

Interval-valued Intuitionistic Q - fuzzy k –ideals in Ternary Semiring

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Abstract: In this paper, the concepts of interval-valued intuitionistic Q -fuzzy k -ideal in ternary semiring are studied. Also observe some important properties. Theorems related to the above concepts are stated and proved.

1 INTRODUCTION

In 1932, Lehmer [16] introduced the notion of ternary algebraic system. The concept of ternary semirings was introduced by Dutta and Kar [6-13] which is generalization of ternary rings and semirings, and they studied some properties of ternary semirings. The concept of fuzzy sets was introduced by Zadeh [19] in 1965. Many papers on fuzzy sets appeared and applied to logic, set theory, group theory, ring theory, real analysis, topology, measure theory, etc. Interval-valued fuzzy sets were introduced independently by Zadeh [20] which is a generalization of fuzzy set. An interval-valued fuzzy set is a fuzzy set whose membership function is many-valued and forms an interval in the membership scale. On the other hand, Atanassov [1] introduced the notion of intuitionistic fuzzy sets as an extension of fuzzy set in which not only a membership degree is given, but also a non-membership degree is involved. Atanassov and Gargov [2] introduced the notion of interval-valued intuitionistic fuzzy sets which is a common generalization of intuitionistic fuzzy sets and interval-valued fuzzy sets. Dutta et al. [13] introduced the notion of interval-valued fuzzy prime ideal of a semiring. Kar et al. [15] introduced the notion of interval-valued prime fuzzy ideal of semigroups. Osman Kazanci, Sultan yamark and Serife yilmaz in [18] have introduced the notion of intuitionistic Q -fuzzification of N -subgroups (subnear-rings) in a near-ring and investigated some related properties. S.Lekkoksung [16] studied intuitionistic Q -fuzzy K -ideals of semirings. Balasubramanian and Raja [3,4] introduced intuitionistic fuzzy k -ideal and interval-valued intuitionistic fuzzy ideal on semi-rings. In this paper, the notion of interval-valued intuitionistic Q -fuzzy k -ideals on semiring is introduced and analyse some interesting properties.

2 PRELIMINARIES

Definition 2.1

A non-empty set S together with two binary operation '+' and '.' is said to be a semiring, if

- i) $(S, +)$ is a commutative semigroup,
- ii) $(S, .)$ is a semigroup,
- iii) $a(b + c) = ab + ac$ and $(a + b)c = ac + bc \forall a, b, c \in S$.

Let $(S, +, .)$ be a semiring. If there exists an element $0_S \in S$ such that $a + 0_S = a = 0_S + a$ and $a \cdot 0_S = 0_S = 0_S \cdot a$ for all $a \in S$; then 0_S is called the zero element of S . If there exists an element $1_S \in S$ such that $a \cdot 1_S = a = 1_S \cdot a$ for all $a \in S$, then 1_S is called the identity element of S .

Note 2: A semiring may or may not have a zero and an identity element.

We say that a semiring S has a zero if there exists an element $0 \in S$ such that $0x = x0 = 0$ and $0 + x = x + 0 = x$ for all $x \in S$.

Definition 2.2

An interval number on $[0,1]$, denoted by \hat{A} , is defined as the closed subinterval of $[0,1]$, where $\hat{A} = [A^-, A^+]$ satisfying $0 \leq A^- \leq A^+ \leq 1$.

For any two interval numbers $\hat{A} = [A^-, A^+]$ and $\hat{B} = [B^-, B^+]$, we define:

- i) $\hat{A} \leq \hat{B}$ if and only if $A^- \leq B^-$ and $A^+ \leq B^+$.
- ii) $\hat{A} = \hat{B}$ if and only if $A^- = B^-$ and $A^+ = B^+$
- iii) $\hat{A} < \hat{B}$ if and only if $\hat{A} \neq \hat{B}$ and $\hat{A} \leq \hat{B}$

