Connected Domination on Anti Fuzzy Graph

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Abstract— In this paper, the concept of connected domination number on anti fuzzy graph is introduced. The bounds on connected domination number of an anti fuzzy graph are obtained. This concept is applied on anti Cartesian product of anti fuzzy graphs and obtained the results on them.

Keywords— **Anti fuzzy graph, Connected Domination number, e-nodal anti fuzzy graph, uninodal anti fuzzy graph.** ***Mathematical Classification:*** **05C62, 05E99, 05C07.**

I. INTRODUCTION

The connected dominating set plays a vital role in networks and theoretical computer science. Now days, it leads a role in medical and in health informatics also. Such problems convert as graph model and get solution by using connected dominating set. The study of dominating sets in graphs was started by Ore and Berge. Further, the concept of domination number was developed by Cockayne and Hedetniemi[2]. E.Sampthkumar and H.B.Walikar[11] defined the dominating set to be a connected dominating set if the induced sub graph of d is connected. A.Somasundaram and S.Somasundaram[13] discussed domination in a fuzzy graph using effective edges. A. Somasundram[14] presented several types domination parameters such as independent domination, total domination, connected domination and domination in Cartesian product and composition of fuzzy graphs. R.Seethalakshmi and R.B.Gnanajothi [12] introduced the definition of anti fuzzy graph. This concept further developed by R.Muthuraj and A. Sasireka[7-9]. They introduced some types of anti fuzzy graphs and applied some operations on them. The concept of domination number and total domination number on anti fuzzy graph were introduced by R.Muthuraj and A. Sasireka[10]. In this paper we introduce the concept of connected domination on an anti fuzzy graph and obtained some bounds on them. We characterized the connected domination number on several types of anti fuzzy graph. Connected domination number is applied on anti Cartesian product of anti fuzzy graph and derived some results and theorems on them.

II. PRELIMINARIES

In this section, basic concepts of an anti fuzzy graph are discussed. Notations and more formal definitions which are followed as in [4, 5, 6].

2.1. Definition [4]

An anti fuzzy graph $G = (\sigma, \mu)$ is a pair of functions $\sigma : V \rightarrow [0,1]$ and $\mu : V \times V \rightarrow [0,1]$, where for all u, v ∈ V, we have $\mu(u,v) \geq \sigma(u) \vee \sigma(v)$ and it is denoted by $G_A(\sigma, \mu)$.

Note

μ is considered as reflexive and symmetric. In all examples σ is chosen suitably. i.e., undirected anti fuzzy graphs are only considered.

Notation:

Without loss of generality let us simply use the letter G_A to denote an anti fuzzy graph.

2.2. Definition [4]

The order p and size q of an anti fuzzy graph $G_A = (V,\sigma,\mu)$ are defined to $p = \sum_{x\in V}\sigma(x)$ and $q = \sum_{xy\in E}\mu(x,y)$ is denoted by O(G) and S(G).

2.3. Definition [4]

Two vertices u and v in G_A are called adjacent if $(1/2)[\sigma(u) \vee \sigma(v)] \leq \mu(u,v)$.

2.4. Definition [6]

The anti complement of anti fuzzy graph $G_A(\sigma,\mu)$ is an anti fuzzy graph $\overline{G_A} = (\bar{\sigma}, \bar{\mu})$ where $\bar{\sigma} = \sigma$ and $\bar{\mu}(u,v) = \mu(u,v) - (\sigma(u) \vee \sigma(v))$ for all u,v in V.

2.5. Definition [4]

An anti fuzzy graph $G_A = (\sigma,\mu)$ is a strong anti fuzzy graph of $\mu(u,v) = \sigma(u) \vee \sigma(v)$ for all $(u,v)\in\mu^*$ and G_A is a complete anti fuzzy graph if $\mu(u,v) = \sigma(u) \vee \sigma(v)$ for all $(u,v) \in \mu^*$ and $u,v\in \sigma^*$. Two vertices u and v are said to be neighbors if $\mu(u,v) > 0$.

