$$x'(t) = Ax(t) + f(t)$$
$$x(0) = x_0.$$

Let A be the infinitesimal generator of a C_0 *semigroup T (t). Let* $x_0 \in X$ *and* $f \in L^1(0, T; X)$*. Then the function* $x \in C([0, T]; X)$ *is given by*

$$x(t) = T(t)x_0 + \int_{t_0}^{t} T(t-s)f(s)ds, \quad 0 \le t \le T$$

is the mild solution of the above initial value problem for $t \in [t_0, T]$.

Definition 2.2. *A semigroup* $\{T(t), t \ge 0\}$ *is said to be uniformly bounded if there exist a constant* $M \ge 1$ *such that*

$$\| T(t) \| \le M, \quad for\ t \ge 0$$

Definition 2.3. *For a given* $T \in (\tau, +\infty)$*, a stochastic process* $\{x(t) \in \mathscr{B}, \tau \le t \le T\}$ *is called a mild solution to equation* (2.1) − (2.3) *in* $(\Omega, P, \{\mathscr{F}_t\})$*, if*

(i) $x(t) \in X$ *is* $\mathscr{F}_t$*– adapted;*

(ii)

$$x(t) = \sum_{k=0}^{+\infty}\left[\prod_{i=1}^{k} b_i(\tau_i)T(t-t_0)x_0 + \sum_{i=1}^{k}\prod_{j=1}^{k} b_j(\tau_j)\int_{\xi_{i-1}}^{\xi_i} T(t-s)f(s, x(s), Ux(s), Vx(s))ds + \int_{\xi_k}^{t} T(t-s)f(s, x(s), Ux(s), Vx(s))ds\right] I_{[\xi_k,\xi_{k+1})}(t), \quad t \in [\tau, T] \qquad (2.4)$$

where, $\prod_{j=m}^{n}(.) = 1$ as $m > n$, $\prod_{j=1}^{k} b_i(\tau_j) = b_k(\tau_k)\, b_{k-1}(\tau_{k-1}) \ldots b_i(\tau_i)$, and $I_A(.)$ is the index function

i.e,

$$I_A(t) = \begin{cases} 1, & if \quad t\epsilon A \\ 0, & if \quad t\epsilon A \end{cases}$$

II. MAIN RESULT

In this section, we establish the existence and uniqueness of the mild solution for the system (2.1) – (2.3) and we have listed out some assumptions which use essential to prove our results.

(H_1) The function f satisfies the Lipschitz condition. That is for $x_1, x_2, x_3, y_1, y_2, y_3 \in X$ and $\tau \le t \le T$, there exist a constant

$L_1, L_2, L_3 > 0$ such that

$$E\| f(t, x_1, x_2, x_3) - f(t, y_1, y_2, y_3)\|^2 \le L_1 E\| x_1 - y_1\|^2 + L_2 E\| x_2 - y_2\|^2 + L_3 E\| x_3 - y_3\|^2$$

$$E\| f(t, 0,0,0)\|^2 \le k,$$

where $k \le 0$ is a constant

(H_2) The condition $\max_{i,k}\{\prod_{j=i}^{k}\|b_j(\tau_j)\|\}$ is uniformly bounded if, there is a constant C > 0 such that

$$\max_{i,k}\left\{\prod_{j=i}^{k}\| b_j(\tau_j)\|\right\} \le C, \qquad \text{for all } \tau_j \in D_j, j = 1,2,3,\ldots$$

(H_3) Denote L= max $\{L_1, L_2, L_3\}$,

$$K^* = \sup_{t\in[t_0,T]}\int_{t_0}^{t} |K(t,s)|^2\, dt < \infty, \qquad \text{and}$$

$$H^* = \sup_{t\in[t_0,T]}\int_{t_0}^{T} |H(t,s)|^2\, dt < \infty.$$

Theorem 3.1. *Let the hypothesis* (H_1) – (H_3) *be hold, if the following inequality*

$$\Gamma = M^2 \max\{1, C^2\}(T-\tau)^2 LT[1 + K^* + H^*] < 1, \tag{3.1}$$

is satisfied , then the system (2.1) – (2.3) *has a unique mild solution in* $\mathscr{B}$

Proof. Let T be an arbitrary number $\tau < T < +\infty$. Let us define the nonlinear operator $S : \mathscr{B} \to \mathscr{B}$ as follows

$$Sx(t) = \sum_{k=0}^{+\infty}\left[\prod_{i=1}^{k} b_i(\tau_i)T(t-t_0)x_0 + \sum_{i=1}^{k}\prod_{j=i}^{k} b_j(\tau_j)\int_{\xi_{i-1}}^{\xi_i} T(t-s)f(s,x(s),Ux(s),Vx(s))ds + \int_{\xi_k}^{t} T(t-s)f(s,x(s),Ux(s),Vx(s))ds\right] I_{[\xi_k,\xi_{k+1})}(t), \quad t\in[\tau,T].$$

