Interval-valued Intuitionistic Q- fuzzy k −ideals in Ternary Semiring

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Abstract: In this paper, the concepts of interval-valued intuitionistic Q-fuzzy k-ideal in ternary semiring are studied. Also observe some important properties. Theorems related to the above concepts are stated and proved.

1 INTRODUCTION

In 1932, Lehmer [16] introduced the notion of ternary algebraic system. The concept of ternary semirings was introduced by Dutta and Kar [6-13] which is generalization of ternary rings and semirings, and they studied some properties of ternary semirings. The concept of fuzzy sets was introduced by Zadeh[19] in 1965. Many papers on fuzzy sets appeared and applied to logic, set theory, group theory, ring theory, real analysis, topology, measure theory, etc. Interval-valued fuzzy sets were introduced independently by Zadeh [20] which is a generalization of fuzzy set. An interval-valued fuzzy set is a fuzzy set whose membership function is many-valued and forms an interval in the membership scale. On the other hand, Atanassov [1] introduced the notion of intuitionistic fuzzy sets as an extension of fuzzy set in which not only a membership degree is given, but also a non-membership degree is involved. Atanassov and Gargov [2] introduced the notion of interval-valued intuionistic fuzzy sets which is a common generalization of intuitionistic fuzzy sets and interval-valued fuzzy sets. Dutta et al. [13] introduced the notion of interval-valued fuzzy prime ideal of a semiring. Kar et al. [15] introduced the notion of interval-valued prime fuzzy ideal of semigroups.Osman Kazanci, Sultan yamark and Serife yilmaz in [18] have introduced the notion of intuionistic Q-fuzzication of N-subgroups (subnear-rings) in a near-ring and investigated some related properties. S.Lekkoksung [16] studied intuitionistic Q-fuzzy K-ideals of semirings. Balasubramanian and Raja [3,4] introduced intuitionistic fuzzy k-ideal and interval-valued intuitionistic fuzzy ideal on semi-rings. In this paper, the notion of interval-valued intuitionistic Q-fuzzy k-ideals on semiring is introduced and analyse some interesting properties.

2 PRELIMINARIES

Definition 2.1

A non-empty set S together with two binary operation '+' and '.' is said to be a semiring. if

i) $(S,+)$ is a commutative semigroup,

ii) $(S,.)$ is a semigroup,

iii) $a(b+c) = ab + ac$ and
$(a+b)c = ac + bc \; \forall a,b,c \in S$.

Let $(S,+,.)$ be a semiring. If there exists an element $0_s \in S$ such that $a + 0_s = a = 0_s + a$ and $a.0_s = 0_s = 0_s.a$ for all $a \in S$; then 0_s is called the zero element of S. If there exists an element $1_s \in S$ such that $a.1_s = a = 1_s.a$ for all $a \in S$, then $1_s$is called the identity element of S.

Note 2: A semiring may or may not have a zero and an identity element.
We say that a semiring$S$has a zero.if there exists an element $0 \in S$ such that $0x = x0 = 0$ and $0 + x = x + 0 = x$ for all $x \in S$.

Definition 2.2

An interval number on [0,1], denoted by $\hat{A}$, is defined as the closed subinterval of [0,1], where $\hat{A} = [A^-, A^+]$ satisfying $0 \leq A^- \leq A^+ \leq 1$.

For any two interval numbers $\hat{A} = [A^-, A^+]$ and $\hat{B} = [B^-, B^+]$, we define:

i) $\hat{A} \leq \hat{B}$ if and only if $A^- \leq B^-$ and $A^+ \leq B^+$.

ii) $\hat{A} = \hat{B}$ if and only if $A^- = B^-$ and $A^+ = B^+$

iii) $\hat{A} < \hat{B}$ if and only if $\hat{A} \neq \hat{B}$ and $\hat{A} \leq \hat{B}$

