Shortest Path in Single Commodity Flow Problem in Dynamic Networks Using Modified Algorithm

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Abstract: The dynamic algorithm of the shortest path single commodity flow problem that simplifies the shortest distance single commodity flow problem is formulated in this paper. This problem is noticeon directed networks with time-expanded capacities, fixed deliverytimes on nodes, and a given time horizon. We consider the distance functions, defined on edges, are nonlinear and depend on time and flow and the demand function also confesson time. The comparable algorithm, based on sinking the dynamic problem to a fixed problem on a time-expanded network, to solve the shortest distance dynamic single commodity flow problem is projected and some details is involve and its complexity discussed.

Keywords: dynamic networks, single commodity flows, dynamic flows, integer programming, flows over time, shortest distance

INTRODUCTION

In network a set of nodes linked by arcs (or branches). The variable for describing a network is (N, A). In real life, there are many every day system and phenomenon which can be readily recognized as networks in their own right since they satisfy the definition of network.

Single commodity flow

Single commodity flows have manyvitalrolesin network optimization, which is the outsized of these models in real world problems. There are many productswhich can be solved by single commodity flow problem comparabledistribution, scheduling planning, telecommunication, transportation, communication, and management problems. (See, for example, [1]). Another commodity into ever convertshas no commodity.So that each one has its own flow safeguarding constraints, but they ostentatiousfor the resources of the mutual network.We can consider single commodity network flow problem needs to find the shortest distancecommodity through a network, where the arcs haveaspecific capacity for commodity, and a common capacity for the commodity. While there is substantial literature on the fixed single commodity flow problem, hardly any results on single commodity dynamic flows are known, although the dynamic single commodity flows are much closer to reality than the fixed ones. Dynamicmodels the flow needs a certain amount of time to travel through each arc, it can be delayed at nodes, flow values on arcs and the network parameters can change with time.

Shortest path

The **shortest path** problem is the problem of finding a **path** between two vertices (or nodes) in a graph such that the sum of the weights of its constituent edges is minimized.This is the most important part of graph theory.

Dynamic network flow

Dynamic flows are generally used to typical different network-structured, decision-making problems terminated time (see, for example, [2, 3]), but because of their convolution, dynamic flow models have not been examined as well as usually flow models. In this paper we study the dynamic description of the shortest distance singlecommodity flow problem on networks withtime-varying capabilities of edges. We agree to take that distance functions, defined one edges, are nonlinear and rest on time and flow and the demand function also depends on time. The shortest distance single commodity dynamic flow problem observes for a likely flow over time with given time horizon, satisfying all supplies and demands with least distance. We suggest an algorithmfor solving this problem, which is based on decreased the dynamic problem to the conventionalfixed problem on a time-expanded network.

Problem formulation

A directed network N = (V,E,k,w,u,τ,d,ϕ) where V is the set of vertices and E is the set of edges and k is the single commodity. Every edges e belongs to set of edges which have a nonnegative time-varying capacity w_e^1(t) which limits the cumulativeon each arc e∈E in every moment of time t ∈T. in this paper we study that every arc e ∈ E has a nonnegative time varying capacity for single commodity, it's also called theconnected capacity u_e(t). Moreover, each edge e ∈ E has anlinked positive shipment time τ_e which define the amount of time it takes for flow to travel from the tail to the head of that edge. The underlying network also consists of demand function d: V ×T→R and cost function ϕ: E×R_+ ×T→R_+, where T ={0,1,2,...,T}.

The demand function d_kv(t) satisfies the following conditions:

a) v∈V for $d_1^v < 0$;

The demand function d_kv(t) satisfies the following conditions:

a) v∈V for $d_1^v < 0$;

b) if d_1^v (t) < 0 ,v ∈ V for commodity k=1 , d_v^1 (t) = 0, t = 1,2,...,T;

In order for the flow to exist we require that $\sum_{t\in T}\sum_{v\in V} d_v^1$ (t) = 0, where k=1, Nodes v ∈ V with $\sum_{t\in T} d_v^1$ (t)<0, k =1 which is known as sources for commodity k, nodes v ∈ V with $\sum_{t\in \in T} d_v^1(t)$ >0, k =1 which is known as sink for commodity k and nodes v ∈V with $\sum_{t\in T} d_v^1 ==$ 0, k=1,which is known as transitional node for commodity k.We denote by V_-^1 and V_+^1 and V_0^1 which is called of sources, sinks and transitional nodes for commodity k, respectively. The sources are nodes through which flow arrives the network and the sinks are nodes concluded which flow prairies the network. The sources and sinks are occasionally called fatal nodes, while the transitional nodes are called non-fatal nodes.

