# Some properties of Anti Q- L-Fuzzy Normal M-Subgroups

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***Abstract***: In this paper we introduced the concept of anti Q-L- fuzzy Normal M-subgroups and discuss some of its properties.

***Keywords***; M-subgroup, anti L-fuzzy M-subgroup, Q- fuzzy normal group andanti Q-L- fuzzyNormal M-subgroup.

## 1. INTRODUCTION

The fundamental concept of fuzzy sets was initiated by Zadeh L [18] in 1965. Fuzzy set theory has been developed in many directions by many scholars and has evoked great interest among mathematicians working in different fields of mathematics such as topological spaces, functional analysis, loop, group, ring, near- ring, vector spaces, and automation. The fuzzy algebraic structures play a prominent role in mathematics with wide applications in many other branches such as theoretical physics, computer science, control engineering, information science, coding theory, group theory, real analyses, hectare theory etc. In 1971, Rosenfeld A [11] first introduced the concept of fuzzy subgroups, which was the first fuzzification of any algebraic such as fuzzy structures. In 1990,Biswas R [3],introduced the concept of Fuzzy subgroups and anti-fuzzy subgroups.The concept of L-Fuzzy Sets is given by Goguen J Ain 1967 [5].Anthony J M and Sherwood H,[1] introduced and defined a new concept of a characterization of fuzzy subgroups in 1982. In 1981, Wu W.M [17] introduced the concept of Normal fuzzy subgroups. The notion of Algebraic fuzzy systems, fuzzy sets and systems are introduced by Swamy U M and ViswanadhaRaju D in 1991[15].Katsaras A K and Liu D B, [6] considered the Fuzzy vector spaces and fuzzy topological vector spaces. Wang-Jin Liu [16] introduced and defined a new Fuzzy invariant subgroups and fuzzy sets and systems in 1981. Garrett Birkhof [4] introduced the concept of Lattice Theory, SatyaSaibaba G S V [12] discussedFuzzy lattice Ordered Groups. AthKehages [2],introduced the concept of the lattice of fuzzy intervals and sufficient conditions for its dis-tibutivity .In 1991, Murali. V [8], defined and studied lattice of fuzzy algebras and closure systems in $I^X$.Makandar. M U andDr A D Lokhande [7], introduced the new concept of Anti Fuzzy Lattice Ordered M-Group in 2013. The concept of Structure Properties of M-Fuzzy Groups is given by Subramanian. S,Nagrajan R and Chellappa [14] in 2012.Soaliraju A and Nagarajan R in 2009 [13], introduced the concept of a New structure and construction of Q- fuzzy groups. Pandiammal P, Natarajan R and Palaniappan N [9], introduced the concept of anti L-fuzzy M-subgroups and anti L-fuzzy normal M-subgroups in 2010. In 2103, Priya T, Ramachandran T and Nagalakshmi K T [10], introduced the concept of On Q-Fuzzy Normal Subgroups. We introduced the concept of anti Q-L- fuzzy Normal M-subgroups and discuss some of its properties.

## 2. PRELIMINARIES

In this section, we review some basic definitions and some results of anti L- fuzzy M-subgroup, homomorphism and anti-homomorphism which will be used in the later sections. Through this section we mean that ( $G,*$)is a group, e is the identity of G and $xy$as $(x * y)$.

*Definition: 2.1 [ Zadeh L A (17) ]*

A mapping $\mu: X \to$[0,1], where $X$is an arbitrary non-empty set and is called a fuzzy set in $X$.

*Definition: 2.2 [ Rosenfeld A (11) ]*

Let G be any group. A mapping $\mu: G \to$[0,1] is a fuzzy group if

(i) $\mu(xy) \geq min\{\mu(x), \mu(y)\}$

(ii) $\mu(x^{-1}) = \mu(x)$, for all $x, y \in G$.

*Definition: 2.3 [ Solairaju A and Nagarajan R (13)]*

Let Q and G a set and a group respectively. A mapping $\mu: G \times Q \to [0,1]$ is called Q- fuzzy set in G. For any Q-fuzzy set $\mu$ in G and $t \in [0,1]$ we define the set $U(\mu; t) = \{x \in G \ / \ \mu(x, q) \geq t, q \in Q\}$ which is called an upper cut of "$\mu$" and can be use to the characterization of $\mu$.