Definition 2.3

Let $X \neq \phi$ be a set and $A \subseteq X$. Then the interval-valued intuitionistic characteristic function $\chi_A = (\tilde{\chi}_{M_A}(x), \tilde{\chi}_{N_A}(x))$ of A is an interval-valued intuitionistic fuzzy subset of X , defined as follows:

$$\hat{\chi}_{M_A}(x) = \begin{cases} \hat{1} & \text{when } x \in A \\ \hat{0} & \text{when } x \notin A \end{cases} \quad \text{and} \quad \hat{\chi}_{N_A}(x) = \begin{cases} \hat{0} & \text{when } x \in A \\ \hat{1} & \text{when } x \notin A \end{cases}$$

Definition 2.4

Let $A = (\hat{M}_A, \hat{N}_A)$ and $B = (\hat{M}_B, \hat{N}_B)$ be two interval-valued intuitionistic fuzzy subsets of a non-empty set X . Then A is said to be subset of B , denoted by $A \subseteq B$, if $\hat{M}_A(x) \leq \hat{M}_B(x)$ and $\hat{N}_A(x) \geq \hat{N}_B(x)$. (i.e) $M_A^-(x) \leq M_B^-(x)$; $M_A^+(x) \leq M_B^+(x)$; $N_A^-(x) \geq N_B^-(x)$; $N_A^+(x) \geq N_B^+(x)$ for all $x \in X$.

Definition 2.5

Let $A = (\hat{M}_A, \hat{N}_A)$ be an interval-valued intuitionistic fuzzy subsets of a non-empty set X and $[\alpha, \beta] \in D[0,1]$. Then the level subset of $A = (\hat{M}_A, \hat{N}_A)$, denoted by $\bar{U}(\hat{M}_A, \hat{N}_A, [\alpha, \beta])$ is defined as: $\bar{U}(\hat{M}_A, \hat{N}_A, [\alpha, \beta]) = \{(x, y) : \hat{M}_A(x) \geq [\alpha, \beta], \hat{N}_A(x) \leq [\alpha, \beta]\}$

Theorem 2.1

If $[\alpha_1, \beta_1]$ and $[\alpha_2, \beta_2]$ be two interval-valued intuitionistic fuzzy number such that $[\alpha_1, \beta_1] > [\alpha_2, \beta_2]$, then $\bar{U}(\hat{M}, \hat{N}, [\alpha_1, \beta_1]) \subseteq \bar{U}(\hat{M}, \hat{N}, [\alpha_2, \beta_2])$.

Definition 2.6

The interval *Min - norm* is a function $Min^i: D[0,1] \times D[0,1] \rightarrow D[0,1]$ defined by $Min^i(\hat{A}, \hat{B}) = [\min(A^-, B^-), \min(A^+, B^+)]$ for all $\hat{A}, \hat{B} \in D[0,1]$, where $\hat{A} = [A^-, A^+]$ and $\hat{B} = [B^-, B^+]$.

Definition 2.7

The interval *Max - norm* is a function $Max^i: D[0,1] \times D[0,1] \rightarrow D[0,1]$ defined by $Max^i(\hat{A}, \hat{B}) = [\max(A^-, B^-), \max(A^+, B^+)]$ for all $\hat{A}, \hat{B} \in D[0,1]$, where $\hat{A} = [A^-, A^+]$ and $\hat{B} = [B^-, B^+]$.

Definition 2.8

A non-empty interval-valued intuitionistic fuzzy subset A of a semiring S is said to be interval-valued intuitionistic fuzzy ideal of S if

1. $M_A(x + y) \geq \min^i\{M_A(x), M_A(y)\}$
2. $N_A(x + y) \leq \max^i\{N_A(x), N_A(y)\}$
3. $M_A(xy) \geq \max^i\{M_A(x), M_A(y)\}$
4. $N_A(xy) \leq \min^i\{N_A(x), N_A(y)\}$ for all $x, y \in S$.