Shipment costs in model, which canvariationcompleted time, we define the cost function $\psi_e^1(x_e^1(t),t)$ it means that flow of commodity k of value ξ = x_e^1(t) edge e entered at time t will justify a shipment cost of $\psi_e^1(\xi, t)$. in this paper the isolated time model, in which all times are vital and limited by possibility T. Time is pompous in isolated steps, so that if one unit of flow leaves node u at time t on arc e = (u,v), then one unit of flow arrives at node v at time t + τ_e where τ_e is the transit time of arc e. The time limit (finite or infinite) is the time pending which the flow can travel in the network and defines the makespan T ={0,1,...,T} of time moments. We start with the definition of fixed single commodity flows. A fixed single commodity flow x on N = (V,E,k,w,u,d,ψ) gives to every arc e ∈ E for k ∈ 1, a non-negative flow value x_e^1, The following flow preservationlimitations are followed.

$$\sum_{e\in E^+} x_e^k - \sum_{e\in E^-} x_e^k = d_v^k, \forall v\in V, k=1$$

where $E^+(v) = \{(u,v) \mid (u,v)\in E\}$, $E^-(v) = \{(v,u) \mid (v,u)\in E\}$. x satisfies in single commodity flow and the demands if single commodity flow x_1, satisfies the demands d_v^1 for all v∈V.x is called feasible in Single commodity flow if it follows the commonabilitylimits:

$$\sum_{k\in K} x_e^1 \le \mu_e \quad \forall e\in E \ldots\ldots\ldots\ldots(1)$$

anddiscretecapabilities of each arc for every commodity:

$$0\le x_e^1 \le \forall e\in E, \forall k \in K \ldots\ldots\ldots (2)$$

Equation 1, 2 also called weak and strong imposing constraints. The total distance of the fixed single commodity flow x is defined as follows:

$$d(x) = \sum_{k\in K}\sum_{e\in E}\psi_e^1(x_e^1)$$

A feasible dynamic flow on N = (V,E,k,w,u,τ,d,ϕ) is a function x:
E×K ×T→R+ that satisfies the following conditions:

$$\sum_{e\in E^+(V)} x_e^1(t-t_e) - \sum_{e\in E^-} x_e^1(t) = d_v^1(t) \quad \forall t\in T, \forall v \in V \ldots\ldots\ldots(3)$$

$$\sum_{k=1} x_e^1(t) \le \mu_e(t), \quad \forall t\in T, \quad \forall e \in E \ldots\ldots\ldots(4)$$

$$0 \le x_e^1(t) \le w_e^1 \; \forall t\in T, \forall e\in E, k=1 \ldots\ldots\ldots(5)$$

$$x_e^1(t)=0, e\in E, \overline{T-t_e}+1, T, \quad k=1 \ldots\ldots\ldots(6)$$

This function x defines the value x_e^1(t) of flow of commodity k which is entered in edge e at time t. It is simple toperceive that the flow does not enter in edge e at time t if it will have to leave the edge after time T; it is sured done condition (6). it means that a feasible dynamic flow in Capability equation (5), w_e^1(t) units of flow of commodity k can enter the arc e at time t. Commoncapability constraints (4) it means that in a feasible dynamic flow, u_e(t) units of flow can enter the arc e at time t. Conditions (3) signify flow preservation constraints. The total distance of the dynamic single commodity flow x is defined as follows:

Shortest distance d(x) = $\sum_{t=0}^{T}\sum_{K=1}\sum_{e\in E}\psi_{e}^{1}\left(x_{e}^{1}(t),t\right)$................................(7)

In shortest-distance single commodity dynamic flow problem if $\tau_e = 0$, ∀e ∈ E and T = 0To find a feasible flow it is minimized to the impartial function (7) that it is very simple to perceive then the formulated problem becomes the fixedshortest-distance single commodity flow problem.

In a directed network the shortest distance single commodity flow problem we observe the path with shortest distance between a specified source and a destination .Numerousforms of the shortest path problem occur and we will formulated and solve little of them.

We consider in a network with *n* nodes and *m* arcs where every node is joint to every other node directly or through a path, this problem is to find the path with shortest distance between two given nodes which is called source (s) and destination (T).Let d_{ij} is the distance between nodes i and node j. Let X_{ij} = 1 if the arc i-j is in the shortest distance. The problem is formulated as follows:

Minimize $\sum_i \sum_j d_{ij}X_{ij}$

Subject to

$$\sum_j X_{Sj} = 1$$

$$-\sum_j X_{ij} + \sum_k X_{jk} = 0 \ \forall j \neq S,T$$

$$\sum_j X_{jT} = 1$$

X_{ij} = 0,1

Consider a network with 6 nodes and 10 arcs shown in fig 1.The arc weights are given in brackets: 1-2 (10), 1-3 (6), 2-3 (5), 2-4 (6), 2-5 (9), 3-4 (7), 3-6 (14), 4-5 (3), 4-6 (6) and 5-6 (5). Find the shortest path between nodes 1 and 6?