*Definition: 2.4[ Solairaju A and Nagarajan R (13)]*

A Q- fuzzy set A is called Q-fuzzy group of G if

(i) $A(xy, q) \geq \min\{A(x, q), A(y, q)\}$

(ii) $A(x^{-1}, q) = A(x, q)$

(iii) $A( x, q) = 1$, for all $x, y \in G$and $q \in Q$.

*Definition:2.5[Biswas R (3)]*

A fuzzy set μofagroup G is called a fuzzy subgroup if $\mu(xy^{-1}) \geq min\{\mu(x), \mu(y)\}$for every $x, y \in G$.

*Definition:2.6[Wu W M (17)]*

A fuzzy subgroupμ ofagroupG is called a fuzzy normal subgroup if $\mu(xyx^{-1}) \geq \mu(y)$ for every $x, y \in G$.

*Definition: 2.7[ Priya T, Ramachandran T and Nagalakshmi K T (10)]*

Any Q-fuzzy group $\mu$ of G is said to be normal if there exists $x \in G$ and $q \in Q$ such that $\mu(x, q) = 1$.

Note that if μ is normal Q- fuzzy group of G, then $\mu(e, q) = 1$ and hence $\mu$ is normal if and only if $\mu(e, q) = 1$.

*Definition: 2.8[Priya T, Ramachandran T and Nagalakshmi K T (10)]*

Let $\mu$be a Q- fuzzy group of G. Then $\mu$ is called Q- fuzzy normal group (QFNG) if $\mu(xy, q) = \mu(yx, q)$, for all $x, y \in G$ and $q \in Q$.

Alternatively, a Q- fuzzy group $\mu$ is said to be Q- fuzzy normal if $\mu(x, q) = \mu(yxy^{-1}, q)$ for $x, y \in G$ and $q \in Q$. the notation $[x, y]$ stands for the expression $x^{-1}y^{-1}xy$.

*Definition: 2.9 [Subramanian S, Nagarajan R and Chellappa B (14)]*

A group with operators is an algebraic system consisting of a group G, a set M and a function defined in the product set $M \times G$ and having the values in G such that if $mx$ denotes the element in G determined by the element x of G and the element m of M, and $m \in M$, then G is called M- group with operators.

*Definition: 2.10 [Subramanian S, Nagarajan R and Chellappa B (14)]*

A subgroup A of an M- group G is said to be the fuzzy subgroup if $mx \in A$ for all $m \in M$ $and$ $x \in A.$

*Definition: 2.11[Subramanian S, Nagarajan R and Chellappa B (14)]*

Let A be a fuzzy set in U and • $:G \times G \rightarrow G$ be a composition law, such that (G, • ) forms M- group. If two conditions $A(m(xy)) \geq min\{A(mx), A(my)\}$ and $A(mx^{-1}) = A(mx)$ for all $x, y$ in A. If the supplementary conditions $A(meG) = 1$ is also satisfied, then this M- fuzzy group is called a standardized M-fuzzy group, where $e$ is an identity of M- group (G, • ).

*Definition: 2.12*

The function $f: G \rightarrow$ G' is said to be a homomorphism if $f(xy) = f(x)f(y)$for all $x, y \in G.$

*Definition: 2,14*

The function $f: G \rightarrow G'$( $G$ and$G'$ are not necessarily commutative) is said to be an anti-homomorphism. If $f(xy) = f(y)f(x)$for all $x, y \in G.$

*Definition: 2.15 [Pandiammal P, Natarajan R and Palaniappan N (9)]*

Let G and G' both be M – groups , f be a homomorphism from G onto G' , $iff$ $(mx) = mf(x)$ for every $m \in M$ , $x \in X,$ thenfis called a M–homomorphism .

*Definition: 2.16[Pandiammal. P, Natarajan. R and Palaniappan. N (9)]*

Let G be M – group and μ be a fuzzy subgroup of G if $\mu(mx) \geq \mu(x)$ for every $x \in G$ , $m \in M$ ,then μ is said to be a fuzzy subgroup with operators of G , we use the pharse μ is an M – fuzzy subgroup of G instead of a fuzzy subgroup with operators ofG. [4]

*Definition: 2.17 [Pandiammal P, Natarajan R and Palaniappan N (9)]*

Let (G, . ) be a M-group. A L-fuzzy subset A of G is said to be anti L-fuzzy M-subgroup (ALFMSG) of G if the following conditions are satisfied:

(i) $\mu_A(mxy) \leq \mu_A(x) \vee \mu_A(y),$

(ii) $\mu_A(x^{-1}) \leq \mu_A(x),$ for all $x$ $and$ $y$ $in$ $G.$

*Definition: 2.18 [Pandiammal P, Natarajan R and Palaniappan N (9)]*

Let (G, . ) and $(G'$, . ) be any two M-groups. Let $f: G \rightarrow G'$ be anyfunctionandletAbeanantiL-fuzzyM-subgroupinG,Vbeananti L-fuzzy M-subgroup in $f(G) = G'$defined by $\mu_v(y) = inf_{x \in f^{-1}(y)} \mu_A(x), \forall x \in G, \forall y \in G'.$ .Then A is called a preimage of V under f and is denoted by $f^{-1}(v).$

*Definition: 2.19[Pandiammal P, NatarajanRand Palaniappan N (9)]*

Let A and B be two L-fuzzy subsets of sets G and H, respectively. The anti-productof A and B, denoted by $A \times B$, is defined as $A \times B = \{ \langle (xy), \mu_{A\times B}(xy) \rangle$ / for all $x$ in G, $y$ in H }, Where $\mu_{A\times B}(xy) = \mu_A(x) \vee \mu_B(y)$, for all $x$ in G and $y$ in H,

*Definition: 2.20[Pandiammal P, Natarajan R and Palaniappan N (9)]*

Let A and B be two anti L-fuzzy M-subgroups of a M-group G. Then A and B are said to be conjugate anti L-fuzzy M-subgroupsof G if for some g in G, $\mu_A(x) = \mu_B(g^{-1} x\, g)$for every $x$ inG .

*Definition: 2.21[SatyaSaibaba G S V (12)]*

A lattice ordered group is a system $G = (G, *, \leq)$ where

(i) $(G, *)$ is a group

(ii) $(G, \leq)$ is a lattice and

(iii) the inclusion is invariant under all translations $x \mapsto a * x * b.$

That is, $x \leq y \Rightarrow a * x * b \leq a * y * b$, for all a,b$\in G$.

*Definition: 2.22 [Goguen J A (5)]*

A L- Fuzzy subset λ of $X$ is a mapping from $X$ into L, where is a complete lattice satisfying from $X$ meet distributive law. If L is the unit interval [ 0, 1] of real numbers, there are the usual fuzzy subset of $X$.

A L-fuzzy subset λ:X → L is said to be a nonempty, if it is not the constant map which assumes the values 0 of L

*Definition: 2.23 [Goguen J A (5)]*

Let λ:X → Lbe a L-fuzzy subset set of X. Then for $t \in L$, the set $\lambda_t = x \in X \,/\, \lambda(x) \leq t$ is called a lower t-cut or t-level set of λ.

*Definition: 2.24 [Goguen. J. A (5)]*

Let λ, $\mu: X \rightarrow$ L be a L-fuzzy sub sets of X. If $\lambda(x) \leq \mu(x)$ for all $x \in X$, then we say that λ is contained in $\mu$ and we write $\lambda \subseteq \mu$.

*Definition: 2.25 [Goguen J A (5)]*

Let λ, $\mu: X \rightarrow$ L be a L-fuzzy sub sets of X. Define $\lambda \cup \mu$, $\lambda \cap \mu$ are L-fuzzy subsets of X by all $x \in X$, $(\lambda \cup \mu)(x) = \lambda(x) \vee \mu(x)$ and $(\lambda \cap \mu)(x) = \lambda(x) \wedge \mu(x)$. Then $\lambda \cup \mu$, $\lambda \cap \mu$ are called union and intersection of λ and $\mu$ respectively.

*Definition: 2.26 [Goguen J A (5)]*

A L-fuzzy subset λ of X is said to have sup property if, for any subset A of X, there exists $a_o \in A$ such that $\lambda(a_o) = \vee_{a \in A} \lambda(a)$.

*Definition: 2.27[Wang-Jin Liu (16)]*

Let $f$ be any function from a set $X$ to set $Y$, and let λ be any L-fuzzy subset of $X$. Then λ is called $f-$ invariant if $f(x) = f(y)$ implies $\lambda(x) = \lambda(y)$, where $x, y \in X$.

*Definition: 2.28[Anthony J M, Sherwood H (1) and Rosenfeld A (11)]*

A L- fuzzy subset λ of G is said to be a L- fuzzy subgroup of G, If for all $x, y \in G$,

(i) $\lambda(xy) \leq \lambda(x) \vee \lambda(y)$,

(ii) $\lambda(x^{-1}) = \lambda(x)$.

