$\mu - \delta\hat{g}$ -Continuous Functions in Supra Topological Spaces

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Abstract: In this paper, we introduce new class of functions in supra topological spaces, namely supra $\delta\hat{g}$ -continuous functions. We investigate its relationship with other type of continuous functions in supra topological spaces. Also we obtain some basic properties of supra $\delta\hat{g}$ -continuous functions. Further we introduce and study a new class of functions namely supra $\delta\hat{g}$ -irresolute.

Keywords and Phrases : supra $\delta\hat{g}$ -continuous, supra $\delta\hat{g}$ -irresolute, supra $\delta\hat{g}$ -closed sets and $\hat{T}^{\mu}_{3/4}$ -space.

1. INTRODUCTION

In 1983, Mashhour et al.[7] introduced supra topological spaces and studied S-continuous maps and S - Continuous maps. In 2008, Devi et al.[4] introduced the concept of supra α-open set, Sα-continuous functions respectively. In 2010, Sayed et al.[11] introduced and investigated several properties of supra b -open sets and supra b -continuity. In 2011, Ravi et al.[5] [9] introduced and investigated a new type of sets called supra g - closed sets and a new class of maps called supra g -continuous maps. The purpose of this present paper is to define a new class of maps called supra $\delta\hat{g}$ -continuous maps and supra $\delta\hat{g}$ -irresolute maps in supra topological spaces and investigate some of the basic properties for this class of functions.

EXPERIMENTAL SECTION

2. PRELIMINARIES

Throughout this paper (X,μ) (or simply X) represent supra topological spaces.

Definition 2.1. [6] A subfamily μ of X is said to be a supra topology on X, if

(i) $X, \phi \in \mu$

(ii) If $A_i \in \mu$ for all $i \in J$, then $\cup A_i \in \mu$.

The pair (X,μ) is called the supra topological space. The elements of μ are called supra open sets in (X,μ) and the complement of a supra open set is called a supra closed set.

Definition 2.2. [6]

(i) The supra closure of a set A is denoted by $cl^{\mu}(A)$ and is defined as $cl^{\mu}(A) = \bigcap\{B : B$ is supra closed and $A \subseteq B\}$.

(ii) The supra interior of a set A is denoted by $\text{int}^{\mu}(A)$ and is defined as $\text{int}^{\mu}(A) = \bigcup\{B : B$ is supra open and $A \supseteq B\}$.

Definition 2.3. [6] Let (X, μ) be a topological space and μ be a supra topology associated with τ, if $\tau \subset \mu$.

Definition 2.4. A subset A of a supra topological space X is called a

(i) supra semi-open set [4] if $A \subseteq cl^{\mu}(\text{int}^{\mu}(A))$.

(ii) supra b-open set [10] if
$A \subseteq cl^{\mu}(\text{int}^{\mu}(A)) \cup \text{int}^{\mu}(cl^{\mu}(A))$.

(iii) supra regular open set [1] if $A = \text{int}^{\mu}(cl^{\mu}(A))$.

The complement of a supra semi-open (resp. supra b-open and supra regular open) set is called supra semi-closed (resp. supra b-closed and supra regular closed).

Definition 2.5. A subset A of a supra topological space (X, μ) is called

(i) a supra generalized closed set (briefly g^{μ}-closed) [4] if $cl^{\mu}(A) \subseteq U$ whenever $A \subseteq U$ and U is supra open in (X, μ).

(ii) a supra semi-generalized closed set (briefly sg^{μ}-closed) [4] if $scl^{\mu}(A) \subseteq U$ whenever $A \subseteq U$ and U is supra semi-open in (X, μ).

(iii) a supra generalized semi-closed set (briefly gs^{μ}-closed) [4] if $scl^{\mu}(A) \subseteq U$ whenever $A \subseteq U$ and U is supra open in (X, μ).

(iv) a supra generalized b-closed set (briefly $g^{\mu}b$-closed) [2] if $bcl^{\mu}(A) \subseteq U$ whenever $A \subseteq U$ and U is supra open set in (X, μ).