The continuity of S can be proved easily. Next we will show that $\mathscr{B}$ is mapped into $\mathscr{B}$ under S

$$\| Sx(t)\|^2 \le \left[\sum_{k=0}^{+\infty}\left[\|\prod_{i=1}^{k} b_i(\tau_i)\|\, T(t-t_0)\|\|x_0\| + \sum_{i=1}^{k}\|\prod_{j=i}^{k} b_j(\tau_j)\|\left\{\int_{\xi_{i-1}}^{\xi_i}\| T(t-s)\, f(s,x(s),Ux(s),Vx(s))\|\, ds\right\} + \int_{\xi_k}^{\xi_i}\| T(t-s)\, f(s,x(s),Ux(s),Vx(s))\|\, ds\right] I_{[\xi_k,\xi_{k+1})}(t)\right]^2$$

$$\le 2\left[\sum_{k=0}^{+\infty}\left[\prod_{i=1}^{k}\| b_i(\tau_i)\|^2\| T(t-t_0)\|^2\| x_0\|^2\, I_{[\xi_k,\xi_{k+1})}(t)\right] + \left[\sum_{k=0}^{+\infty}\left[\sum_{i=1}^{k}\|\prod_{j=i}^{k} b_j(\tau_j)\|\left\{\int_{\xi_{i-1}}^{\xi_i}\| T(t-s)\, f(s,x(s),Ux(s),Vx(s))\|\, ds\right\}\right.\right.\right.$$

$$+\int_{\xi_k}^{t} \| T(t-s) \| \| f(s, x(s), Ux(s), Vx(s)) \| ds \Big] I_{[\xi_k, \xi_{k+1})}(t) \Big]^2 \Big]$$

$$\leq 2M^2 \max_k \left\{ \| \prod_{i=1}^{k} b_i(\tau_i) \|^2 \right\} \| x_0 \|^2$$

$$+ 2M^2 \left[\max_{i,k} \left\{ 1, \| \prod_{j=i}^{k} b_j(\tau_j) \| \right\} \right]^2$$

$$\times \left(\int_{t_0}^{t} \| f(s, x(s), Ux(s), Vx(s) \| ds \, I_{[\xi_k, \xi_{k+1})}(t) \right)^2$$

$$\leq 2M^2C^2 \| x_0 \|^2 + 2M^2 \max\{1, C^2\} \left(\int_{t_0}^{t} \| f(s, x(s), Ux(s), Vx(s) \| ds \right)^2$$

$$\leq 2M^2C^2 \| x_0 \|^2 + 2M^2 \max\{1, C^2\}(t-t_0) \int_{t_0}^{t} \| f(s, x(s), Ux(s), Vx(s) \|^2 ds$$

$$E \| (Sx)(t) \|^2$$

$$\leq 2M^2C^2 \| x_0 \|^2 + 2M^2 \max\{1, C^2\}(T-\tau) \int_{t_0}^{t} E \| f(s, x(s), Ux(s), Vx(s) \|^2 ds$$

$$\leq$$

$$2M^2C^2 \| x_0 \|^2 + 4M^2 \max\{1, C^2\} (T-\tau)^2 k + 4M^2 \max\{1, C^2\}(T-\tau)$$

$$\left\{ L_1 \int_{t_0}^{t} E \| x(s) \|^2 + L_2 \int_{t_0}^{t} E \| Ux(s) \|^2 + L_3 \int_{t_0}^{t} E \| Vx(s) \|^2 \right\}$$

Hence,

$$\sup_{t\in[\tau,T]} E \| (Sx)(t) \|^2 \leq 2M^2C^2 \| x_0 \|^2 + 4M^2 \max\{1, C^2\} (T-\tau)^2 k$$

$$+ 4M^2 \max\{1, C^2\}(T-\tau)$$

$$\left\{ L_1 \int_{t_0}^{t} \sup_{s\in[\tau,t]} E \| x(s) \|^2 + L_2 \int_{t_0}^{t} \sup_{s\in[\tau,t]} E \| Ux(s) \|^2 \right.$$

$$\left. + L_3 \int_{t_0}^{t} \sup_{s\in[\tau,t]} E \| Vx(s) \|^2 \right\}$$

$$\leq 2M^2C^2 \| x_0 \|^2 + 4M^2 \max\{1, C^2\} (T-\tau)^2 k + 4M^2 \max\{1, C^2\}(T-\tau)^2$$

$$\left\{ L_1 \sup_{t\in[\tau,T]} E \| x(t) \|^2 + L_2 \sup_{t\in[\tau,T]} E \| Ux(t) \|^2 + L_3 \sup_{t\in[\tau,T]} E \| Vx(t) \|^2 \right\}$$

For $t \in [\tau, T]$, therefore S maps $\mathscr{B}$ into itself.