*Definition: 2.29[ Biswas R (3)]*

A L- fuzzy subset λ of G is said to be an anti L- fuzzy subgroup of G, If for all $x, y \in G$,

(i) $\lambda(xy) \leq \lambda(x) \vee \lambda(y)$,

(ii) $\lambda(x^{-1}) = \lambda(x)$.

**Remark:**$\lambda(e) \leq \lambda(x)$, for all $x \in G$.

*Definition: 2.30[ Biswas R (3)]*

A L-fuzzy subset λ of G is said to be an anti L- fuzzy subgroup of G if and only if $\lambda_t$ is a subgroup of G for all $\lambda(G) \cup t \in L \,/\, \lambda(e) \leq t$.

*Definition: 2.31[Makandar U M and ,Dr A D Lokhande (7)]*

Let Q and X be ant two sets and λ be a L-fuzzy subset of X. A Q-L-fuzzy subset λ of X is a mapping from of $Q \times X$ into L, where L is a complete lattice satisfying the infinite meet distributive law. If L is the unit interval [0,1] of real numbers, there are the usual Q-fuzzy subset of X. A Q- L- fuzzy subset $\lambda: X \times Q \rightarrow$ L is said to be a nonempty, if it is not the constant map which assumes the values 0 of L.

*Definition: 2.32[Makandar U M and ,Dr A D Lokhande (7)]*

Let λ, $\mu: X \times Q \rightarrow$ L be a Q-L-fuzzy subsets of X. If $\lambda(x, q) \leq \mu(x, q)$ for all $x \in G$ and $q \in Q$ then we say that λ is contained in $\mu$ and we write $\lambda \subseteq \mu$.

*Definition: 2.33 [Makandar U M and ,Dr A D Lokhande (7)]*

Let λ, $\mu: X \times Q \rightarrow$ L be a Q-L-fuzzy subsets of X. . Define λ∪ $\mu$, λ∩ $\mu$ are Q- L-fuzzy subsets of $X$ by all $x \in X$, $(\lambda \cup \mu)(x,q) = \lambda(x,q) \vee \mu(x,q)$ and $(\lambda \cap \mu)(x,q) = \lambda(x,q) \wedge \mu(x,q)$ for all $x \in G \; and \; q \in Q$. Then λ∪ $\mu$, λ∩ $\mu$ are called union and intersection of λ and $\mu$ respectively

*Definition: 2.34[Pandiammal P, Natarajan R and Palaniappan N (9)]*

Let(G, .)be a M-group. An anti L-fuzzy M-subgroup A of G is said to be an anti L-fuzzy normal M-subgroup (ALFNMSG) of G if ()=() , for all and in G.

*Definition: 2.35[Pandiammal P, Natarajan R and Palaniappan N (9)]*

Let A be an L-fussy subset in asset S,the anti-strongest L-fuzzy relation on A is V given by () = ()∨() for all and in S.

*Definition: 2.36[Pandiammal P, Natarajan R and Palaniappan N (9)]*

Let (G,.) be a M-group. An anti L-fuzzy M-subgroup A of G is said to be an anti L-fuzzy characteristic M-subgroup (ALFCMSG) of G if ()=(($x$)),for all $X$ in G and $f$ in $AutG$.

*Definition: 2.37[Pandiammal P, Natarajan R and Palaniappan N (9)]*

A L-fuzzy subset A of a set $X$ is said to be anti-normalized if there exist $x$ in $X$ such that $\mu_{A(X)=}0$.

## 3. ANTI Q-L- FUZZY NORMAL M-SUBGROUPS

*Definition: 3.1*

Let(G, .)be a M-group. An anti Q-L-fuzzy M-subgroup A of G is said to be an anti L-fuzzy normal M-subgroup (ALQFNMSG) of G if $\alpha_A(xy,q) = \alpha_A(yx,q)$ , for all $x$ and $y$ in G and q in Q.

*Definition: 3.2*

Let A be an Q- L-fussy subset in a set S, the anti-strongest L-fuzzy relation on A is V given by $\alpha_V((x,y),q)) = \alpha_A(x,q) \vee \alpha_A(y,q)$,for all $x$ and $y$ in S and q in Q.

*Definition: 3.3*

Let (G,.) be a M-group. An anti L-fuzzy M-subgroup A of G is said to be an anti L-fuzzy characteristic M-subgroup (ALQFMSG) of G if $\mu_{A(x,q)=}\mu_{A(f(x,q))}$,for all X in G and f in AutG.

