BIPOLAR FUZZY GRAPHS BASED ON ECCENTRICITY NODES

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Abstract: The distance and related concepts like eccentricity, radius, diameter,center,periphery are already defined and used in many applications of fuzzy graph theory. In this paper, we define a new distance called Bipolar fuzzy distance in Bipolar fuzzy graphs. Using this distance,bipolar fuzzy eccentricity,bipolar fuzzy center are defined and the relation between bipolar fuzzy radius and bipolar fuzzy diameter is established. Also the concept of fuzzy self centered graphs is generalized to bipolar fuzzy self centered graphs and a necessary condition for a bipolar fuzzy graph to be a bipolar fuzzy sellf centered graph is obtained.

Keywords: Bipolar fuzzy graph, eccentricity,radial nodes, peripheral nodes.

1.Introduction

In 1965,L.A.Zadeh[14] initiated the concept of bipolar fuzzy sets as a generalization of fuzzy sets. Bipolar fuzzy sets are an extension of fuzzy sets whose membership degree range is [-1,1]. In a bipolar fuzzy set, the membership degree 0 of an element means that the element is irrelevant to the corresponding property, the membership degree (0,1] of an element indicates that the element somewhat satisfies the property, and the membership degree

[-1,0) of an element indicates that the element somewhat satisfies the implicit counter-property. Although bipolar fuzzy sets and intuitionistic fuzzy sets look similar to each other, they are essentially different sets. In many domains, it is important to be able to deal with bipolar fuzzy information. It is noted that positive information represents what is granted to be possible, while negative information represents what is considered to be impossible. This domain has recently motivated new research in several direction. Akram[1] introduced the concept of bipolar fuzzy graphs and defined different operations on it. By a bipolar fuzzy graph, we mean a pair G = (A,B) where $A = (\mu_A^P, \mu_A^N)$ is a bipolar fuzzy set in V and $B = (\mu_B^P, \mu_B^N)$ is a bipolar fuzzy relation on E such that $\mu_B^P\ (x,y) \leq \min(\mu_A^P(x), \mu_A^P(y))$ and $\mu_B^N\ (x,y) \geq \max(\mu_A^N(x), \mu_A^N(y))$ for all$(x,y) \in E.$ We call A the bipolar fuzzy vertex set of V, B the bipolar fuzzy edge set of E respectively. Note that B is symmetric bipolar fuzzy relation on A. we use the notation xy for an element of E. Bipolar fuzzy graphs are precise models of all kinds of networks. In this paper, we define a new distance called Bipollar fuzzy distance in Bipolar fuzzy graphs. Using this distance,bipolar fuzzy eccentricity,bipolar fuzzy center are defined and the relation between bipolar fuzzy radius and bipolar fuzzy diameter is established. Also the concept of fuzzy self centered graphs is generalized to bipolar fuzzy self centered graphs and a necessary condition for a bipolar fuzzy graph to be a bipolar fuzzy sellf centered graph is obtained.

2. BIPOLAR FUZZY DISTANCE

Definition 2.1: A path P of length n is a sequence of distinct nodes $u_0, u_1, u_2, \ldots u_n$ such that $\mu(u_{i-1}, u_i) > 0, i = 1,2,3, \ldots. n$ and the degree of membership of a weakest edge is defined as its strength. The strength of connectedness between two nodes x and y is defined as the maximum of the strengths of all paths between x and y and is denoted by $CONN_G(x, y)$. An x-y path P is called a strongest x-y path if its strength equals $CONN_G(x, y)$.

Definition 2.2: By a bipolar fuzzy graph, we mean a pair G = (A,B) where $A = (\mu_A^P, \mu_A^N)$ is a bipolar fuzzy set in V and $B = (\mu_B^P, \mu_B^N)$ is a bipolar fuzzy relation on E such that $\mu_B^P\ (x, y) \le \min(\mu_A^P(x), \mu_A^P(y))$ and $\mu_B^N\ (x, y) \ge \max(\mu_A^N(x), \mu_A^N(y))$ for all$(x, y) \in E.$ We call A the bipolar fuzzy vertex set of V, B the bipolar fuzzy edge set of E respectively. Note that B is symmetric bipolar fuzzy relation on A. we use the notation xy for an element of E. Thus,
G = (A,B) is a bipolar fuzzy graph of $G^* = (V, E)$ if $\mu_B^P\ (xy) \le \min(\mu_A^P(x), \mu_A^P(y))$ and
$\mu_B^N\ (xy) \ge \max(\mu_A^N(x), \mu_A^N(y))$ for all $xy \in E.$

Definition 2.3: Let G:(A,B) be a bipolar fuzzy graph on $G^*: (V, E)$ then the biolar fuzzy distance between two nodes $(\mu_A^P(u), \mu_A^N(u))$ & $(\mu_A^P(v), \mu_A^N(v))$ in G is defined and denoted by
$d_{bf}^p(u, v) = min[L(P) * S(P)]/P$ and $d_{bf}^N(u, v) = max[L(P) * S(P)]/P$ is a u – v path,
L(P) is the length and S(P) is the strength of P and * represents ordinary product.