2.6. Definition [4]

An edge e = {u, v} of an anti fuzzy graph G_A is called an effective edge if $\mu(u,v) = \sigma(u) \vee \sigma(v)$.

2.7. Definition [4]

u is a vertex in an anti fuzzy graph G_A then N(u) = {v: (u,v) is an effective edge} is called the neighborhood of u and $N[u] = N(u) \cup \{u\}$ is called closed neighborhood of u.

2.8. Definition [4]

A path P_A in an anti fuzzy graph is a sequence of distinct vertices $u_0, u_1, u_2, \ldots, u_n$ such that $\mu(u_{i-1},u_i)>0$, $1\leq i \leq n$. Here $n \geq 0$ is called the length of the path P_A. The consecutive pairs (u_{i-1}, u_i) are called the edges of the path.

2.9. Definition [4]

A cycle in G_A is said to be an anti fuzzy cycle if it contains more than one weakest edge.

2.10. Definition [5]

Let $G_A^* = G_{A_1}^* \times G_{A_2}^* = (V, E')$ be the anti cartesian product of anti fuzzy graphs where $V= V_1 \times V_2$ and $E' = \{(u_1,u_2),(u_1,v_2) / u_1\in V_1, (u_2,v_2)\in E_2\}\cup\{(u_1,w_2),(v_1,w_2) / w_2\in V_2, (u_1,v_1)\in E_1\}$. Then the anti cartesian product of two anti fuzzy graphs, $G_A = G_{A_1} \times G_{A_2}$: $(\sigma_1\times\sigma_2, \mu_1\times\mu_2)$ is an anti fuzzy graph and is defined by

$(\sigma_1 \times \sigma_2)(u_1, u_2) = \max\{\sigma_1(u_1), \sigma_2(u_2)\}$ for all $(u_1, u_2)\in V$

$(\mu_1 \times \mu_2)((u_1, u_2), (u_1, v_2)) = \max\{\sigma_1(u_1), \mu_2(u_2, v_2)\}$ for all $u_1\in V_1$ and $(u_2, v_2)\in E_2$

$(\mu_1 \times \mu_2)((u_1, w_2), (v_1, w_2)) = \max\{\sigma_2(w_2), \mu_1(u_1, v_1)\}$ for all $w_2\in V_2$ and $(u_1, v_1) \in E_1$,

Then the fuzzy graph $G_A = (\sigma_1\times\sigma_2, \mu_1\times\mu_2)$ is said to be the anti cartesian product of $G_{A_1} = (\sigma_1, \mu_1)$ and $G_{A_2} = (\sigma_2, \mu_2)$.

2.11. Definition [6]

Every vertex in an anti fuzzy graph G_A has unique fuzzy values then G_A is said to be v-nodal anti fuzzy graph. i.e. $\sigma(u) = c$ for all $u\in V(G_A)$.

2.12. Definition [6]

Every edge in an anti fuzzy graph G_A has unique fuzzy values then G_A is said to be e-nodal anti fuzzy graph. i.e. $\mu(u,v) = c$ for all $uv \in E(G_A)$

2.13. Definition [6]

Every vertices and edges in an anti fuzzy graph G_A have the unique fuzzy value then G_A is called as uninodal anti fuzzy graph.

2.14. Definition

The strong neighborhood of an edge e_i in an anti fuzzy graph G_A is $N_s(e_i) = \{e_j \in E(G) / e_j$ is an effective edge with $\vee N(e_i)$ in G_A and adjacent to $e_i\}$.

2.15. Definition

An edge $e = \{u,v\}$ of an anti fuzzy graph G_A is called an weak edge if $\mu(u,v) \neq \sigma(u) \vee \sigma(v)$.

2.16. Definition

G_A is an anti fuzzy graph and $u,v \in V(G_A)$. If v is said to be a support vertex to u then v is adjacent to atleast one end vertex u in G_A.

III. CONNECTED DOMINATION ON ANTI FUZZY GRAPH

In this section, we define the definition of connected domination number on anti fuzzy graph G_A. These concepts are applied on some types of simple anti fuzzy graph G_A, few elementary bounds on total domination number are described, the corresponding theorems and results are illustrated.