Definition 2.9

An interval-valued intuitionistic Q -fuzzy subset $A = (\tilde{M}_A, \tilde{N}_A)$ in S is called an interval-valued intuitionistic Q -fuzzy right ideal of S if

1. $\tilde{M}_A(x + y, q) \geq \min^i\{\tilde{M}_A(x, q), \tilde{M}_A(y, q)\}$
2. $\tilde{M}_A(xyz, q) \geq \tilde{M}_A(x, q)$
3. $\tilde{N}_A(x + y, q) \leq \max^i\{\tilde{N}_A(x, q), \tilde{N}_A(y, q)\}$
4. $\tilde{N}_A(xyz, q) \leq \tilde{N}_A(x, q)$ for all $x, y, z \in S, q \in Q$.

Definition 2.10

An interval-valued intuitionistic Q -fuzzy subset $A = (\tilde{M}_A, \tilde{N}_A)$ is called an intuitionistic Q -fuzzy ideal of S if it is interval-valued intuitionistic Q -fuzzy right, interval-valued intuitionistic Q -fuzzy lateral and interval-valued intuitionistic Q -fuzzy left ideal of S .

Definition 2.11

Let $A = (\tilde{M}_A, \tilde{N}_A)$ be an interval-valued intuitionistic Q -fuzzy subset of S and let $\tilde{s}, \tilde{t} \in D[0,1]$. Then the set $S_A^{(s,t)} = \{x \in S / \tilde{M}_A(x, q) \geq \tilde{s}, \tilde{M}_A(x, q) \leq \tilde{t}, q \in Q\}$ is called a (s, t) -level set of A . The set $\{(\tilde{s}, \tilde{t}) \in Im(\tilde{M}_A) \times Im(\tilde{N}_A) = \tilde{s} + \tilde{t} \leq \tilde{1}\}$ is called image of $A = (\tilde{M}_A, \tilde{N}_A)$. Clearly $S_A^{(s,t)} = U(\tilde{M}_A, \tilde{s}) \cap L(\tilde{N}_A, \tilde{t})$, where $U(\tilde{M}_A, \tilde{s})$ and $L(\tilde{N}_A, \tilde{t})$ are upper and lower level subsets of \tilde{M}_A and \tilde{N}_A respectively.

Theorem 2.2 [5]

If an interval-valued Q -fuzzy subset \tilde{M} is a interval-valued Q -fuzzy right (left, lateral) ideal of a ternary semi-ring S if and only if \tilde{M}^c is an interval-valued anti Q -fuzzy right (left, lateral) ideal of S .

Theorem 2.3 [5]

An interval-valued intuitionistic Q -fuzzy set $A = (\tilde{M}_A, \tilde{N}_A)$ in S is an intuitionistic Q -fuzzy right (left, lateral) ideal of S if and only if any level set $S_A^{(s,t)}$ is a right (left, lateral) ideal of S for $\tilde{s}, \tilde{t} \in D[0,1]$ whenever non-empty.

Theorem 2.4 [5]

An interval-valued intuitionistic Q -fuzzy set $A = (\tilde{M}_A, \tilde{N}_A)$ in S is an interval-valued intuitionistic Q -fuzzy right (left, lateral) ideal of S if and only if the interval-valued Q -fuzzy subsets \tilde{M}_A and \tilde{N}_A^c are interval-valued Q -fuzzy right (left, lateral) ideals of S .

3 Interval-valued Intuitionistic Q-fuzzy k-ideal of Ternary Semirings

Definition 3.1. An interval-valued Q -fuzzy right (left, lateral) ideal $\tilde{M} = [M^-, M^+]$ of a ternary semiring S is said to be an interval-valued Q -fuzzy right (left, lateral) k -ideal of S if for all $x, y \in S, q \in Q$,

$$M^-(x, q) \geq \min \{M^-(x + y, q), M^-(y, q)\}.$$

$$M^+(x, q) \geq \min \{M^+(x + y, q), M^+(y, q)\},$$

Definition 3.2. An interval-valued anti Q -fuzzy right (left, lateral) ideal $\tilde{M} = [M^-, M^+]$ of a ternary semiring S is said to be an interval-valued anti Q -fuzzy right (left, lateral) k -ideal of S if for all $x, y \in S, q \in Q$,

$$M^+(x, q) \leq \max \{M^+(x + y, q), M^+(y, q)\},$$

$$M^-(x, q) \leq \max \{M^-(x + y, q), M^-(y, q)\}.$$

Definition 3.3. An interval-valued intuitionistic Q -fuzzy right (left, lateral) ideal $A = (\tilde{M}_A, \tilde{N}_A)$ in S is called an interval-valued intuitionistic Q -fuzzy right (left, lateral) k -ideal of S if