Example 2.1: Define $G = (A, B)$ by $\left(\mu_A{}^P(a), \mu_A{}^N(a)\right) = (0.5, -0.9)$, $\left(\mu_A{}^P(b), \mu_A{}^N(b)\right) = (0.6, -0.8)$, $\left(\mu_A{}^P(c), \mu_A{}^N(c)\right) = (0.5, -0.6)$, $\left(\mu_A{}^P(d), \mu_A{}^N(d)\right) = (0.4, -0.3)$ and $\left(\mu_B{}^P(ab), \mu_B{}^N(ab)\right) = (0.3, -0.2)$, $\left(\mu_B{}^P(bc), \mu_B{}^N(bc)\right) = (0.5, -0.4)$, $\left(\mu_B{}^P(cd), \mu_B{}^N(cd)\right) = (0.2, -0.1)$, $\left(\mu_B{}^P(ad), \mu_B{}^N(ad)\right) = (0.3, -0.2)$, $\left(\mu_B{}^P(bd), \mu_B{}^N(bd)\right) = (0.4, -0.3)$.
$\left(d_{bf}^P(a, b), d_{bf}^N(a, b)\right) = (0.3, -0.2)$,$\left(d_{bf}^P(a, c), d_{bf}^N(a, c)\right) = (0.4, -0.2)$, $\left(d_{bf}^P(a, d), d_{bf}^N(a, d)\right) = (0.3, -0.2)$,$\left(d_{bf}^P(b, c), d_{bf}^N(b, c)\right) = (0.4, -0.2)$, $\left(d_{bf}^P(b, d), d_{bf}^N(b, d)\right) = (0.4, -0.2)$ and $\left(d_{bf}^P(c, d), d_{bf}^N(c, d)\right) = (0.2, -0.1)$.

Definition2.4: Let G:(A,B) be a bipolar fuzzy graph on $G^*: (V, E)$ then the biolar fuzzy eccentricity of a node $(\mu_A^P(u), \mu_A^N(u)) \in V(G)$ is defined and denoted by
$e_{bf}^P(u) = \max_{v \in V(G)} d_{bf}^P\ (u, v)$ and $e_{bf}^N(u) = \min_{v \in V(G)} d_{bf}^N\ (u, v)$

Definition2.5: The minimum of the fuzzy positive eccentricities and maximum of the fuzzy negative eccentricities of all nodes is called the bipolar fuzzy radius of the graph G. It is deoted as $r_{bf}(G)$.
Thus $r_{bf}^P(G) = \min_{u \in V(G)} e_{bf}^P\ (u)$ and $r_{bf}^N(G) = \max_{u \in V(G)} e_{bf}^N\ (u)$

Definition 2.6: The maximum of the fuzzy positive eccentricities and minimum of the fuzzy negative eccentricities of all nodes is called the bipolar fuzzy diameter of the graph G. It is deoted as $D_{bf}(G)$.
That is $D_{bf}^P(G) = \max_{u \in V(G)} e_{bf}^P\ (u)$ and $D_{bf}^N(G) = \min_{u \in V(G)} e_{bf}^N\ (u)$

In example 2.1, $\left(e_{bf}^P(a),\ e_{bf}^N(a)\right) = (0.4, -0.2)$, $\left(e_{bf}^P(b),\ e_{bf}^N(b)\right) = (0.4, -0.2)$, $\left(e_{bf}^P(c),\ e_{bf}^N(c)\right) = (0.4, -0.2)$, $\left(e_{bf}^P(d),\ e_{bf}^N(d)\right) = (0.4, -0.2)$.

Definition 2.7: A node , $\left(\mu_A{}^P(v), \mu_A{}^N(v)\right) \in V(G)$ is called a bipolar fuzzy eccentric node of another node , $\left(\mu_A{}^P(u), \mu_A{}^N(u)\right)$ if $e_{bf}^P(u) = d_{bf}^P(u, v)$ and $e_{bf}^N(u) = d_{bf}^N(u, v)$

Definition 2.8: Nodes with minimum fuzzy positive eccentricity and nodes with maximum fuzzy negative eccentricity are called bipolar fuzzy central nodes or bipolar fuzzy radial nodes.