*Definition: 3.4*

A L-fuzzy subset A of a set X is said to be anti-normalized if there exist x in X such that $\mu_{A(x,q)=}0$.

*Theorem: 3.5*

If A and B are two anti Q-L-fuzzy normal M-subgroups of a M-group G, then their union AUB is an anti Q -L-fuzzy normal M-subgroup of G.

*Proof:*

Let x and y belong to G and $A = \{ (x,q), \; \alpha_A(x,q) \; / \; x\epsilon G \; and \; q\epsilon Q\}$and $B = \{ (x,q), \alpha_B(x,q) \; / \; x\epsilon G \; and \; q\epsilon Q\}$be an anti Q-L-fuzzy normal M-subgroup of a M-group G. Let C = A□ B and $C = \{ (x,q), \alpha_C(x,q) \; / \; x\epsilon G \; and \; q\epsilon Q\}$.

Then, Cleary C is anti Q-L-fuzzy M-subgroups of a M-group G, since A and B are two anti Q-L-fuzzy M-subgroups of a M-groupG.

$$\alpha_C(xy,q) = \alpha_A(xy,q) \vee \alpha_B(xy,q) = \alpha_A(yx,q) \vee \alpha_B(yx,q) = \alpha_C(yx,q).$$

Therefore,$\alpha_C(xy,q) = \alpha_C(yx,q)$., for all $x \; and \; y$in G and q in Q.

Hence $A \cup B$ is an anti Q-L-fuzzy normal M-subgroup of a M-group G.

*Theorem: 3.6*

The union of a family of an anti Q- L-fuzzy normal M-subgroup of a M-group G is an anti Q-L-fuzzy normal M-subgroup of a M-groupG.

*Proof:*

Let $\{ A_i \}_{i \in I}$ be an family of an anti Q-L-fuzzy normal M-subgroup of a M-group G and let A = ∪$A_i$ .

Then for $x$ $and$ $y$belongs to G and q in Q.

We know that, the union of a family of an anti Q-L-fuzzy M-subgroup of a M-group G is an anti Q-L-fuzzy M-subgroup of a M-group G.

$$\alpha_A(xy, q) = Sup_{i \in I} \alpha_{Ai}(xy, q)$$

$= Sup_{i \in I} \alpha_{Ai}(yx, q)$for all $x$ $and$ $y$in G and q in Q.

$= \alpha_{Ai}(yx, q)$

Therefore,$\alpha_A(xy, q) = \alpha_A(yx, q)$ , for all$x$ $and$ $y$ in G and q in Q.

Hence the union of a family of an anti Q- L-fuzzy M-subgroup of a M-group G is an anti Q-L-fuzzy M-subgroup of G.

*Theorem: 3.7*

If A is an anti Q- L-fuzzy characteristic M-subgroup of a M-group G, then A is an anti Q-L-fuzzy normal M-subgroup of a M-groupG.

*Proof:*

Let A be an anti Q-L-fuzzy characteristic M-subgroup of a M-group G and let $x$ and $y$ inG and q in Q.

*Consider the map* $f : G \rightarrow G$ *defined by*

$f(x) = yxy^{-1}$. We know that, $f$ in AutG.

Now, $\alpha_A(xy, q) = \alpha_A(f(xy, q)), as\ A\ is\ an\ AQLFCMSG\ of\ a\ M - groupG.$

$= \alpha_A(y(xy)y^{-1}, q) = \alpha_A(yx, q).$

Therefore, $\alpha_A(xy, q) = \alpha_A(yx, q)$, for all $x$ $and$ $y$ in G and q in Q.

Hence A is an anti Q- L-fuzzy normal M-subgroup of a M-group G.

*Theorem: 3.8*

An anti Q-L-fuzzy M-subgroup A of a M-group G is an anti Q- L-fuzzy normal M-subgroup of G if and only if A is constant on the conjugate classes of G.

*Proof:*

Suppose that A is an anti Q- L-fuzzy normal M-subgroup of a M-group G and let $x$ $and$ $y$ inG and q in Q,

Now, $\alpha_A(y^{-1}xy, q) = \alpha_A(xyy^{-1}, q)$, sinceA is an AQLFNMSG of $G$

$= \alpha_A(x, q).$

Therefore, $\alpha_A(y^{-1}xy, q) = \alpha_A(x, q).$

for all $x$ $and$ $y$in G and q in Q.