1. $\tilde{M}_A(x, q) \geq \min \{\tilde{M}_A(x + y, q), \tilde{M}_A(u, q)\}$
2. $\tilde{N}_A(x, q) \leq \max \{\tilde{N}_A(x + y, q), \tilde{N}_A(u, q)\}$ for all $x, y \in S, q \in Q$.

Theorem 3.6. If an interval-valued Q -fuzzy subset \tilde{M} is an interval-valued Q -fuzzy right (left, lateral) k -ideal of a ternary semiring S if and only if \tilde{M}^c is an interval-valued anti Q -fuzzy right (left, lateral) k -ideal of S .

Proof. Let \tilde{M} be an interval-valued Q -fuzzy right k -ideal of a ternary semiring S . By Theorem 2.2,

$$\tilde{M}^c \text{ is an interval-valued anti } Q\text{-fuzzy right ideal of } S.$$

Let $x, y \in S, q \in Q$.

$$\text{Then } \tilde{M}(x, q) \geq \min^i \{\tilde{M}(x + y, q), \tilde{M}(y, q)\}$$

$$\Rightarrow -\tilde{M}(x, q) \leq -\min^i \{\tilde{M}(x + y, q), \tilde{M}(y, q)\}$$

$$\Rightarrow 1 - \tilde{M}(x, q) \leq 1 - \min^i \{\tilde{M}(x + y, q), \tilde{M}(y, q)\}$$

$$\Rightarrow 1 - \tilde{M}(x, q) \leq \max \{1 - \tilde{M}(x + y, q), 1 - \tilde{M}(y, q)\}$$

$$\Rightarrow \tilde{M}^c(x, q) \leq \max^i \{\tilde{M}^c(x + y, q), \tilde{M}^c(y, q)\}$$

Therefore \tilde{M}^c is an interval-valued anti Q -fuzzy right ideal of S .

Similarly, we can prove the converse.

Theorem 3.7 An interval-valued intuitionistic Q -fuzzy set A in S is an interval-valued intuitionistic Q -fuzzy right (left, lateral) k -ideal of S if and only if any level set $S_A^{(s,t)}$ is a right (left, lateral) k -ideal of S for $s, t \in [0,1]$ whenever nonempty.

Proof:

Let A be an interval-valued intuitionistic Q -fuzzy right k -ideal of S .

Let $S_A^{(s,t)}$ is a right ideal of S .

If there exists $x, y \in S, q \in Q$ such that $x + y, y \in S_A^{(s,t)}$ and $x \notin S_A^{(s,t)}$, then

$$\min^i \{\tilde{M}_A(x + y, q), \tilde{M}_A(u, q)\} \geq s > \tilde{M}_A(x, q) \text{ and}$$

$$\max^i \{\tilde{N}_A(x + y, q), \tilde{N}_A(u, q)\} \geq t > \tilde{N}_A(x, q), \text{ which is a contradiction.}$$

Therefore $S_A^{(s,t)}$ is a right k -ideal in S .

Conversely,

if there exists $x, y \in S, q \in Q$ such that $\tilde{M}_A(x, q) < s = \min \{ \tilde{M}_A(x + y, q), \tilde{M}_A(y, q) \}$ and $\tilde{N}_A(x, q) > t = \max \{ \tilde{N}_A(x + y, q), \tilde{N}_A(y, q) \}$ then $x + y, y \in S_A^{(s,t)}$ and $x \notin S_A^{(s,t)}$ which is a contradiction.