3.1. Definition [8]

A set $D \subseteq V(G_A)$ is said to be a dominating set of an anti fuzzy graph G_A if for every vertex $v \in V(G_A) \setminus D$ there exists u in D such that v is a strong neighborhood of u with $\mu(u,v) = \sigma(u) \vee \sigma(v)$ otherwise it dominates itself.

A dominating set D is called a minimal dominating set if no proper subset of D is a dominating set.

The maximum fuzzy cardinality taken over all minimal dominating set in G_A is called a domination number of anti fuzzy graph G_A and is denoted by $\gamma(G_A)$ or γ_A. ie, $|D|_f = \sum_{v \in D} \sigma(v)$.

3.2. Definition

A dominating set $D \subseteq V(G_A)$ is said to be a connected dominating set of an anti fuzzy graph G_A if for every vertex $v \in V(G_A) \setminus D$ is adjacent to atleast one strong neighbourhood vertex in D and the induced subgraph of D is connected.

A connected dominating set D is called a minimal connected dominating set if no proper subset D' of D is a dominating set.

The maximum fuzzy cardinality taken over all minimal connected dominating set is called connected domination number of G_A and it is denoted by □$_c(G_A)$ or □$_{Ac}$.

3.3. Example

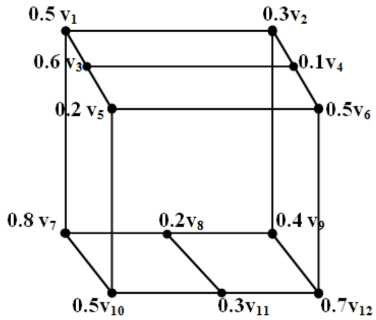

Fig. 1. Anti Fuzzy Graph GA

In figure 1, the connected dominating sets are, $D_1=\{v_1,v_3,v_7,v_{10},v_{11},v_{12}\}$, $D_2=\{v_3,v_5,v_6,v_7,v_8,v_9,v_{12}\}$, $D_3=\{v_1,v_2,v_3,v_7, v_9,v_{12}\}$. Therefore, minimal connected dominating set of G_A is $D_1=\{v_1,v_3,v_7,v_{10},v_{11},v_{12}\}$ and □$_c(G_A)=3.4$

3.4. Theorem

If a connected dominating set D is a minimal connected dominating set of a simple connected anti fuzzy graph G_A then each $u\in D$, atleast one of the following conditions are hold.

i. There exists a vertex $v\in V\backslash D$ such that $N[v]\cap D=\{u\}$

ii. u is a cut vertex in D

iii. u is leaf in D.

3.5. Theorem

Let G_A be a connected anti fuzzy graph and D be connected dominating set of G_A. Then D contains all cut vertices of G_A(if there exists a cut vertices in G_A).

Proof

Let D be a connected dominating set of connected anti fuzzy graph G_A. Let us assume that there exists a cut vertices u,v in G_A. i.e., $u,v\in V(G_A)$. Suppose $u\notin D$ then $D\subset V(G_A) \setminus u$ and G_A-u is disconnected. Which is contradiction to D is connected dominating set of G_A. hence D is a connected dominating set which contains all cut vertices of G_A.

3.6. Theorem

For any connected anti fuzzy graph G_A, □$_A$ ≤ □$_{Ac}$.

Proof

G_A is a simple connected anti fuzzy graph. D is a dominating set and D_c is a connected dominating set of G_A respectively. Dominating set contain vertices which are adjacent to atleast one vertex in $V\backslash D$. D may contain more than one component with some isolated vertices but the connected dominating set D_c has no isolated vertices and has exactly one component. Therefore, $|D| \leq |D_c|$. Hence □$_A$ ≤ □$_{Ac}$.

3.7. Corollary

For any connected anti fuzzy graph G_A, D is a minimal connected dominating set if and only if <D> is either a tree or there exists a cycle.

3.8. Proposition

For any connected anti fuzzy graph G_A, 0 < □$_{Ac}$ ≤ p.