Definition 2.9: Nodes with maximum fuzzy positive eccentricity and nodes with minimum fuzzy negative eccentricity are called bipolar fuzzy diametral nodes or bipolar fuzzy peripherel nodes.

In example 2.1, $\left(r_{bf}^{P}(G),\ r_{bf}^{N}(G)\right) = (0.4, -0.2)\ and\ \left(D_{bf}^{P}(G),\ D_{bf}^{N}(G)\right) = (0.4, -0.2),$

Here all nodes are both bipolar fuzzy central as well as bipolar fuzzy diametral. Now consider the following example.

Example 2.2:

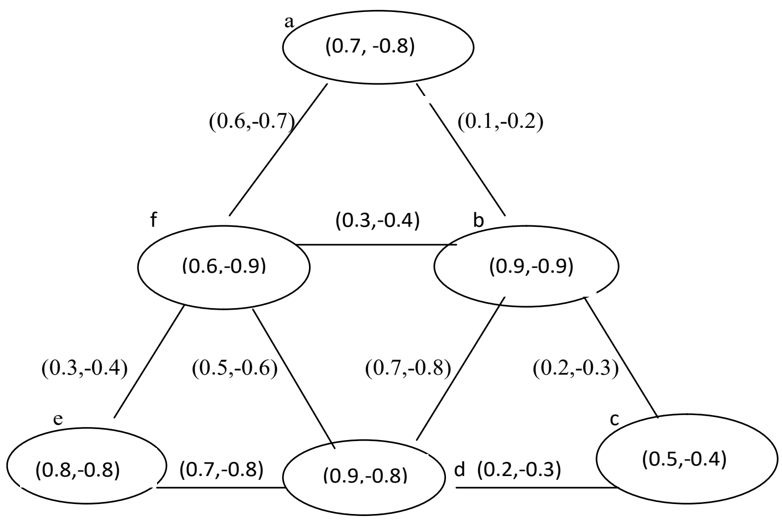

Figure 1. Bipolar fuzzy eccentricity and center

$\left(d_{bf}^{P}(a,b), d_{bf}^{N}(a,b)\right) = (0.1, -0.2)$, $\left(d_{bf}^{P}(a,c), d_{bf}^{N}(a,c)\right) = (0.2, -0.4)$, $\left(d_{bf}^{P}(a,d), d_{bf}^{N}(a,d)\right) = (0.2, -0.4)$, $\left(d_{bf}^{P}(a,e), d_{bf}^{N}(a,e)\right) = (0.3, -0.6)$, $\left(d_{bf}^{P}(a,f), d_{bf}^{N}(a,f)\right) = (0.2, -0.4)$, $\left(d_{bf}^{P}(b,c), d_{bf}^{N}(b,c)\right) = (0.2, -0.3)$ $\left(d_{bf}^{P}(b,d), d_{bf}^{N}(b,d)\right) = (0.3, -0.6)$, $\left(d_{bf}^{P}(b,e), d_{bf}^{N}(b,e)\right) = (0.3, -0.6)$, , $\left(d_{bf}^{P}(b,f), d_{bf}^{N}(b,f)\right) = (0.2, -0.4)$, $\left(d_{bf}^{P}(c,d), d_{bf}^{N}(c,d)\right) = (0.2, -0.3)$, $\left(d_{bf}^{P}(c,e), d_{bf}^{N}(c,e)\right) = (0.4, -0.6)$, $\left(d_{bf}^{P}(c,f), d_{bf}^{N}(c,f)\right) = (0.3, -0.6)$, $\left(d_{bf}^{P}(d,e), d_{bf}^{N}(d,e)\right) = (0.4, -0.8)$, $\left(d_{bf}^{P}(d,f), d_{bf}^{N}(d,f)\right) = (0.3, -0.6)$ and $\left(d_{bf}^{P}(e,f), d_{bf}^{N}(e,f)\right) = (0.3, -0.4)$.

