Flexural Analysis of Three lamina beam with sine load

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Abstract: The present research incorporates an solution for the flexural investigation of basically simply supported three layer laminated beams with sine load by using trigonometric shear deformation theory. These beam numerical are acquired by executing higher shear deformation theory having displacement field over the beam cross-section. Plan depends on arrangement of a beam represented by rule of virtual work strategy through the thickness of a laminate beam. Governing differential conditions and limit conditions for beam are gotten by utilizing the rule of virtual work. Articulations for transverse displacement of beams are acquired and commitment because of shear deformation. The numerical outcomes have been processed for different lengths to thickness proportions of the beam with MATLAB coding. The consequences of the present research are contrasted and those of other shear deformation beam theories to confirm the exactness of the result.

Keywords- Displacement field, uniform thickness, laminated beam, principle of virtual work

I. INTRODUCTION

Beams are adaptable and inventive development material utilized generally in business and private activities. A beam is an auxiliary component that essentially opposes loads connected along the side to the shaft's hub. Its method of avoidance is fundamentally by twisting. The heaps connected to the beam result in response powers at the beams help focuses. The aggregate impact of the considerable number of powers following up on the beam is to deliver shear powers and twisting minutes inside the shaft, that thusly instigate interior burdens, strains and redirections of the beam. beams are portrayed by their way of help, profile (state of cross-segment), length, and their material. Bars are customarily depictions of building auxiliary components, yet any structures, for example, car outlines, air ship segments, machine outlines, and other mechanical or basic frameworks contain shaft structures that are intended to convey parallel burdens are broke down in a comparable manner. Levinson presented new theory for beams of rectangular cross-section which includes warping of the cross-sections is presented in the present work. By satisfying the shear-free conditions on the lateral surfaces of the beam a pair of coupled equations of motion are obtained such that no arbitrary shear coefficient is required. It is shown that the uncoupled equation for the transverse displacement is the same as the corresponding equation in Timoshenko beam theory provided that for the Timoshenko equation the shear coefficient is taken to be 5/6; this value lies within the range of values, 0·822–0·870, appearing in the literature for the beam of rectangular cross-section.[2] Sayyad and Ghugal study the flexural analysis of single-layer fibrous composite beams using several displacement-based shear deformation theories. The theories accounts for the parabolic variation of shear stress through the thickness of a beam, so that the shear correction factor is not needed. The number of unknown variables in all the theories is the same as in the first-order shear deformation theory [11]. Karama M study the new higher order shear deformable laminated composite plate theory is proposed. It is constructed from 3-D elasticity bending solutions by using an inverse method. Present theory exactly satisfies stress boundary conditions on the top and the bottom of the plate [9].

II. METHODOLOGY

The beam as shown in Fig. 1 under consideration consists of three layers: layer 1, layer 2 and layer 3.

Layer 1 (0^0 layer) occupies the region:

$$0 \le x \le L;\ -b/2 \le y \le b/2;\ -h/2 \le z \le -h/6$$

Layer 2 (90^0 layer) occupies the region:

$$0 \le x \le L;\ -b/2 \le y \le b/2;\ -h/6 \le z \le h/6$$

Layer 3 (0^0 layer) occupies the region:

$$0 \le x \le L;\ -b/2 \le y \le b/2;\ h/6 \le z \le h/2$$

where x, y, z are Cartesian coordinates, L is the length, b is the width and h is the total depth of beam. The beam is subjected to transverse load of intensity q(x) per unit length of the beam. The beam (Fig.1) can have meaningful boundary conditions and loading conditions.