Definition 2.3

Let $X \neq \phi$ be a set and $A \subseteq X$. Then the interval-valued intuitionistic characteristic function $\chi_A = \left(\tilde{\chi}_{M_A}(x), \tilde{\chi}_{N_A}(x)\right)$ of A is an interval-valued intuitionistic fuzzy subset of X, defined as follows:

$$\hat{\chi}_{M_A}(x) = \begin{cases} \hat{1} when x \in A \\ \hat{0} when x \notin A \end{cases} \text{ and } \hat{\chi}_{N_A}(x) = \begin{cases} \hat{0} when x \in A \\ \hat{1} when x \notin A \end{cases}$$

Definition 2.4

Let $A = (\widehat{M}_A, \widehat{N}_A)$ and $B = (\widehat{M}_B, \widehat{N}_B)$ be two interval-valued intuitionistic fuzzy subsets of a non-empty set X. Then A is said to be subset of B, denoted by $A \subseteq B$, if $\widehat{M}_A(x) \leq \widehat{M}_B(x)$ and $\widehat{N}_A(x) \geq \widehat{N}_B(x)$. (i.e) $M_A^-(x) \leq M_B^-(x)$; $M_A^+(x) \leq M_B^+(x)$; $N_A^-(x) \geq N_B^-(x)$; $N_A^+(x) \geq N_B^+(x)$ for all $x \in X$.

Definition 2.5

Let $A = (\widehat{M}_A, \widehat{N}_A)$ be an interval-valued intuitionistic fuzzy subsets of a non-empty set X and $[\alpha, \beta] \in D[0,1]$. Then the level subset of $A = (\widehat{M}_A, \widehat{N}_A)$, denoted by $\overline{U}\left(\widehat{M}_A, \widehat{N}_A, [\alpha, \beta]\right)$ is defined as: $\overline{U}\left(\widehat{M}_A, \widehat{N}_A, [\alpha, \beta]\right) = \left\{(x, y) \ : \ \widehat{M}_A(x) \geq [\alpha, \beta], \ \widehat{N}_A(x) \leq [\alpha, \beta]\right\}$

Theorem 2.1

If $[\alpha_1, \beta_1]$ and $[\alpha_2, \beta_2]$ be two interval-valued intuitionistic fuzzy number such that $[\alpha_1, \beta_1] > [\alpha_2, \beta_2]$, then $\overline{U}\left(\widehat{M}, \widehat{N}, [\alpha_1, \beta_1]\right) \subseteq \overline{U}\left(\widehat{M}, \widehat{N}, [\alpha_2, \beta_2]\right)$.

Definition 2.6

The interval $Min - norm$ is a function $Min^i : D[0,1] \times D[0,1] \rightarrow D[0,1]$ defined by $Min^i\left(\hat{A}, \hat{B}\right) = [\min(A^-, B^-), \min(A^+, B^+)]$ for all $\hat{A}, \hat{B} \in D[0,1]$, where $\hat{A} = [A^-, A^+]$ and $\hat{b} = [B^-, B^+]$.

Definition 2.7

The interval $Max - norm$ is a function $Max^i : D[0,1] \times D[0,1] \rightarrow D[0,1]$ defined by $Max^i\left(\hat{A}, \hat{B}\right) = [\max(A^-, B^-), \max(A^+, B^+)]$ for all $\hat{A}, \hat{B} \in D[0,1]$, where $\hat{A} = [A^-, A^+]$ and $\hat{B} = [B^-, B^+]$.

Definition 2.8

A non-empty interval-valued intuitionistic fuzzy subset A of a semiring S is said to be interval-valued intuitionistic fuzzy ideal of S if

1. $M_A(x+y) \geq min^i\{M_A(x), M_A(y)\}$
2. $N_A(x+y) \leq max^i\{N_A(x), N_A(y)\}$
3. $M_A(xy) \geq max^i\{M_A(x), M_A(y)\}$
   $N_A(xy) \leq min^i\{N_A(x), N_A(y)\}$ for all $x, y \in S$.