3.9. Observation

i. Every connected anti fuzzy graph has a connected dominating set.

ii. Every connected dominating set is not an independent set.

3.10. Theorem

If G_A is an uninodal anti fuzzy graph then |p-q| ≤ □$_{Ac}$ ≤ p-δ.

Proof

Let D be a connected dominating set of an uninodal anti fuzzy graph G_A. By the definition of uninodal anti fuzzy graph, every vertices and edges has same fuzzy value. It obvious that |p-q| ≤ □$_{Ac}$.

For upper bound, the connected dominating set contains the maximum vertex which have maximum degree Δ and its support vertices which make as single component. Therefore,

⇒□□$_{Ac}$ +δ ≤ p

⇒□□$_{Ac}$ ≤ p-δ

Hence |p-q| ≤ □$_{Ac}$ ≤ p-δ.

3.11. Theorem

For an uninodal anti fuzzy graph,

i. □$_{Ac}$ +δ ≤ p.

ii. $\overline{\gamma_{AC}}$ ≤ p-δ for p=q

iii. $\frac{2p}{3} \leq \overline{\gamma_{AC}} \leq$ p

3.12. Proposition

i. If G_A is an uninodal anti fuzzy graph then □$_{Ac}$ ≤ $\frac{q}{\Delta}$

ii. If G_A is an uninodal anti fuzzy graph then □$_c(G_A)$ + □$_c(\overline{G_A})$ ≤ 2p.

3.13. Theorem

G_A is an e-nodal anti fuzzy graph with an odd cycle then in the connected dominating set exists a fuzzy path.

3.14. Proposition

i. If G_A is an e-nodal anti fuzzy graph then p-Δ ≤ □$_{Ac}$.

ii. If G_A is an e-nodal anti fuzzy graph with effective edges then □$_{Ac}$ ≥ $\frac{p}{1+\Delta}$.

iii. If G_A is an e-nodal anti fuzzy graph with weak edges then □$_{Ac}$ ≤ $\frac{p}{1+\Delta}$.

3.15. Theorem

If G_A is an anti fuzzy path with m≥3 vertices then the connected dominating set contains m-2 vertices and □$_{Ac}$= p - σ(u_1) - σ(u_m) where u_1,u_m are leaf.

Proof

Let us consider that G_A be an anti fuzzy path with m≥3 vertices and D be a connected dominating set of G_A. consider a path $u_1,u_2,u_3,..,u_i,u_{i+1}, \ldots u_m$ be a vertices on path. u_1 and u_m contain only one neighbour. Then they dominate by u_2 and u_{m-1}. The remaining vertices $u_2,u_3,\ldots,u_i,..u_{m-1}$ have two neighbours. Since they dominated by atleast any one of its neighbours. But D is a connected dominating set and <D> is connected.

Thus to construct D for a path, there exists $u_2,u_3,\ldots,u_i,..u_{m-1}$ vertices. Then <D> is connected otherwise it is disconnected. Hence the connected dominating set contains m-2 vertices. Therefore, $\square_{Ac}= p - \sigma(u_1) - \sigma(u_m)$.

3.16. Example

Let $G_A=(\sigma,\mu)$ be an anti fuzzy path with the vertex set $V(G_A)=\{a, b, c, d, e\}$, $E(G_A)= \{ab, bc, cd, de \}$ and defined by $\sigma(a)=0.5$, $\sigma(b)=0.4$, $\sigma(c)=0.6$, $\sigma(d)=0.3$, $\sigma(e)=0.2$ and $\mu(a,b)=0.5$, $\mu(b,c)=0.6$, $\mu(c,d)=0.6$, $\mu(d,e)=0.3$. Then the connected dominating set is {b,c,d}and the corresponding connected domination number is 1.3. Here p=2 and a & e are end vertices with fuzzy value0.5 and 0.2. Therefore, $\square_{Ac}= 2 -0.5 - 0.2 =1.3$

Remark

1. If D is connected dominating set then V\D may be disconnected anti fuzzy graph.
2. If G_A is an anti fuzzy path and D is a connected dominating set then V\D contains two isolated vertices only.
3. If G_A is a connected anti fuzzy graph with $n\geq4$ vertices then $\square_c(G_A) \neq \square_{Ac}(\overline{G_A})$.