(v) a supra $\hat{g}$-closed set (briefly $\hat{g}^{\mu}$-closed) [3] if $cl^{\mu}(A) \subseteq U$ whenever $A \subseteq U$ and U is supra semi-open in (X, μ).

(vi) a supra $\delta\hat{g}$-closed set (briefly $\mu - \delta\hat{g}$-closed) [8] if $cl^{\mu}_{\delta}(A) \subseteq U$ whenever $A \subseteq U$ and U is supra open in (X, μ).

The complement of a g^{μ}-closed (resp. sg^{μ}-closed, gs^{μ}-closed, $g^{\mu}b$-closed, $\hat{g}^{\mu}$-closed and $\mu - \delta\hat{g}$-closed) set is called g^{μ}-open (resp. sg^{μ}-open, gs^{μ} -open, $g^{\mu}b$-open, $\hat{g}^{\mu}$-open and $\mu - \delta\hat{g}$-open) set.

Definition 2.6. Let (X, τ) and (Y, σ) be two topological spaces and μ be an associated supra topology with τ. A function $f : (X, \tau) \to (Y, \sigma)$ is called

(i) supra continuous [7] if the inverse image of each open set in Y is supra open in X.

(ii) supra semi-continuous [4] if $f^{-1}(V)$ is supra semi-open in X for every open set V of Y.

(iii) supra g-continuous (briefly g^{μ}-continuous) [9] if the inverse image of each open set in Y is g^{μ}-open in X.

(iv) supra sg -continuous (briefly sg^{μ}-continuous) [5] if the inverse image of each open set in Y is sg^{μ}-open in X.

(v) supra gs-continuous (briefly gs^{μ}-continuous) [5] if the inverse image of each open set in Y is gs^{μ}-open in X.

(vi) supra gb-continuous (briefly $g^{\mu}b$-continuous) [7] if $f^{-1}(V)$ is $g^{\mu}b$-closed in (X, μ) for every closed set V of (Y, σ).

(vii) supra $\hat{g}$-continuous (briefly $\hat{g}^{\mu}$ -continuous) [3] if $f^{-1}(V)$ is $\hat{g}^{\mu}$-closed in (X, μ) for every closed set V of (Y, σ).

RESULTS AND DISCUSSION

3. SUPRA $\delta\hat{g}$ –CONTINUOUS FUNCTIONS

We introduce the following definitions.

Definition 3.1. Let (X,τ) and (Y,σ) be two topological spaces and μ be an associated supra topology with τ. A function $f:(X,\tau)\to(Y,\sigma)$ is called

(i) supra δ-continuous if $f^{-1}(V)$ is supra δ-closed in (X,μ) for every closed set V of (Y,σ).

(ii) supra δg-continuous if $f^{-1}(V)$ is supra δg-closed in (X,μ) for every closed set V of (Y,σ).

Definition 3.2. Let (X,τ) and (Y,σ) be two topological spaces and μ be an associated supra topology with τ. A function $f:(X,\tau)\to(Y,\sigma)$ is called supra $\delta\hat{g}$-continuous (briefly $\mu-\delta\hat{g}$-continuous) if $f^{-1}(V)$ is supra $\delta\hat{g}$-closed in (X,μ) for every closed set V of (Y,σ).

Theorem 3.4. Every supra δ-continuous function is supra $\delta\hat{g}$-continuous.

Proof. Let $f:(X,\tau)\to(Y,\sigma)$ be a supra δ-continuous function with associated supra topology μ and A be any closed set in Y. Then $f^{-1}(A)$ is supra δ-closed set in (X,μ). Since every supra δ-closed set is supra $\delta\hat{g}$-closed, $f^{-1}(A)$ is supra $\delta\hat{g}$-closed in (X,μ). Hence f is supra $\delta\hat{g}$-continuous.

Remark 3.5. The converse of the above theorem is not true in general as shown in the following example.