Then let us show that S is a contraction mapping

$$\|(Sx)(t)-(Sy)(t)\|^2$$

$$\leq \left[\sum_{k=0}^{+\infty}\left\{\sum_{i=1}^{k}\prod_{j=i}^{k}\|b_i(\tau_j)\| \times \int_{\xi_{i-1}}^{\xi_i}\|T(t-s)\|\,\big\|f(s,x(s),Ux(s),Vx(s)) - f(s,y(s),Uy(s),Vy(s))\big\|ds + \int_{\xi_k}^{t}\|T(t-s)\|\,\big\|f(s,x(s),Ux(s),Vx(s)) - f(s,y(s),Uy(s),Vy(s))\big\|ds\right\} I_{[\xi_k,\xi_{k+1})}(t)\right]^2$$

$$\leq M^2\left[\max_{i,k}\left\{1,\prod_{j=i}^{k}\|b_j(\tau_j)\|\right\}\right]^2 \times \left(\int_{t_0}^{t}\big\|f(s,x(s),Ux(s),Vx(s)) - f(s,y(s),Uy(s),Vy(s))\big\|ds\, I_{[\xi_k,\xi_{k+1})}(t)\right)^2$$

$$\leq M^2\max\{1,C^2\}(t-t_0)\left(\int_{t_0}^{t}\|f(s,x(s),Ux(s),Vx(s)) - f(s,y(s),Uy(s),Vy(s))\|^2 ds\right)$$

Then $E\|S(x)(t)-Sy(t)\|^2$

$$\leq M^2\max\{1,C^2\}(t-t_0)\int_{t_0}^{t}E\|f(s,x(s),Ux(s),Vx(s)) - f(s,y(s),Uy(s),VY(s))\|^2 ds$$

$$\leq M^2\max\{1,C^2\}(T-\tau)\left[L_1\int_{t_0}^{t}E\|x(s)-y(s)\|^2 ds + L_2\int_{t_0}^{t}E\|Ux(s)-Uy(s)\|^2 ds + L_3\int_{t_0}^{t}E\|Vx(s)-Vy(s)\|^2 ds\right]$$

That is,

$$E\|S(x)(t)-Sy(t)\|^2 \leq M^2\max\{1,C^2\}(T-\tau)[L_1A+L_2B+L_3C] \quad (3.2)$$

Where,

$$A = \int_{t_0}^{t} E \parallel x(s) - y(s) \parallel^2 ds$$

$$B = \int_{t_0}^{t} E \parallel Ux(s) - Uy(s) \parallel^2 ds$$

$$C = \int_{t_0}^{t} E \parallel Vx(s) - Vy(s) \parallel^2 ds$$

Consider B

$$\int_{t_0}^{t} E \parallel Ux(s) - Uy(s) \parallel^2 ds \leq L_2 \int_{t_0}^{t}\int_{t_0}^{s} \parallel K(s,\tau) \parallel^2 \parallel x(\tau) - y(\tau) \parallel^2 d\tau ds$$

$$\leq L_2 \int_{t_0}^{t} \parallel x(\tau) - y(\tau) \parallel^2 \int_{t_0}^{s} \parallel K(s,\tau) \parallel^2 d\tau ds$$

Taking supremum over t, we get

$$\leq L_2 \parallel x - y \parallel^2 \int_{t_0}^{t} \parallel K^* \parallel^2 ds$$

$$\leq L_3 \|x - y\|^2 K^* T \qquad (3.3)$$

Simillarly (from (3.3)) we get

$$\int_{t_0}^{t} E\|Vx(s) - Vy(s)\|^2 \leq L_3 \|x - y\|^2 H^* T \qquad (3.4)$$

$$\int_{t_0}^{t} E\|x(s) - y(s)\|^2 \leq L_1 \|x - y\|^2 T \qquad (3.5)$$

Substituting (3.3) , (3.4) and (3.5) in (3.2) we get

$$\|(Sx)(t) - (Sy)(t)\|^2 \leq M^2 \max\{1, C^2\} (T - \tau)^2 [L_1 \|x - y\|^2 T + L_2 \|x - y\|^2 K^* T + L_3 \|x - y\|^2 H^* T]$$

Using the definition of L, we have

$$\|(Sx)(t) - (Sy)(t)\|^2 \leq M^2 \max\{1, C^2\} (T - \tau)^2 L[\|x - y\|^2 T + \|x - y\|^2 K^* T + \|x - y\|^2 H^* T]$$

$$\leq M^2 \max\{1, C^2\} (T-\tau)^2 LT[1 + K^* + H^*]\|x-y\|^2$$

$$\leq \Gamma\|x-y\|^2$$

since $0 < \Gamma < 1$. Thus we get that the operator S satisfies the contraction mapping principle and it implies, S has a unique fixed point which is the mild solution of the system (2.1) − (2.3) . This completes the proof.