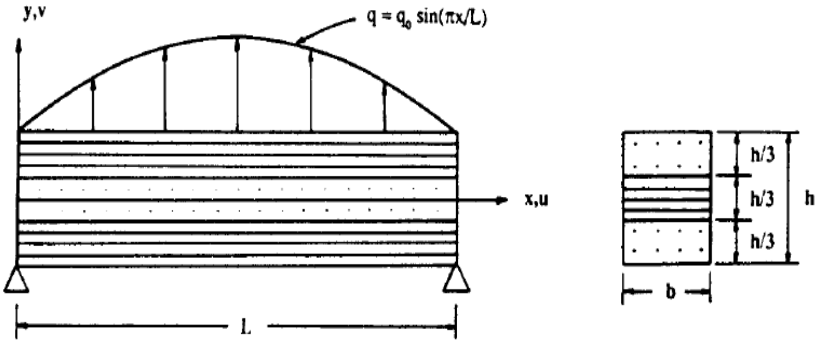

Fig.1 Three Lamina beam with sine load

Assumptions made in theoretical formulation

1. Axial displacement consists of
   (a) Displacement given by Elementary beam bending theory (ETB).
   (b) Displacements due to shear deformation, which is assumed to be sinusoidal in nature with respect to thickness, coordinate
2. The transverse displacement is assumed to be functions of longitudinal length coordinate of beam.
3. The displacements are small compared to beam thickness.
4. The layers are perfectly bonded to each other.
5. The stacking sequence of layers is such that there is no bending-twisting coupling.
6. The body forces are ignored in the analysis.
7. One dimensional constitutive law are assumed for each layer
8. The beam is subjected to lateral load only.

Displacement Field

The displacement field based on above assumptions of present layerwise trigonometric shear deformation theory is given as below

$$u^{(1)}(x,z) = -z\frac{dw}{dx} + \left[\frac{h}{\pi} + \sin\left(\frac{\pi z}{h}\right)\right]\phi(x)$$

$$u^{(2)}(x,z) = -z\frac{dw}{dx} + \left[\frac{h}{\pi} + \sin\left(\frac{\pi z}{h}\right)\right]\phi(x)$$

$$u^{(3)}(x,z) = -z\frac{dw}{dx} + \left[\frac{h}{\pi} + \sin\left(\frac{\pi z}{h}\right)\right]\phi(x)$$

$$w(x,z) = w(x)$$

Here $u^{(1)}$, $u^{(2)}$ and $u^{(3)}$ are the axial displacement components in the x direction, superscripts 1,2 and 3 refer to layer 1 ,layer 2 and layer 3. w(x) is the transverse displacement in the z direction. The function $\phi(x)$ is a rotation function or the warping function of the cross-section of the beam.

Example: A simply supported beam with three laminations (0^0 / 90^0 / 0^0) and subjected to a sinusoidal distributed load as shown in Fig. 1.

Beam is compared with three equal thickness unidirectionally reinforced laminations of graphite- epoxy and has a length to depth of four (L/h = 4) . The direction of graphite fibres is longitudinal in bottom and top layers (Layers 1 and 3 respectively) and normal to the longitudinal axis of the beam in middle layer (Layer 2). The numerical values of the properties are EL = E1 = 172 Gpa, ET = E2 = 6.9 Gpa, GLT = 3.4 Gpa, GTT = 1.4 Gpa, µLT = µTT = 0.25

$\overline{D}$, D_2, $\overline{D}_3$, D_1, Ω are constants appeared in governing equations.

Transverse Displacement $\overline{w}$

$$\overline{w} = \left[100\ \frac{E_1}{E_2}\ \frac{D_2}{D_3}\ \frac{1}{\overline{D}}\ \frac{1}{L^2\pi^2}\ \frac{1}{\Omega}\ \text{Sin}\ \frac{\pi x}{L}\right] + \left[100\ \frac{E_1}{E_2}\ \frac{1}{\overline{D}}\ \frac{1}{\pi^4}\ \frac{1}{\Omega}\ \text{Sin}\ \frac{\pi x}{L}\right]$$

Axial Displacement $\bar{u}$

$$\bar{u} = \left[-\frac{E_1}{E_2}\frac{1}{\pi}\frac{z}{h}\frac{D_2}{\bar{D}_3}\frac{1}{\bar{\bar{D}}}\frac{1}{h^2}\frac{L}{h}\frac{1}{\Omega}\text{Cos}\frac{\pi x}{L} \right] + \left[-\frac{E_1}{E_2}\frac{z}{h}\frac{1}{\bar{\bar{D}}}\frac{1}{\pi^3}\frac{L^3}{h^3}\frac{1}{\Omega}\text{Cos}\frac{\pi x}{L} \right] +$$

$$\left[\frac{E_1}{E_2}\frac{L}{h}\frac{1}{h^2}\frac{1}{\pi^2}\frac{1}{\bar{\bar{D}}}\frac{1}{\bar{D}_3}\frac{1}{\Omega}\text{Sin}\frac{\pi z}{h}\text{Cos}\frac{\pi x}{L} \right]$$

Axial stresses $\overline{\sigma_x}$

$$\overline{\sigma_x} = \left[\frac{1}{D_1}\frac{Z}{h}\frac{1}{\bar{\bar{D}}}\frac{L^2}{h^2}\frac{1}{\pi^2}\text{Sin}\frac{\pi x}{L} \right]$$