Therefore by Theorem 2.3,

$A = (\tilde{M}_A, \tilde{N}_A)$ is an interval-valued intuitionistic Q -fuzzy right k -ideal of S .

Corollary 3.1

An interval-valued intuitionistic fuzzy set $A = (\tilde{M}_A, \tilde{N}_A)$ in S is an interval-valued intuitionistic Q -fuzzy right (left, lateral) k -ideal of S if and only if for every $\tilde{s}, \tilde{t} \in D[0; 1]$ such that $\tilde{s} + \tilde{t} \leq 1$ all non-empty $U(\tilde{M}_A, \tilde{s})$ and $L(\tilde{N}_A, \tilde{t})$ are right (left, lateral) k -ideals of S .

Theorem 3.8. An interval-valued intuitionistic fuzzy set $A = (\tilde{M}_A, \tilde{N}_A)$ in S is an interval-valued intuitionistic Q -fuzzy right (left, lateral) k -ideal of S if and only if the interval-valued Q -fuzzy subsets \tilde{M}_A and \tilde{N}_A^c are interval-valued Q -fuzzy right (left, lateral) k -ideals of S .

Proof. If $A = (\tilde{M}_A, \tilde{N}_A)$ is an interval-valued intuitionistic Q -fuzzy right k -ideal of S , then clearly \tilde{M}_A is an interval-valued Q -fuzzy right k -ideal of S .

For all $x, y \in S, q \in Q$,

$$\begin{aligned} \tilde{N}_A^c(x, q) &= 1 - g_A(x, q) \geq 1 - \max^i \{ \tilde{N}_A(x + y, q), \tilde{N}_A(y, q) \} \\ &= \min^i \{ 1 - \tilde{N}_A(x + y, q), 1 - \tilde{N}_A(y, q) \} \\ &= \min \{ \tilde{N}_A^c(x + y, q), \tilde{N}_A^c(y, q) \}. \end{aligned}$$

Thus \tilde{N}_A^c is an interval-valued intuitionistic Q -fuzzy right k -ideal of S .

Conversely assume that \tilde{M}_A and \tilde{N}_A^c are interval-valued Q -fuzzy right k -ideals of S , then by Theorem 2.4,

$A = (\tilde{M}_A, \tilde{N}_A)$ is an interval-valued intuitionistic Q -fuzzy right ideal of S . Now for all $x, y \in S, q \in Q$,

$$\begin{aligned} 1 - \tilde{N}_A(x, q) &= \tilde{N}_A^c(x, q) \geq \min \{ \tilde{N}_A^c(x + y, q), \tilde{N}_A^c(y, q) \} = \min \{ 1 - \tilde{N}_A(x + y, q), 1 - \tilde{N}_A(y, q) \} \\ &= 1 - \max \{ \tilde{N}_A(x + y, q), \tilde{N}_A(y, q) \} \end{aligned}$$

which implies $-\tilde{N}_A(x, q) \geq -\max \{ \tilde{N}_A(x + y, q), \tilde{N}_A(y, q) \}$ implies

$$\tilde{N}_A(x, q) \leq \max \{ \tilde{N}_A(x + y, q), \tilde{N}_A(y, q) \}.$$

Therefore $A = (\tilde{M}_A, \tilde{N}_A)$ is an interval-valued intuitionistic Q -fuzzy right k -ideal of S .

Corollary 3.2. Let $A = (\tilde{M}_A, \tilde{N}_A)$ be an interval-valued intuitionistic fuzzy set in S . Then an interval-valued intuitionistic Q -fuzzy set $A = (\tilde{M}_A, \tilde{N}_A)$ is an interval-valued intuitionistic Q -fuzzy right (left, lateral) k -ideal of S if and only if interval-valued intuitionistic Q -fuzzy set $A_1 = (\tilde{M}_A, \tilde{M}_A^c)$ and IQFS $A_2 = (\tilde{N}_A^c, \tilde{N}_A)$ are interval-valued intuitionistic Q -fuzzy right (left, lateral) k -ideals of S .

Proof: It is straightforward by Theorem 3.6 and Theorem 3.8

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