Example 3.6. Let $X=\{a,b,c\}=Y$ with topologies $\tau=\{\phi,\{a,b\},X\}$, $\sigma=\{\phi,\{a\},Y\}$ respectively. The supra topology μ is defined as follows: $\mu=\{\phi,\{a,b\},\{b,c\},X\}$. Let $f:(X,\tau)\to(Y,\sigma)$ be a function defined by $f(a)=b, f(b)=a$ and $f(c)=c$. Clearly f is supra $\delta\hat{g}$-continuous but not supra δ-continuous function. Because the inverse image of the closed set $\{b,c\}$ in (Y,σ) is $\{a,c\}$ which is not supra δ-closed in (X,μ).

Theorem 3.7. Every supra $\delta\hat{g}$-continuous function is supra δg-continuous.

Proof. Let $f:(X,\tau)\to(Y,\sigma)$ be a supra $\delta\hat{g}$-continuous function with associated supra topology μ and A be any closed set in Y. Then $f^{-1}(A)$ is supra $\delta\hat{g}$-closed set in (X,μ). Since every supra $\delta\hat{g}$-closed set is supra δg-closed, $f^{-1}(A)$ is supra δg-closed in (X,μ). Hence f is supra δg-continuous.

Remark 3.8. The converse of the above theorem is not true in general as shown in the following example.

Example 3.9. Let $X=\{a,b,c\}=Y$ with topologies $\tau=\{\phi,\{b,c\},X\}$, $\sigma=\{\phi,\{c\},Y\}$ respectively. The supra topology μ is defined as follows: $\mu=\{\phi,\{a\},\{b,c\},X\}$. Let $f:(X,\tau)\to(Y,\sigma)$ be the identity function. Clearly f is supra δg-continuous but not supra $\delta\hat{g}$-continuous function. Because the inverse image of the closed set $\{a,b\}$ in (Y,σ) is $\{a,b\}$ which is not supra $\delta\hat{g}$-closed in (X,μ).

Theorem 3.10. Every supra $\delta\hat{g}$-continuous function is g^{μ}-continuous.

Proof. It is true that every supra $\delta\hat{g}$-closed set is g^{μ}-closed.

Remark 3.11. The converse of the above theorem is not true in general as shown in the following example.

Example 3.12. Let $X=\{a,b,c\}=Y$ with topologies $\tau=\{\phi,\{a,b\},X\}$, $\sigma=\{\phi,\{c\},\{a,c\},Y\}$ respectively. The supra topology μ is defined as follows: $\mu=\{\phi,\{a,b\},\{b,c\},X\}$. Let $f:(X,\tau)\to(Y,\sigma)$ be a function defined by $f(a)=a, f(b)=c$ and $f(c)=b$. Clearly f is g^{μ}-continuous but not supra $\delta\hat{g}$-continuous function. Because the inverse image of the closed set $\{b\}$ in (Y,σ) is $\{c\}$ which is not supra $\delta\hat{g}$-closed in (X,μ).

Theorem 3.13. Every supra $\delta\hat{g}$-continuous function is gs^{μ}-continuous.

Proof. It is true that every supra $\delta\hat{g}$-closed set is gs^{μ}-closed.

Remark 3.14. The converse of the above theorem is not true in general as shown in the following example.

Example 3.15. Let $X = \{a, b, c\} = Y$ with topologies $\tau = \{\phi, \{b, c\}, X\}$, $\sigma = \{\phi, \{b\}, \{a, b\}, \{b, c\}, Y\}$ respectively. The supra topology μ is defined as follows: $\mu = \{\phi, \{a, c\}, \{b, c\}, X\}$. Let $f : (X, \tau) \to (Y, \sigma)$ be a function defined by $f(a) = a$, $f(b) = c$ and $f(c) = b$. Clearly f is gs^{μ}-continuous but not supra $\delta\hat{g}$-continuous function. Because the inverse image of the closed set $\{c\}$ in (Y, σ) is $\{b\}$ which is not supra $\delta\hat{g}$-closed in (X, μ).

Theorem 3.16. Every supra $\delta\hat{g}$-continuous function is $g^{\mu}b$-continuous.

Proof. It is true that every supra $\delta\hat{g}$-closed set is $g^{\mu}b$-closed.

Remark 3.17. The converse of the above theorem is not true in general as shown in the following example.