III. EXAMPLE

Next we see an application for the problem (2.1) − (2.3), consider a one dimensional ironic pipe of length π whose ends are kept at 0º and whose sides are insulated. Suppose there is an exothermic reaction taking place inside the pipe and heat produced is proportional to the temperature t − d (for the sake of simplicity, we assume the delay d ≥ 0 is constant). Consequently, the temperature in the pipe may be modelled to satisfy

$$(*) = \begin{cases} \dfrac{\partial u(t,x)}{\partial t} = \dfrac{\partial^2 u(t,x)}{\partial x^2} + \gamma\, u(x, t-d, E), \quad 0 < x < \pi, t > 0, \\ u(t,0) = u(\pi, t) \\ u(x,t) = \varrho(x,t) \quad -d \leq t \leq 0, 0 \leq x \leq \pi \end{cases}$$

Where E represents the energy of the pipe and $E(t) = \int_0^t u(x,t)dx$ and γ depends on the rate of reaction and ϱ: $[-d,0]\times[0,\pi] \to \Re$ is a given function. Also we can see that in the absence of heat production (i.e, $\gamma = 0$), the problem $(*)$ has solution given by

$$u(x,t) = \sum_{n=1}^{\infty} a_n e^{-n^2 t} \sin nx,$$

$$\text{where } d = 0\,, \varrho\,(x,0) = \sum_{n=1}^{\infty} a_n \sin nx,$$

However, it often occurs that the exothermic reaction can be related with random impulses. In some cases, the equation $(*)$ may be written in the generalized form with d=0

$$(**) = \begin{cases} \dfrac{\partial u(t,x)}{\partial t} = \dfrac{\partial^2 u(t,x)}{\partial x^2} + \gamma\, u(x,t,E), \quad 0 < x < \pi, t > 0, t \neq \xi_k \\ u(x,\xi_k) = q(k)\tau_k u(x, \xi_k^-), t = \xi_k\,, \\ u(t,0) = u(\pi,t) = 0, \\ u(x,t) = \varrho(x,t) \quad 0 \leq x \leq \pi. \end{cases}$$

And setting X= L^2 [0, π] and the operator A= $\frac{\partial^2}{\partial x^2}$ with the domain

$$D(A) = \left\{ u \in X \mid u \text{ and } \frac{\partial u}{\partial t} \text{ are absolutely continuous}, \frac{\partial^2 U}{\partial x^2} \in X\,, u(0) = u(\pi) = 0 \right\}$$

It is well known that a strongly continuous, compact, analytic and self adjoint semigroup T(t) can be generated using A,

$$\| T(t) \| \leq M, \quad \text{for } t \geq 0\,, \text{ where } M > 0$$

And hence T(t) is bounded.

Moreover, suppose that the impulsive nature satisfy the following condition

$$E\,[\max_{i,k}\left\{\prod_{j=i}^{k} \| q(j)(\tau_j) \|^2\right\}] < \infty$$

Under these condition, the function f and b_k can be defined as

$$f(t, x(t), U(x(t)) = \gamma u(x, t, E) \text{and } b_k(\tau_k) = q(k)\tau_k$$

Then the problem $(**)$ can be modeled as the abstract random impulsive differential equations of the form (2.1) – (2.3). The next result is consequence of Theorem 3.1

Proposition 4.1. *Let the hypotheses* $(H_1) - (H_3)$ *be hold. Then there exist a unique mild solution u of the system* $(*)$ *provided,*

$$M^2 \max\{1, C^2\} (T - \tau)^2 LT[\,1 + K^* + H^*] < 1$$

is satisfied.

IV. CONCLUSIONS

In this paper, we have investigated the existence and uniqueness of special type impulsive differential equations with random coefficients. By introducing new terms in non homogeneous part and defining its properties, the impulsive differential equations with random coefficients which is already defined has been transformed into the existence of the solution of a new equation. Sufficient conditions for the existence of solution of the system have been obtained by using contraction mapping principle. Similarly, other kinds of impulsive differential equations with random coefficients can be studied by similar techniques. We can extended the obtained results to other field of impulsive differential equations.

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