Here, $\left(e_{bf}^{P}(a),\ e_{bf}^{N}(a)\right) = (0.3, -0.6),\ \left(e_{bf}^{P}(b),\ e_{bf}^{N}(b)\right) = (0.3, -0.6),$

$$\left(e_{bf}^{P}(c),\ e_{bf}^{N}(c)\right) = (0.4, -0.6), \left(e_{bf}^{P}(d),\ e_{bf}^{N}(d)\right) = (0.4, -0.8),$$

$$\left(e_{bf}^{P}(e),\ e_{bf}^{N}(e)\right) = (0.4, -0.8), \left(e_{bf}^{P}(f),\ e_{bf}^{N}(f)\right) = (0.3, -0.6)\ and$$

$$\left(r_{bf}^{P}(G),\ r_{bf}^{N}(G)\right) = (0.3, -0.6), \qquad \left(D_{bf}^{P}(a),\ D_{bf}^{N}(a)\right) = (0.4, -0.8).$$

Note that a,b and f are not fuzzy positive eccentric nodes of any other nodes and a,b,c and f are not fuzzy negative eccentric nodes of any other nodes.

Definition 2.10: The subgraph induced by the set of all bipolar fuzzy central nodes is called the centre of the bipolar fuzzy graph G and the subgraph induced by the set of all bipolar fuzzy diametral nodes is called the periphery of the bipolar fuzzy graph G.

Remark 2.1: The bipolar fuzzy center of a bipolar fuzzy graph need not be the same as the center of its underlying graph.

3. BIPOLAR FUZZY SELF CENTERED GRAPHS

In this section, we shall discuss the properties of self centered bipolar fuzzy graphs with respect to the new distance.

Definition 3.1: A bipolar fuzzy graph G is called bipolar fuzzy self centered, if it is isomorphic with its bipolar fuzzy center.

Example 3.1: A bipolar fuzzy graph and its underlying graph are shown below.

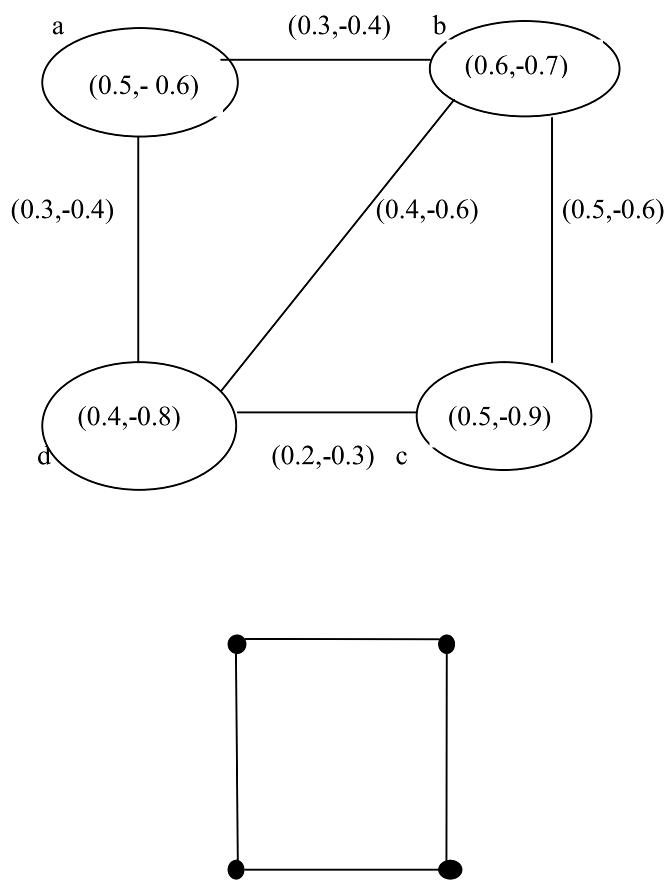

Figure 2. Bipolar fuzzy graph and its underlying graphs

$\left(d_{bf}^{P}(a,b), d_{bf}^{N}(a,b)\right) = (0.3, -0.4)$,$\left(d_{bf}^{P}(a,c), d_{bf}^{N}(a,c)\right) = (0.4, -0.6)$,

$\left(d_{bf}^{P}(a,d), d_{bf}^{N}(a,d)\right) = (0.3, -0.4)$, $\left(d_{bf}^{P}(b,c), d_{bf}^{N}(b,c)\right) = (0.4, -0.6)$

$\left(d_{bf}^{P}(b,d), d_{bf}^{N}(b,d)\right) = (0.4, -0.6)$, $\left(d_{bf}^{P}(c,d), d_{bf}^{N}(c,d)\right) = (0.2, -0.3)$and

$$\left(e_{bf}^{P}(a),\ e_{bf}^{N}(a)\right) = (0.4, -0.6), \qquad \left(e_{bf}^{P}(b),\ e_{bf}^{N}(b)\right) = (0.4, -0.6),$$
$$\left(e_{bf}^{P}(c),\ e_{bf}^{N}(c)\right) = (0.4, -0.6), \qquad \left(e_{bf}^{P}(d),\ e_{bf}^{N}(d)\right) = (0.4, -0.6)$$

Also note that d (a,b) =1, d (a,c) =2, d (a,d) =1, d (b,c) =1, d (b,d) =1, d (c,d) =1 and e (a) = 2, e (b) = 1, e (c) = 2, e (d) = 1.