Example 3.18. Let $X = \{a, b, c\} = Y$ with topologies $\tau = \{\phi, \{a, c\}, X\}$, $\sigma = \{\phi, \{a, b\}, Y\}$ respectively. The supra topology μ is defined as follows: $\mu = \{\phi, \{a, b\}, \{b, c\}, \{a, c\}, X\}$. Let $f : (X, \tau) \to (Y, \sigma)$ be the identity function. Clearly f is $g^{\mu}b$-continuous but not supra $\delta\hat{g}$-continuous function. Because the inverse image of the closed set $\{c\}$ in (Y, σ) is $\{c\}$ which is not supra $\delta\hat{g}$-closed in (X, μ).

Remark 3.19. The following examples show that supra $\delta\hat{g}$-continuous is independent from supra continuous, supra semi continuous, sg^{μ}-continuous and $\hat{g}^{\mu}$-continuous.

Example 3.20. Let $X = \{a, b, c\} = Y$ with topologies $\tau = \{\phi, \{a, b\}, X\}$, $\sigma = \{\phi, \{b, c\}, Y\}$ respectively. The supra topology μ is defined as follows: $\mu = \{\phi, \{a, b\}, \{b, c\}, X\}$. Let $f : (X, \tau) \to (Y, \sigma)$ be the identity function. Clearly f is supra continuous, supra semi-continuous, sg^{μ}-continuous and $\hat{g}^{\mu}$-continuous but not supra $\delta\hat{g}$-continuous function. Because the inverse image of the closed set $\{a\}$ in (Y, σ) is $\{a\}$ which is not supra $\delta\hat{g}$-closed in (X, μ).

Example 3.21. Let $X = \{a, b, c, d\} = Y$ with topologies $\tau = \{\phi, \{b, c, d\}, X\}$, $\sigma = \{\phi, \{c\}, \{a, b, c\}, Y\}$ respectively. The supra topology μ is defined as follows: $\mu = \{\phi, \{a\}, \{a, b\}, \{b, c, d\}, X\}$. Let $f : (X, \tau) \to (Y, \sigma)$ be the identity function. Clearly f is supra $\delta\hat{g}$-continuous function but not supra continuous, supra semi-continuous, sg^{μ}-continuous and $\hat{g}^{\mu}$-continuous. Because the inverse image of the closed set $\{a, b, d\}$ in (Y, σ) is $\{a, b, d\}$ which is not supra closed, supra semi-closed, sg^{μ}-closed and $\hat{g}^{\mu}$-closed but it is a supra $\delta\hat{g}$-closed in (X, μ).

Theorem 3.22. Let (X, τ) and (Y, σ) be two topological spaces and μ be the associated supra topology with τ. A function $f : (X, \tau) \to (Y, \sigma)$ is supra $\delta\hat{g}$-continuous iff $f^{-1}(U)$ is supra $\delta\hat{g}$-open in (X, μ) for every open set U in (Y, σ).

Proof. Let $f : (X, \tau) \to (Y, \sigma)$ be a supra $\delta\hat{g}$-continuous function and U be an open set in (Y, σ). Then $f^{-1}(U^c)$ is supra $\delta\hat{g}$-closed set in (X, μ). But $f^{-1}(U^c) = [f^{-1}(U)]^c$ and hence $f^{-1}(U)$ is supra $\delta\hat{g}$-open in (X, μ). Conversely $f^{-1}(U)$ is supra $\delta\hat{g}$-open in (X, μ) for every open set U in (Y, σ). U^c is closed set in (Y, σ). Then $[f^{-1}(U)]^c$ is supra $\delta\hat{g}$-closed in (X, μ). But $[f^{-1}(U)]^c = f^{-1}(U^c)$ and hence $f^{-1}(U^c)$ is supra $\delta\hat{g}$-closed set in (X, μ). Therefore f is supra $\delta\hat{g}$-continuous.

Remark 3.23. The composition of two supra $\delta\hat{g}$-continuous functions need not be supra $\delta\hat{g}$-continuous as the following example shows.