Transverse shear stresses $\tau_{zx}^{\overline{CR}}$ using constitutive relationship

$$\tau_{zx}^{\overline{CR}} = \left[\frac{2G_1}{G_1+G_2}\frac{L}{h}\frac{1}{\pi}\text{Cos}\frac{\pi x}{L} \right]$$

Transverse shear stresses $\tau_{zx}^{\overline{EE}}$ using equilibrium equations

$$\tau_{zx}^{\overline{EE}} = \left[\frac{1}{\bar{\bar{D}}}\frac{1}{8}\frac{L}{h}\frac{1}{\pi}\left(4\frac{z^2}{h^2}-1\right)\text{Cos}\frac{\pi x}{L} \right]$$

III. RESULTS

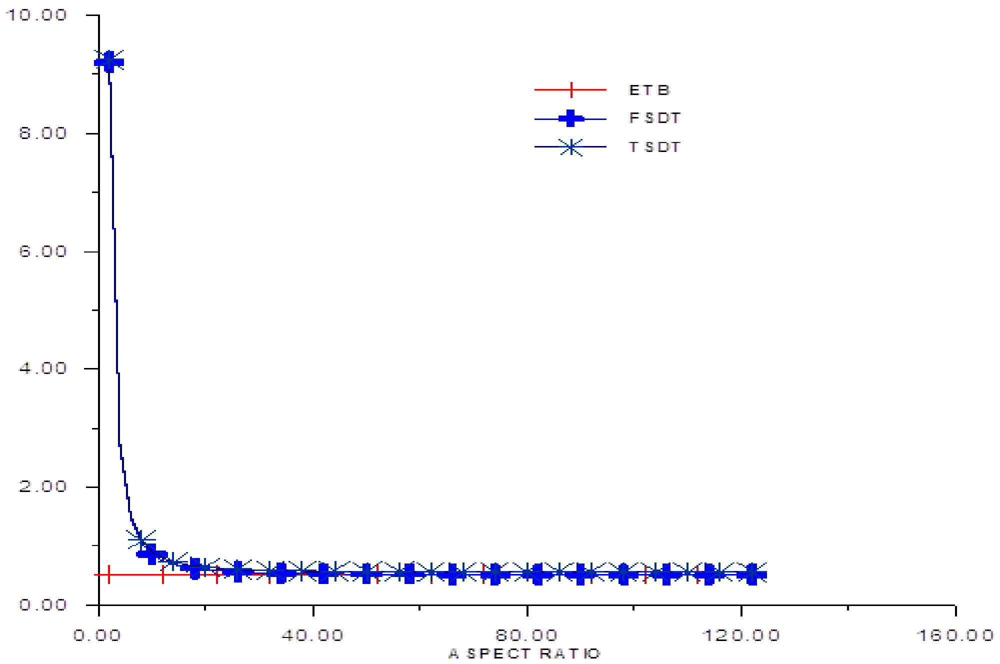

Fig: 2 Transverse Displacement $\bar{w}$

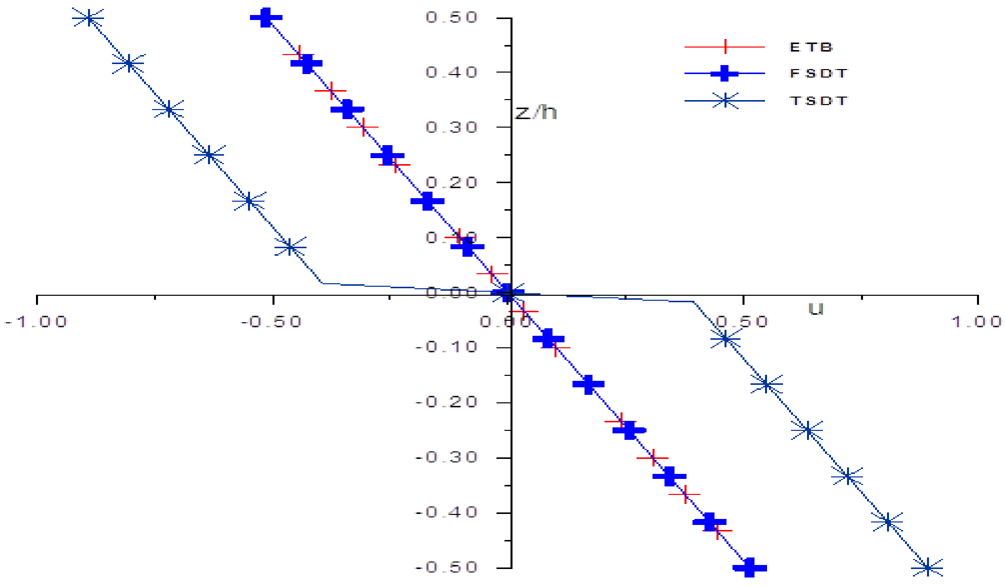

Fig: 3 Axial Displacement $\bar{u}$

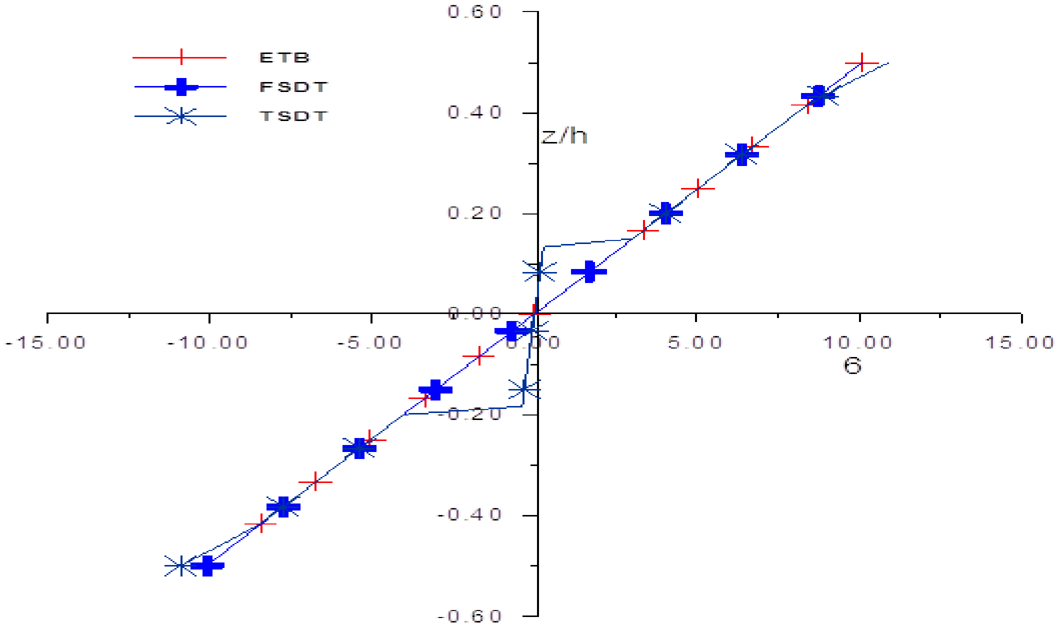