Hence $(x, q) = \{ y^{-1}xy, q \,/\, y \in G\ and\ q \in Q \}.$

Hence A is constant on the conjugate classes of G.

Conversely, suppose that A is constant on the conjugate classes of G. Then, $\alpha_A(xy, q) = \alpha_A(xyxx^{-1}, q)$ $= \alpha_A(x(yx)x^{-1}, q)$, as A is constant on the conjugate classes of $G$.

$= \alpha_A(yx, q).$

Therefore,$\alpha_A(xy, q) = \alpha_A(yx, q)$, for all $x$ $and$ $y$ in G and q in Q.

Hence A is an anti Q-L-fuzzy normal M-subgroup of a M-group G.

*Theorem: 3.9*

Let A be an anti Q-L-fuzzy normal M-subgroup of a M-group G. Then for any y in G, we have $\alpha_A(yxy^{-1},q) = \alpha_A(y^{-1}xy,q)$, for every x inG.

*Proof:*

Let A be an anti Q- L-fuzzy normal M-subgroup of a M-group G. For any y in G, we have,

$\alpha_A(yxy^{-1},q) = \alpha_A(yy^{-1}x,q)$

$= \alpha_A(x,q)$, since A is an ALQFNMSG of G

$= \alpha_A(xyy^{-1},q)$, since A is an ALQFNMSG of G

$= \alpha_A(y^{-1}xy,q)$.

Therefore, $\alpha_A(yxy^{-1},q) = \alpha_A(y^{-1}xy,q)$, for all $x$ *and* $y$ in G and q in Q.

*Theorem: 3.10*

An anti Q-L-fuzzy M-subgroup A of a M-group G is anti-normalized if and only if $\alpha_A(e,q) = 0$, where e is the identity element of the M-groupG.

*Proof:*

If A is anti-normalized, then there exists x in G such that $\alpha_A(x,q) = 0$, but by properties of an anti Q-L-fuzzy M-subgroup A of the M-group G, $\alpha_A(x,q) \geq \alpha_A(e,q)$, for every $x$ inG and $q$ in $Q$.

Since $\alpha_A(x,q) = 0$ and $\alpha_A(x,q) \geq \alpha_A(e), 0 \geq \alpha_A(e)$.

Hence$\alpha_A(e,q) = 0$.

Conversely, if $\alpha_A(e,q) = 0$, then by the definition of anti-normalized fuzzy subset, A is anti-normalized.

*Theorem: 3.11*

Let A and B be anti Q-L-fuzzy M-subgroups of M-groups G and H, respectively. If A and B are anti Q- L-fuzzy normal M-subgroups, then anti-product $A \times B$ is an anti Q- L-fuzzy normal M-subgroup of$G \times H$.

*Proof:*

Let A and B be anti Q-L-fuzzy normal M-subgroups of the M-groups G and Hrespectively.

We know that anti-product $A \times B$ is an anti Q- L-fuzzy M-subgroup of $G \times H$.

Let $x_1$ *and* $x_2$be in G, $y_1$ *and* $y_2$be in H and q in Q.

Then $(x_1, y_1)$ and $(x_2, y_2)$are in $G \times H$ and q in Q

Now,

$$\alpha_{A\times B}[((x_1,y_1)(x_2,y_2),q)] = \alpha_{A\times B}[(x_1x_2, y_1y_2),q]$$
$$= \alpha_A(x_1x_2,q) \vee \alpha_B(y_1y_2,q)$$
$$= \alpha_A(x_2x_1,q) \vee \alpha_B(y_2y_1,q),$$
$$= \alpha_{A\times B}[(x_2x_1, y_2y_1),q]$$
$$= \alpha_{A\times B}[(x_2,y_2)(x_1,y_1),q].$$

Therefore, $\alpha_{A\times B}[((x_1,y_1)(x_2,y_2),q)] = \alpha_{A\times B}[(x_2,y_2)(x_1,y_1),q]$,

for all $x_1, x_2$ in G and $y_1, y_2$ *in* H and q in Q.

Hence anti-product $A \times B$ is an anti Q-L-fuzzy normal M-subgroup of $G \times H$.