3.17. Theorem

If G_A is an anti fuzzy cycle then the connected dominating set contains m -2 vertices and $\square_{Ac}= p - \wedge\, \sigma(u_i) - \sigma(u_{i\pm1})$ for i= 1 to m.

Proof

Let G_A be an anti fuzzy cycle with m vertices and D be a connected dominating set of G_A. We know that in a cycle there exist m-2 vertices in connected dominating set. To construct a minimal connected dominating set, choose a vertex which have minimum fuzzy value in $V(G_A)$ (say u_i). $N(u_i)=\{u_{i+1}, u_{i-1}\}$. If $\sigma(u_{i+1}) > \sigma(u_{i-1})$ then $u_{i+1} \subseteq D$ and this vertex dominate u_i. Therefore, $u_i,u_{i-1} \in V(G_A)\backslash D$. Therefore, $\square_{Ac}= p - \wedge\, \sigma(u_i) - \sigma(u_{i-1})$.

Similarly, If $\sigma(u_{i-1}) > \sigma(u_{i+1})$ then $u_{i-1} \subseteq D$ and this vertex dominate u_i. Therefore $u_i,u_{i+1} \in V(G_A)\backslash D$. Therefore, $\square_{Ac}= p - \wedge\, \sigma(u_i) - \sigma(u_{i+1})$. Hence $\square_{Ac}= p - \wedge\, \sigma(u_i) - \sigma(u_{i\pm1})$.

3.18. Theorem

If G_A is an anti fuzzy cycle (with $m\geq4$ vertices) and D is connected dominating set then <D> is a tree.

Proof

In a cycle, connected dominating set contains all the vertices except the vertex which have minimum fuzzy value and its neighbour. Also these vertices must be dominated by atleast one vertex in D then there is no cyclic in D. Hence <D> is a tree.

3.19. Theorem

If G_A is an complete anti fuzzy graph with n vertices then

i. connected dominating set contains two vertices and $\square_{Ac}= \vee\, \sigma(u_i) + s$, where s is any one of $\sigma\,[N_s(u_i)]$.

ii. $\square_c(\overline{G_A}) =0$

Proof

i. G_A is a complete anti fuzzy graph and D is connected dominating set. We know that, every vertex is adjacent to each other in G_A. Thus a single vertex may dominate all the remaining vertices in G_A. To construct D, choose a vertex which has maximum fuzzy value say u_i. Therefore $D=\{u_i\}$. By the definition of connected dominating set, <D> is connected. Let $s= \vee\, \{\sigma\,[N_s(u_i)]\}$ and say $s=\sigma(u_r)$ Thus, to get maximal cardinality of D, choose a vertex u_r. Hence $D=\{u_i, u_r\}$ and $\square_{Ac}= \vee\, \sigma(u_i) + s$.

ii. If G_A is complete anti fuzzy graph then all edges are effective and $\overline{G_A}$ is a null anti fuzzy graph. Therefore, $\square_c(\overline{G_A}) =0$.

3.20. Theorem

If T is an anti fuzzy tree, u∈V(T) is a support vertex and W is the set of all pendent vertices in T then D=V(T)\W is a connected dominating set of T and $\square_{Ac}$=p - Σσ(u_i) if and only if ⟨D⟩≅P_s.

Proof

Let T be an anti fuzzy tree and u∈V(T) be a support vertex. Let v≠u be any vertex in T and D be a connected dominating set of G_A. W is the set of all pendent vertices of G_A. Thus every pendent vertices should be adjacent to a support vertex or may be a root vertex of G_A. since no pendent vertices should be contained in D. Hence D=V(T)\W, which is a connected dominating set. Therefore, |W|=Σσ(u_i). Hence γ_{AC}=p-Σσ(u_i).