Example3.24. Let $X=\{a,b,c\}$, $Y=\{p,q,r\}$, $Z=\{x,y,z\}$ with topologies $\tau=\{\phi,\{b,c\},X\}$, $\sigma=\{\phi,\{q\},Y\}$ and $\gamma=\{\phi,\{x\},\{y\},\{x,y\},Z\}$. The supra topologies μ and η associated with the topologies τ and σ respectively are defined as follows: $\mu=\{\phi,\{a,b\},\{b,c\},X\}$ and $\eta=\{\phi,\{p\},\{q\},\{p,q\},\{q,r\},\{p,r\},Y\}$. Let $f:(X,\tau)\to(Y,\sigma)$ be a function defined by $f(a)=p, f(b)=q$ and $f(c)=r$. Let $g:(Y,\sigma)\to(Z,\gamma)$ be a function defined by $g(p)=x, g(q)=y$ and $g(r)=z$. Clearly f and g are supra $\delta\hat{g}$-continuous functions. $g\circ f:(X,\tau)\to(Z,\gamma)$ is not supra $\delta\hat{g}$-continuous function. Since $(g\circ f)^{-1}(\{y,z\})=f^{-1}[g^{-1}(\{y,z\})]=f^{-1}(\{q,r\})=\{b,c\}$ is not supra $\delta\hat{g}$-closed set of (X,μ) where $\{y,z\}$ is a closed set of (Z,γ).

Theorem 3.25. Let (X,τ), (Y,σ) and (Z,γ) be three topological spaces. Let μ and η be the associated supra topologies with τ and σ respectively. If a function $f:(X,\tau)\to(Y,\sigma)$ is supra $\delta\hat{g}$-continuous and $g:(Y,\sigma)\to(Z,\gamma)$ is continuous, then $g\circ f:(X,\tau)\to(Z,\gamma)$ is supra $\delta\hat{g}$-continuous.

Proof. Let F be closed set in (Z,γ). Then $g^{-1}(F)$ is closed in (Y,σ). Since f is supra $\delta\hat{g}$-continuous, $(g\circ f)^{-1}(F)=f^{-1}(g^{-1}(F))$ is supra $\delta\hat{g}$-closed set of (X,μ) and so $g\circ f$ is supra $\delta\hat{g}$-continuous function.

Theorem 3.26. Let (X,τ) and (Y,σ) be two topological spaces and μ be the associated supra topology with τ. A function $f:(X,\tau)\to(Y,\sigma)$ be supra continuous and supra δ-closed. Then for every supra $\delta\hat{g}$-closed subset A of (X,μ), $f(A)$ is supra $\delta\hat{g}$-closed in (Y,σ).

Proof. Let A be supra $\delta\hat{g}$-closed in (X,μ). Let $f(A)\subseteq U$ where U is open in (Y,σ). Since $A\subseteq f^{-1}(U)$ is supra open in (X,μ), $f^{-1}(U)$ is $\hat{g}^{\mu}$-open in (X,μ). Also since A is supra $\delta\hat{g}$-closed and $f^{-1}(U)$ is $\hat{g}^{\mu}$-open in (X,μ), $cl_{\delta}^{\mu}(A)\subseteq f^{-1}(U)$. Thus $f(cl_{\delta}^{\mu}(A))\subseteq U$.

Hence $cl_{\delta}^{\mu}(f(A))\subseteq cl_{\delta}^{\mu}(f(cl_{\delta}^{\mu}(A))=f(cl_{\delta}^{\mu}(A))\subseteq U$, since f is supra δ-closed. Hence $f(A)$ is supra $\delta\hat{g}$-closed in (Y,σ).

4. SUPRA $\delta\hat{g}$ –IRRESOLUTE FUNCTIONS

Definition 4.1. Let (X,τ) and (Y,σ) be two topological spaces and μ and η be the associated supra topologies with τ and σ respectively. A function $f:(X,\tau)\to(Y,\sigma)$ is called supra $\delta\hat{g}$-irresolute if $f^{-1}(V)$ is supra $\delta\hat{g}$-closed in (X,μ) for every supra $\delta\hat{g}$-closed set V of (Y,η).