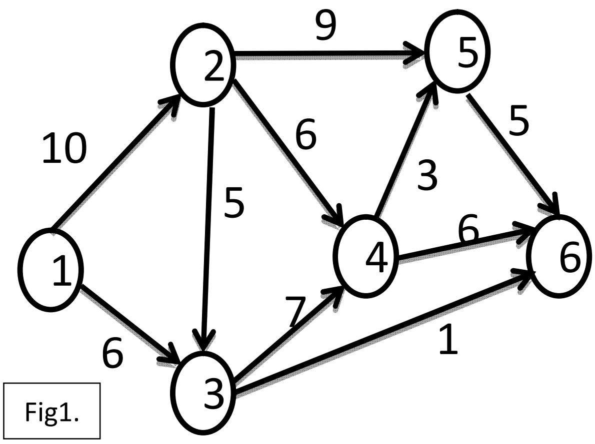


Fig1.

The objective is to Minimize $10X_{12} + 6X_{13} + 5X_{23} + 6X_{24} + 9X_{25} + 7X_{34} + 14X_{36} + 3X_{45} + 6X_{46} + 5X_{56}$

Subject to

$X_{12} + X_{13} = 1$

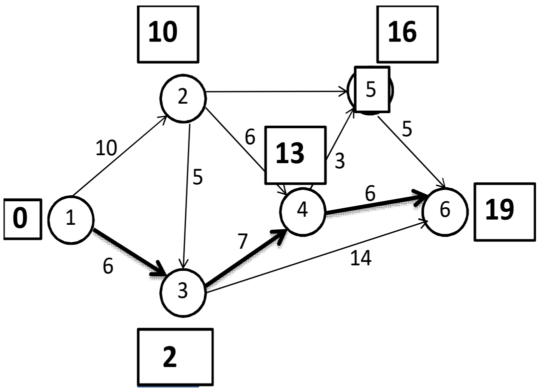
$-X_{12} + X_{23} + X_{24} + X_{25} = 0$

$-X_{13} – X_{23} + X_{34} + X_{36} = 0$

$-X_{24} – X_{34} + X_{45} + X_{46} = 0$

$-X_{25} - X_{45} + X_{56} = 0$
$X_{36} + X_{46} + X_{56} = 1$
$X_{ij} = 0, 1$

The optimum solution to the binary IP is given by $X_{13} = X_{34} = X_{46} = 1$ with Z = **19**. From the solution we observe that we travel from node 1 to node 3 and then to node 4 and then to node 6. The path is shown in Figure 2.



Shortest distance $x_{13=} x_{34=} x_{46} =19$

RESULTS

We propose an approach for solving the formulated problem, this problem founded on its dropped to a fixed flow problem. We show that the shortest distance single commodity flow problem on dynamic network N can be compact to the shortestdistance fixed flow problem on fixed network N^T it the time-expanded network. In this paper we study in a distance problem and solved by dynamic programming. And discuss the integer programming and find the many routes but results are very good.The dynamic flow problem in a given network with shipment times on the arcs can be converted into acorrespondingfixed flow problem in the corresponding time-expanded network. A discrete dynamic flow in the given network can be interpreted as a fixed flow in the consistent time-expanded network. The improvement of problem is that it opportunities the methodof defining an optimal flow over time into a usualfixed network flow problem in the time-expanded network. The time-expanded network is a fixeddemonstration of the dynamic network. Such a time-expanded network comprisesreplicas of the node set of the fundamental network for each isolatedbreak of time, building a time layer. Copies of an arc of the measured network link copies of its end-nodes in time coats whose distances like the shipment time of that arc. We define this network as obeyed:

1. $V^T: =\{v(t) \mid v \in V, t \in T\}$;
2. $E^T: =\{e(t) = (v(t),w(t+\tau_e)) \mid e = (v,w) \in E, 0 \le t \le T-\tau_e\}$;
3. $u^T_{e(t)} := u_e(t)$ for $e(t) \in E^T$;
4. $w^{1\,T}_{e(t)} := w^1_{e(t)}(t)$ for $e(t) \in E^T$, $k \in K$.
5 $\psi^1_{e(t)}(x^1_{e(t)}) := \psi^1_e(x_e(t),t)$ for $e(t) \in E^T$, $k \in K$;
6. $d^{1\,T}_{v(t)} := d^1_v(t)$ for $v(t) \in V^T$, $k \in K$.

The spirit of the time-expanded network that is it encloses a copy of the vertices of the dynamic network for each time t ∈ T, and the shipment times and flows are contained in the edges relating those copies.

CONCLUSION

The dynamic networks on the shortest distance single commodity flow problem can be solved by fixed flow reckonings in the consistent time-expanded network on dynamic network. If the dynamic network is direct with respect to flow of the distance function, Time expanded network will be linear. we can apply well-known methods for shortest distance flow problems, with linear and the distance function of the dynamic network is concave with esteem to flow, then the distance programming algorithms, combinatorial algorithms, as well as other progresses. If there exactly one source function of the time-expanded network will be concave. If the distance function of dynamic network is convex with regard to flow, then the distance function of the time-expanded network will be convex. In this case we can apply methods from convex programming and the specialization of such methods for shortest distance flow problems.

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