Next, we prove that ⟨D⟩≅P_s. Suppose T is a tree with m≥4 vertices with $\square_{Ac}$=p - Σσ(u_i). Then there exists atleast two pendent vertices in T. D contains only the support vertices and which forms a path. Hence ⟨D⟩≅P_s.

3.21. Theorem

If G_A is an anti fuzzy star then $\square_{Ac}$= ∨ σ(r) + f, where f= ∨ σ(s) and s=N_s(r).

Proof

In an anti fuzzy star G_A, a root vertex r is adjacent to all pendent vertices and also it dominate all vertices in G_A. Therefore r⊆D and it is isolated. Let s be the set strong neighbours of r. To avail maximal cardinality for D, consider f is the maximum fuzzy value of set of elements of s. Therefore, $\square_{Ac}$= ∨ σ(r) + f.

3.22. Theorem

If G_A is an complete bipartite graph with r and s vertices then $\gamma_c(G_A) \leq$ min{r,s}.

Proof

Let V_1, V_2 be the bipartition of the vertex set of $G_{A_{r,s}}$with V_1={u_1,u_2,…,u_r} and V_2={v_1,v_2,…,v_s}. Let us assume that r≤s. Let D be a dominating set of G_A. and D=V_2. We claim that ⟨D⟩ is connected. Let u_1,u_2∈D. If u_1,u_2∈V_2 then u_1 and u_2 are not adjacent vertices in V_2. Then there exists a u_1-u_2 path asu_1v_1, u_2v_1. Therefore, ⟨D⟩ is connected. Hence $\square(G_{A_{r,s}}) \leq$ r=min{r,s}.

Let D be a connected dominating set of $G_{A_{r,s}}$. Since every vertex in D dominates minimum number of S vertices in$G_{A_{r,s}}$. Also $G_{A_{r,s}}$ contains r+s number of vertices with rs number of edges. D contains atleast r vertices. Hence |D| ≤ r. Therefore, $\gamma_c(G_{A_{r,s}})$ =r ≤ min{r,s}.

IV. CONNECTED DOMINATION ON ANTI CARTESIAN PRODUCT OF ANTI FUZZY GRAPHS

In this section, we apply connected domination number on anti cartesian product on same types of anti fuzzy graphs such as cycle, path and complete anti fuzzy graph. To the resulting anti fuzzy graph obtain the bounds on them. In this paper, to derive the theorems consider that the number of vertices in G_{A_1} (say m_1) should be greater than or equal to the number of vertices in G_{A_2} (say m_2).

4.1. Theorem [5]

Let G_A be an anti cartesian product of anti fuzzy graphs $G_{A_1}$and G_{A_2} where G_{A_1}= (σ_1, μ_1) and G_{A_2}= (σ_2, μ_2) then G_A = ($\sigma_1\times\sigma_2$, $\mu_1\times\mu_2$) is an anti fuzzy graph.

4.2. Theorem

A connected dominating set exists for an anti cartesian product of anti fuzzy graphs G_{A_1} and G_{A_2} if and only if G_{A_1} and G_{A_2} are connected.

4.3. Proposition

$\square\ (G_{A_1} \times G_{A_2}) \leq \square_c (G_{A_1} \times G_{A_2})$.

4.4. Theorem

Let G_{A_1} and G_{A_2} be any same types of anti fuzzy graphs and $G_A = (G_{A_1} \times G_{A_2})$ is an anti fuzzy graph then $\frac{2p_1p_2}{1+\Delta(G_A)} \leq \square_c (G_{A_1} \times G_{A_2})$.