Theorem 4.2. Let (X,τ), (Y,σ) and (Z,γ) be three topological spaces and μ , η and ν be the associated supra topologies with τ, σ and γ respectively. Let $f:(X,\tau)\to(Y,\sigma)$ and $g:(Y,\sigma)\to(Z,\gamma)$ be two functions. Then

(i) $g\circ f:(X,\tau)\to(Z,\gamma)$ is supra $\delta\hat{g}$-irresolute, if g is supra $\delta\hat{g}$-irresolute and f is supra $\delta\hat{g}$- irresolute.

(ii) $g\circ f:(X,\tau)\to(Z,\gamma)$ is supra $\delta\hat{g}$-continuous, if g is supra $\delta\hat{g}$-continuous and f is supra $\delta\hat{g}$-irresolute.

Proof. (i) Let F be supra $\delta\hat{g}$-closed set in (Z,ν). Then $g^{-1}(F)$ is supra $\delta\hat{g}$- closed in (Y,η). Since f is supra $\delta\hat{g}$-irresolute, $(g\circ f)^{-1}(F)=f^{-1}(g^{-1}(F))$ is supra $\delta\hat{g}$-closed set of (X,μ) and so $g\circ f$ is supra $\delta\hat{g}$-irresolute function.

(ii) Let F be any closed set in (Z,γ). Then $g^{-1}(F)$ is supra $\delta\hat{g}$-closed in (Y,η). Since f is supra $\delta\hat{g}$-irresolute, $(g\circ f)^{-1}(F)=f^{-1}(g^{-1}(F))$ is supra $\delta\hat{g}$-closed set of (X,μ) and so $g\circ f$ is supra $\delta\hat{g}$-continuous function.

Remark 4.3. supra $\delta\hat{g}$ -continuity and supra $\delta\hat{g}$ -irresoluteness are independent notions as seen in the following examples.

Example 4.4. Let $X=\{a,b,c\}=Y$ with topologies $\tau=\{\phi,\{a\},\{a,b\},X\}$, $\sigma=\{\phi,\{a,c\},Y\}$ respectively. The supra topologies μ and η are defined as follows: $\mu=\{\phi,\{a\},\{a,b\},\{b,c\},X\}$ and $\eta=\{\phi,\{a,b\},\{b,c\},\{a,c\},Y\}$. Let $f:(X,\tau)\to(Y,\sigma)$ be the identity function. Clearly f is supra $\delta\hat{g}$ -irresolute but not supra $\delta\hat{g}$ -continuous function. Because the inverse image of the closed set $\{b\}$ in (Y,σ) is $\{b\}$ which is not supra $\delta\hat{g}$ -closed in (X,μ).

Example 4.5. Let $X=\{a,b,c\}=Y$ with topologies $\tau=\{\phi,\{a\},\{a,b\},X\}$, $\sigma=\{\phi,\{a\},\{b\},\{a,b\},\{b,c\},Y\}$ respectively. The supra topologies μ and η are defined as follows: $\mu=\{\phi,\{a\},\{a,b\},\{b,c\},X\}$ and $\eta=\{\phi,\{a\},\{b\},\{a,b\},\{b,c\},\{a,c\},Y\}$. Let $f:(X,\tau)\to(Y,\sigma)$ be the identity function. Clearly f is supra $\delta\hat{g}$ -continuous but not supra $\delta\hat{g}$ -irresolute function. Because the inverse image of the supra $\delta\hat{g}$ -closed set $\{b\}$ in (Y,η) is $\{b\}$ which is not supra $\delta\hat{g}$ -closed in (X,μ).

5. APPLICATIONS

Definition 5.1. [8] A space (X,μ) is called $\hat{T}^{\mu}_{3/4}$ -space if every supra $\delta\hat{g}$ -closed set in it is supra δ -closed.

Theorem 5.2. Let (X,τ) and (Y,σ) be two topological spaces and μ and η be the associated supra topologies with τ and σ respectively. A function $f:(X,\tau)\to(Y,\sigma)$ be supra $\delta\hat{g}$ -irresolute. Then f is supra δ -continuous if (X,μ) is $\hat{T}^{\mu}_{3/4}$ -space.