*Theorem: 3.12*

Let an anti Q-L-fuzzy normal M-subgroup A of a M-group G be conjugate to an anti Q-L-fuzzy normal M-subgroup M of G and an anti Q-L-fuzzy normal M-subgroup B of a M-group H be conjugate to an anti Q-L-fuzzy normal M-subgroup N of H. Then an anti Q- L-fuzzy normal M-subgroup $A \times B$ of a M-group $G \times H$is conjugate to an anti Q-L-fuzzy normal M-subgroup $M \times N\ of G \times H.$

*Proof:*

It is trivial.

*Theorem: 3.13*

Let A and B be Q-L-fuzzy subsets of the M-groups G and H, respectively. Suppose that e and eꞁ are the identity element of G and H, respectively. If anti-product $A \times B$is an anti Q- L-fuzzy normal M-subgroup of $G \times H$, then at least one of the following two statementsmusthold.

(i) $\alpha_B(e', q) \leq \alpha_A(x, q)$,for all $x$ inG and q in Q.

$(ii)\alpha_A(e, q) \leq \alpha_B(y, q)$,for all $y$ inH and q in Q.

*Proof:*

It is trivial.

*Theorem: 3.14*

Let A and B be Q-L-fuzzy subsets of the M-groups G and H, respectively and anti-product $A \times B$ is an anti Q-L-fuzzy normal M-subgroup of $G \times H.$ Then the following aretrue:

(i) if $\alpha_A(x, q) \geq \alpha_B(e', q)$,then A is an anti Q-L-fuzzy normal M-subgroup of G,

(ii) if $\alpha_B(x, q) \geq \alpha_A(e, q)$, then B is an anti Q-L-fuzzy normal M-subgroup ofH,

(iii) either A is a anti Q-L-fuzzy normal M-subgroup of G or B is an anti Q-L-fuzzy normal M-subgroup ofH.

*Proof:*

It is trivial.

## 4. ANTI Q-L- FUZZY NORMAL M- SUBGROUPS OF M- GROUPS UNDER HOMOMORPHISM AND ANTI- HOMOMORPHISM

*Theorem: 4.1*

Let $(G, \cdot)$ and $(G', \cdot)$ be any two M- groups. The homomorphic image of an anti Q- L- fuzzy normal M- subgroup of G is an anti Q-L- fuzzy normal M- subgroup of $G'$.

*Proof:*

Let $(G, \cdot)$ and $(G', \cdot)$ be any two M- groups.

Let $f: G \to G'$ be a homomorphism.

That is $f((xy), q) = f(x, q)f(y, q)$,

$f(mx, q) = mf(x, q)$, for all $x\ and\ y\ in\ G, m \in M\ and\ q \in Q$.

Let V= $f(A, q)$ where A is an anti Q- L- fuzzy normal M- subgroup of G.

We have to prove that V is an anti Q- L- fuzzy normal M-subgroup of $G'$.

Now, for $f(x, q)\ and\ f(y, q)$ in $G'\ and\ q\ in\ Q$ we have

clearly V is an anti Q-L-fuzzy M-subgroup of a M-group $G'$ .

since A is an anti Q- L-fuzzy M-subgroup of a M-group G.

Now, $\alpha_v(f(x, q)f(y, q)) = \alpha_v(f(xy), q)$, as f is a homomorphism

$$\leq \alpha_A((xy), q)$$

$= \alpha_A((yx), q)$, as A is an AQLFNMSG of G

$\geq \alpha_v( f(yx), q )$

$= \alpha_v( f(y, q)f(x, q) )$, as f is a homomorphism,

which implies that $\alpha_v( f(x, q)f(y, q) ) = \alpha_v( f(y, q)f(x, q) )$,

for all $x$ *and* $y$ in G and q in Q.

Hence V is an anti Q-L-fuzzy normal M-subgroup of a M-group $G'$.

*Theorem: 4.2*

Let ( G, · ) and ($G'$ · ) be any two M-groups. The homomorphic pre-image of an anti Q- L-fuzzy normal M-subgroup of $G'$is an anti Q-L-fuzzy normal M-subgroup ofG.

*Proof:*

Let ( G, · ) and ($G'$, · ) be any two M-groups.

Let $f: G \rightarrow G'$ be a homomorphism.

That is $(f(xy), q) = f(x, q)f(y, q)$,

$f(mx, q) = mf(x, q)$, for all $x$ *and* $y$ in G , m in M and q in Q.

Let V=f(A), where V is an anti Q-L-fuzzy normal M-subgroup of $G'$.

We have to prove that A is an anti Q- L-fuzzy normal M-subgroup of G.