Proof

D is connected dominating set of $G_{A_1} \times G_{A_2}$ and V_1 and V_2 are the vertex set of G_{A_1} and G_{A_2}. Since,

$\Delta(G_A)\ \ |D|_f \geq 2\ |(V_1 \times V_2)(G_A)|_f - |D|_f$

$\Delta(G_A)\ \square_{\square c} \geq 2\ p_1\ p_2 - \square_{\square c}$

$\Delta(G_A)\ \square_{\square c} + \square_{\square c} \geq 2\ p_1\ p_2$

$(1+\Delta(G_A))\square\square_{\square c} \geq 2\ p_1\ p_2$

$\square_{\square c} \geq \frac{2p_1p_2}{1+\Delta(G_A)}$

4.5. Example

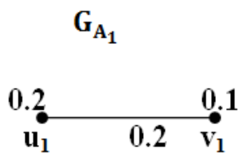

Fig. 2. Anti Fuzzy Graph(P1)

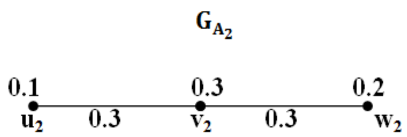

Fig. 3. Anti Fuzzy Graph(P2)

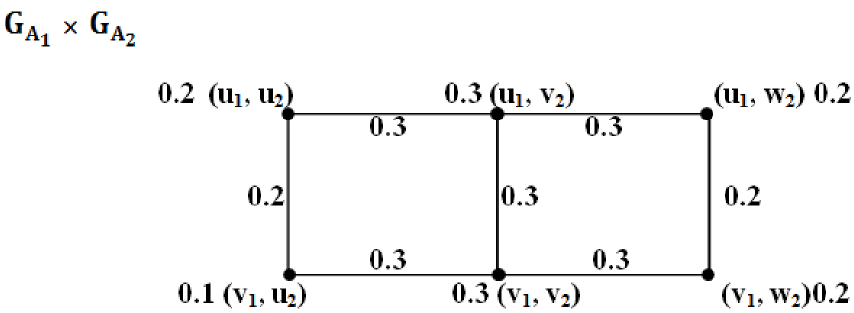

Fig.4. Anti Cartesian product of anti fuzzy graphs G_{A_1} and G_{A_2}

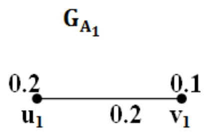

Fig.2 Anti Fuzzy Graph(P1)

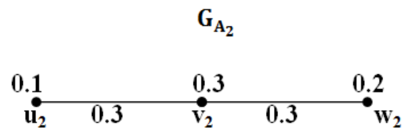

Fig.3 Anti Fuzzy Graph(P2)

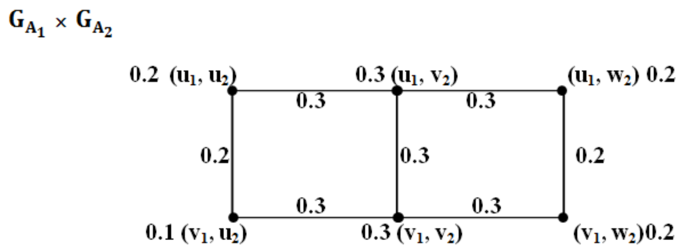

Fig.4 Anti cartesian product of anti fuzzy graphs G_{A_1} and G_{A_2}

From Figure 2, p_1=0.6.
From Figure 3, p_2=0.3
From Figure 4, $\Delta(G_A)$=0.9 and the minimal connected dominating set is, D={(u_1,v_2), (v_1,v_2)}.
Therefore, $\gamma_c(G_A)$=0.6
By theorem 4.4, $\frac{2p_1p_2}{1+\Delta(G_A)} = \frac{2(0.6)(0.3)}{1+0.9} = 0.19 \leq 0.6$

Therefore, $\square_{\square c} \geq \frac{2p_1p_2}{1+\Delta(G_A)}$

4.6. Theorem

For $m_1,m_2 \geq 2$, G_A is an anti cartesian product of anti fuzzy graph with m_1m_2 vertices then there exists atleast one Hamiltonian fuzzy path.