Fig: 4 Axial stresses $\overline{\sigma_x}$

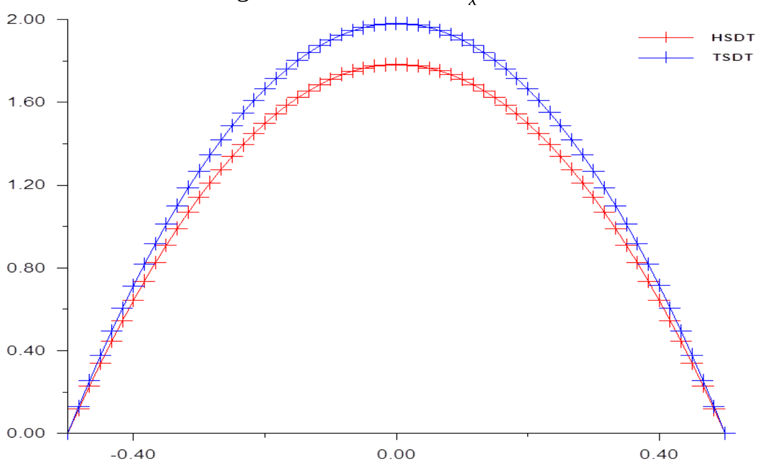

Fig: 5 Transverse shear stresses $\overline{\tau_{zx}^{EE}}$

IV.CONCLUSION

In this paper trigonometric beam theory has been developed and presented for laminated beam. Governing differential conditions and limit conditions for beam are gotten by utilizing the rule of virtual work. Bending problems of laminated beam to be solve by present theory. The results are compared with other shear deformation theory and present theory give accurate results for bending behavior of laminated beam.

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