Let $x$ *and* $y$ in G. Then, we know that

A is an anti Q-L-fuzzy M-subgroup of a M-group G,

since V is an anti Q-L-fuzzy M-subgroup of a M-group $G'$.

Now,

$\alpha_A((xy), q) = \alpha_V( f(xy), q )$,since$\alpha_A(x, q) = \alpha_V( f(x, q) )$

$= \alpha_V( f(x, q)f(y, q) )$, as f is a homomorphism

$= \alpha_V( f(y, q)f(x, q) )$,as V is an AQLFNMSG of $G'$.

$= \alpha_V( f(yx), q )$, as f is a homomorphism

$= \alpha_A((yx), q)$ *since* $\alpha_A(x, q) = \alpha_V( f(x, q) )$,

which implies that $\alpha_A\big((xy), q\big) = \alpha_A((yx), q)$ , for all $x$ *and* $y$ in G.

Hence A is an anti Q-L-fuzzy normal M-subgroup of a M-group G.

*Theorem: 4.3*

Let( G, · ) and ($G'$ · )be any two M-groups. The anti-homomorphic image of an anti Q-L-fuzzy normal M-subgroup of G is an anti Q-L-fuzzy normal M-subgroup of$G'$.

*Proof:*

Let ( G, · ) and ($G'$, · ) be any two M-groups.

Let $f : G \rightarrow G'$ be an anti-homomorphism.

That is $f(xy, q) = f(y, q)f(x, q), f(mx, q) = mf(x, q)$, for all $x$ and $y$ in G and m in M and q in Q.

Let $V = f(A, Q)$, where A is an anti Q-L-fuzzy normal M-subgroup of G.

We have to prove that V is an anti Q- L-fuzzy normal M-subgroup of $G'$.

For $f(x, q)$ and $f(y, q)$ *in* $G'$, we know that

V is an anti Q-L-fuzzy M-subgroup of a M-group $G'$.

since A is an anti Q-L-fuzzy M-subgroup of a M-group G.

Now, $\alpha_V( f(x, q)f(y, q) ) = \alpha_V( f((yx), q) )$, as f is an anti-homomorphism

$\leq \alpha_V((yx), q)$

$= \alpha_A((xy), q)$,as A is an AQLFNMSG of G

$\leq \alpha_V( f((yx), q) )$

$= \alpha_V( f(y, q)f(x, q) )$,as f is an anti-homomorphism,

which implies that $\alpha_V( f(x, q)f(y, q) ) = \alpha_V( f(y, q)f(x, q) )$ , for all $x$ and $y$ in G and $q$ in $Q$.

Hence V is an anti Q-L-fuzzy normal M-subgroup of a M-group .

*Theorem: 4.4*

Let ( G, · ) and ($G'$, · ) be any two M-groups. The anti-homomorphic pre-image of an anti Q-L-fuzzy normal M-subgroup of G| is an anti Q-L-fuzzy normal M-subgroup ofG.

*Proof:*

Let ( G, · ) and ($G'$, · ) be any two M-groups.

Let $f : G \rightarrow G'$ be an anti-homomorphism.

That is $f((xy), q) = f(y, q)f(x, q), f(mx, q) = mf(x, q)$, for all $x$ and $y$in G ,$m$ in $M$and q in Q.

Let V=f(A,Q) , where V is an anti Q-L-fuzzy normal M-subgroup of $G'$.

We have to prove that A is an anti Q-L-fuzzy normal M-subgroup of G.

Let $x$ and $y$ in G, we haveA is an anti Q-L-fuzzy M-subgroup of a M-group G,

since V is an anti Q-L-fuzzy M-subgroup of a M-group $G'$.

Now, $\alpha_A\big((xy), q\big) = \alpha_V\Big( f\big((xy), q\big)\Big)$,

since $\alpha_A(x, q) = \alpha_V( f(x, q))$

$= \alpha_V( f(y, q)f(x, q) )$,as f is an anti homomorphism

$= \alpha_V( f(x, q)f(y, q) )$,as V is an ALQFNMSG of G|

$= \alpha_V( f((yx), q) )$, as f is an antihomomorphism

$= \alpha_A((yx), q)$,since$\alpha_A(x, q) = \alpha_V( f(x, q) )$,

which implies that $\alpha_A\big((xy), q\big) = \alpha_A((yx), q)$ , for all $x$ and $y$ in G and q in Q.

Hence A is an anti Q-L-fuzzy normal M-subgroup of a M-group G.

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