Note

From the example 4.5, the Figure 4 is a anti cartesian product of anti fuzzy graph with 6 vertices. And there exists three Hamiltonian fuzzy paths such as

i. Path 1- (u_1,u_2), (u_1,v_2), (u_1,w_2), (v_1,w_2), (v_1,v_2), (v_1,u_2)
ii. Path 2- (u_1,u_2), (v_1,u_2), (v_1,v_2), (u_1,v_2), (u_1,w_2), (v_1,w_2)
iii. Path 3- (v_1,u_2), (u_1,u_2), (u_1,v_2), (v_1,v_2), (v_1,w_2), (u_1,w_2)

4.7. Theorem

If $G_A = (G_{A_1} \times G_{A_2})$ is an anti cartesian product of anti fuzzy paths $G_{A_1}$and G_{A_2} with m_1(=2,3) and n vertices ($m_1 \leq m_2$ and $\sigma_1(u_i)<\sigma_2(v_j)$) then the connected dominating set is isomorphic to path

Proof

$G_{A_1} \times G_{A_2}$ is a anti fuzzy graph with m_1m_2 vertices. Let us consider that $\sigma_1(u_i)<\sigma_2(v_j)$. Thus every vertex in G_A has fuzzy value as $\sigma_2(v_j)$for j=1 to n. Hence $G_{A_1}^{u_2}$ contained in connected dominating set which is a acyclic graph. Hence the connected dominating set is isomorphic to path.

4.8. Theorem

If $G_A = (G_{A_1} \times G_{A_2})$ is an anti Cartesian product of anti fuzzy cycles G_{A_1} and G_{A_2} then the connected dominating set is isomorphic to tree or a cycle.

Proof

G_{A_1} and G_{A_2} are an anti fuzzy cycles with the vertices u_i for i=1 to m_1 and v_j for j= 1 to m_2. G_A is an anti cartesian product of anti fuzzy cycles G_{A_1} and G_{A_2} . D is a connected dominating set of G_A. In $G_{A_1} \times G_{A_2}$, every $G_{A_1}^{u_i}$ and $G_{A_2}^{v_j}$ are an anti fuzzy cycle and atleast one vertex of $G_{A_1}^{u_i}$ contained in connected dominating set which yields that D is acyclic graph. Therefore, D is isomorphic to tree. In some cases in G_A, $G_{A_1}^{u_i}$ contained in connected dominating set which gives that D is cyclic with m_2 vertices. Hence the connected dominating set is isomorphic to cycle.

Note

The connected dominating set is not isomorphic to G_{A_2} because in G_A, $\sigma(u_i,v_j)= \max(\sigma(u_i), \sigma(v_j))$

4.9. Proposition

If G_A is an anti cartesian product of anti fuzzy cycles G_{A_1} and G_{A_2} with order p_1 and p_2 then $□_{Ac} \geq p_1 \vee p_2$.

4.10. Theorem

If G_{A_1} and G_{A_2} are a complete anti fuzzy graphs with $m_1(\leq m_2)$ and m_2 vertices of order p_1 and p_2. $G_A = (G_{A_1} \times G_{A_2})$ is an anti cartesian product of complete anti fuzzy graphs with m_1m_2 vertices. D is a connected dominating set which are isomorphic to K_{m_1} and $□_{Ac}=p_1$.

Proof

Let D be a minimal connected dominating set of an anti fuzzy graph $G_A = (G_{A_1} \times G_{A_2})$. The vertex set of G×H can be partitioned into disjoint m_1 components with n vertices. Such as $(u_1,v_1),(u_1,v_2),(u_1,v_3),\ldots(u_1,v_{m_2})$, $(u_2,v_1),(u_2,v_2),(u_2,v_3),\ldots(u_2, v_{m_2}),\ldots\ldots (u_m, v_{m_2})$. Since each component should be a complete on itself. Suppose in every component $\sigma(u_i,v_j)_{i=1 \text{ to } m1}$ has maximum fuzzy value then it is dominated the remaining vertices in components. Therefore, D= $\{(u_1,v_j), (u_2,v_j),\ldots. (u_{m_1},v_j) \}$. And D is isomorphic to K_{m_1}. Hence $□_{Ac}=p_1$.

V. CONCLUSION

In this paper, connected domination number is defined on anti fuzzy graphs. The definition of connected domination number is applied on various types of anti fuzzy graph and also applied on anti cartesian product of anti fuzzy graphs such as anti fuzzy path, anti fuzzy cycle and complete anti fuzzy graph obtained the bounds on them.

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