This bipolar fuzzy graph is bipolar fuzzy self centered but the underlying graph is not self centered.

In the following theorem we present a necessary condition for a bipolar fuzzy grah G to be bipolar fuzzy self centered.

Theorem 3.1: If a connected bipolar fuzzy graph G is bipolar fuzzy self centered, then each node of G is bipolar fuzzy eccentric.

Proof.

Assume that the bipolar fuzzy graph G is biolar fuzzy self centered. We have to prove that each node of G is bipolar fuzzy eccentric. Let $(\mu_A^P(u), \mu_A^N(u))\ be$ any arbitrary nodes of G.

By the definition of a bipolar fuzzy eccentric node, $\left(e_{bf}^{P}(u),\ e_{bf}^{N}(u)\right) = \left(d_{bf}^{P}(u,v), d_{bf}^{N}(u,v)\right)$.

But since G is bipolar fuzzy self centered, $\left(e_{bf}^{P}(u),\ e_{bf}^{N}(u)\right) = \left(e_{bf}^{P}(v),\ e_{bf}^{N}(v)\right)$ and hence$\left(e_{bf}^{P}(v),\ e_{bf}^{N}(v)\right)$ $= \left(d_{bf}^{P}(u,v), d_{bf}^{N}(u,v)\right) == \left(d_{bf}^{P}(v,u), d_{bf}^{N}(v,u)\right)$.

Thus $(\mu_A^P(u), \mu_A^N(u))$ is a bipolar fuzzy eccentric node of $(\mu_A^P(v), \mu_A^N(v))$. Hence every node of G is bipolar fuzzy eccentric.

The next result is also a necessary condition for bipolar fuzzy self centered graphs.

Theorem 3.2: If a connected bipolar fuzzy graph G is bipolar fuzzyself centered then for every pair of nodes $\left(\mu_A^P(u), \mu_A^N(u)\right)$, $\left(\mu_A^P(v), \mu_A^N(v)\right)$ such that whenever $\left(\mu_A^P(u), \mu_A^N(u)\right)$ is a bipolar fuzzy eccentric node of $\left(\mu_A^P(v), \mu_A^N(v)\right)$ then $\left(\mu_A^P(v), \mu_A^N(v)\right)$ should be one of the bipolar fuzzy eccentric nodes of $\left(\mu_A^P(u), \mu_A^N(u)\right)$.

Proof.

Assume that G is bipolar fuzzy self centered also assume that $(\mu_A^P(u), \mu_A^N(u))$ is a bipolar fuzzy eccentric nodes of $(\mu_A^P(v), \mu_A^N(v))$. This means $\left(e_{bf}^{P}(v),\ e_{bf}^{N}(v)\right) = \left(d_{bf}^{P}(v,u), d_{bf}^{N}(v,u)\right)$ since G is bipolar fuzzy self centered, all nodes will be having the same bipolar fuzzy eccentricity. Therefore $\left(e_{bf}^{P}(v),\ e_{bf}^{N}(v)\right) = \left(e_{bf}^{P}(u),\ e_{bf}^{N}(u)\right)$

From the above 2 equations,

$$\left(e_{bf}^{P}(u),\ e_{bf}^{N}(u)\right) = \left(d_{bf}^{P}(v,u), d_{bf}^{N}(v,u)\right) = \left(d_{bf}^{P}(u,v), d_{bf}^{N}(u,v)\right)$$

That is $(\mu_A^P(v), \mu_A^N(v))$ is a bipolar fuzzy eccentric nodes of $(\mu_A^P(u), \mu_A^N(u))$.

This result is not sufficient, as we are not able to prove the bipolar fuzzy eccentricity of a third node is equal to the common bipolar fuzzy eccentricity of the selected pair of nodes.

4.CONCLUSION

Bipolar fuzzy graphs are precise models of all kinds of networks. In this paper,a genuine effort is made to generalize the concept of distance. The concept of bipolar fuzzy distance is relevant as it represents the net flow between a given pair of nodes of a bipolar fuzzy graph.In this paper the concept of bipolar fuzzy center and bipolar fuzzy self centered graphs are introduced. Facility location in a bipolar fuzzy network model can be made easy by using the concept of bipolar fuzzy center and bipolar fuzzy self centered graphs.

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