Proof. Let V be a supra δ -closed subset of (Y,η). Since every supra δ -closed set is supra $\delta\hat{g}$ -closed and f is supra $\delta\hat{g}$ -irresolute, $f^{-1}(V)$ is supra $\delta\hat{g}$ -closed in (X,μ). Since X is $\hat{T}^{\mu}_{3/4}$ -space, $f^{-1}(V)$ is supra δ -closed in (X,μ). Thus f is supra δ -continuous.

Theorem 5.3. Let (X,τ), (Y,σ) and (Z,γ) be three topological spaces and μ and η be the associated supra topologies with τ and σ respectively. Let Y be $\hat{T}^{\mu}_{3/4}$ -space. If a function $f:(X,\tau)\to(Y,\sigma)$ is supra $\delta\hat{g}$ -continuous and $g:(Y,\sigma)\to(Z,\gamma)$ is supra $\delta\hat{g}$ -continuous, then $g\circ f:(X,\tau)\to(Z,\gamma)$ is supra $\delta\hat{g}$ -continuous.

Proof. Let G be any closed set in (Z,γ). Then $g^{-1}(G)$ is supra $\delta\hat{g}$ -closed in (Y,η). Since Y is $\hat{T}^{\mu}_{3/4}$, $g^{-1}(G)$ is supra closed in (Y,η). Also since f is supra $\delta\hat{g}$ -continuous, $(g\circ f)^{-1}(G)=f^{-1}(g^{-1}(G))$ is supra $\delta\hat{g}$ -closed of (X,μ). Hence $g\circ f$ is supra $\delta\hat{g}$ -continuous function.

Theorem 5.4. Let (X,τ) and (Y,σ) be two topological spaces and μ and η be the associated supra topologies with τ and σ respectively. Let a function $f:(X,\tau)\to(Y,\sigma)$ be onto, supra $\delta\hat{g}$ -irresolute and supra δ -closed. If X is a $\hat{T}^{\mu}_{3/4}$ -space then Y is also a $\hat{T}^{\mu}_{3/4}$ -space.

Proof. Let B be a supra $\delta\hat{g}$ -closed subset of (Y,η). Since f is supra $\delta\hat{g}$ -irresolute, $f^{-1}(B)$ is supra $\delta\hat{g}$ -closed set in (X,μ). Also since X is $\hat{T}^{\mu}_{3/4}$ -space, then $f^{-1}(B)$ is supra δ -closed in X. Since f is surjective, B is supra δ -closed in Y. Hence Y is $\hat{T}^{\mu}_{3/4}$ -space.

Theorem 5.5. Let (X,τ) and (Y,σ) be two topological spaces and μ and η be the associated supra topologies with τ and σ respectively. If $f:(X,\tau)\to(Y,\sigma)$ is bijection, supra open and supra $\delta\hat{g}$-continuous, then f is supra $\delta\hat{g}$-irresolute.

Proof. Let V be supra $\delta\hat{g}$-closed in (Y,η) and let $f^{-1}(V)\subseteq U$ where U is supra open in (X,μ). Since f is supra open, $f(U)$ is supra open in (Y,η). Every supra open set is $\hat{g}^{\mu}$-open and hence $f(U)$ is $\hat{g}^{\mu}$-open. Clearly $V\subseteq f(U)$. Then $cl_{\delta}^{\mu}(V)\subseteq f(U)$ and thus $f^{-1}(cl_{\delta}^{\mu}(V))\subseteq U$. Since f is supra $\delta\hat{g}$-continuous and $cl_{\delta}^{\mu}(V)$ is a supra closed subset of (Y,η), $cl_{\delta}^{\mu}(f^{-1}(V))\subseteq cl_{\delta}^{\mu}(f^{-1}(cl_{\delta}^{\mu}(V))\subseteq U$. Thus we have $cl_{\delta}^{\mu}(f^{-1}(V))\subseteq U$ whenever $f^{-1}(V)\subseteq U$ and U is $\hat{g}^{\mu}$-open set in (X,μ). This shows that $f^{-1}(V)$ is supra $\delta\hat{g}$-closed in (X,μ). Hence f is supra $\delta\hat{g}$-irresolute.

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