Definition 2.9

An interval-valued intuitionistic Q-fuzzy subset $A = (\widetilde{M}_A, \widetilde{N}_A)$ in S is called an interval-valued intuitionistic Q-fuzzy right ideal of S if

1. $\widetilde{M}_A(x + y, q) \geq min^i\{\widetilde{M}_A(x, q), \widetilde{M}_A(y, q)\}$
2. $\widetilde{M}_A(xyz, q) \geq \widetilde{M}_A(x, q)$
3. $\widetilde{N}_A(x + y, q) \leq max^i\{\widetilde{N}_A(x, q), \widetilde{N}_A(y, q)\}$
4. $\widetilde{N}_A(xyz, q) \leq \widetilde{N}_A(x, q)$ for all $x, y, z \in S, q \in Q$.

Definition 2.10

An interval-valued intuitionistic Q-fuzzy subset $A = (\widetilde{M}_A, \widetilde{N}_A)$ is called an intuitionistic Q-fuzzy ideal of S if it is interval-valued intuitionistic Q-fuzzy right, interval-valued intuitionistic Q-fuzzy lateral and interval-valued intuitionistic Q-fuzzy left ideal of S.

Definition 2.11

Let $A = (\widetilde{M}_A, \widetilde{N}_A)$ be an interval-valued intuitionistic Q-fuzzy subset of S and let $\tilde{s}, \tilde{t} \in D[0,1]$. Then the set $S_A^{(s,t)} = \{x \in S/\ \widetilde{M}_A(x, q) \geq \tilde{s}, \widetilde{M}_A(x, q) \leq \tilde{t}, q \in Q\}$ is called a (s, t)-level set of A. The set $\{(\tilde{s}, \tilde{t}) \in Im(\widetilde{M}_A) \times Im(\widetilde{N}_A) = \tilde{s} + \tilde{t} \leq \tilde{1}\}$ is called image of $A = (\widetilde{M}_A, \widetilde{N}_A)$. Clearly $S_A^{(s,t)} = U(\widetilde{M}_A, \tilde{s}) \cap L(\widetilde{N}_A, \tilde{t})$, where $U(\widetilde{M}\_A, \tilde{s})$ and $L(\widetilde{N}_A, \tilde{t})$ are upper and lower level subsets of $\widetilde{M}_A$ and $\widetilde{N}_A$ respectively.

Theorem 2.2 [5]

If an interval-valued Q-fuzzy subset $\widetilde{M}$ is a interval-valued Q-fuzzy right (left, lateral) ideal of a ternary semi-ring S if and only if $\widetilde{M}^c$ is an interval-valued anti Q-fuzzy right (left, lateral) ideal of S.

Theorem 2.3 [5]

An interval-valued intuitionistic Q-fuzzy set $A = (\widetilde{M}_A, \widetilde{N}_A)$ in S is an intuitionistic Q-fuzzy right (left, lateral) ideal of S if and only if any level set $S_A^{(s,t)}$ is a right (left, lateral) ideal of S for $\tilde{s}, \tilde{t} \in$ D[0,1] whenever non-empty.

Theorem 2.4 [5]

An interval-valued intuitionistic Q-fuzzy set $A = (\widetilde{M}_A, \widetilde{N}_A)$ in S is an interval-valued intuitionistic Q-fuzzy right (left, lateral) ideal of S if and only if the interval-valued Q-fuzzy subsets $\widetilde{M}_A$ and $\widetilde{N}_A^c$ are interval-valued Q-fuzzy right (left, lateral) ideals of S.

3 Interval-valued Intuitionistic Q-fuzzy k-ideal of Ternary Semirings

Definition 3.1. A interval-valued Q −fuzzy right (left, lateral) ideal $\widetilde{M} = [M^-, M^+]$ of a ternary semiring S is said to be a interval-valued Q −fuzzy right (left, lateral) k −ideal of S if for all $x, y \in S,\ q \in Q,$
$M^-(x,q) \geq \min\{M^-(x + y, q), M^-(y,q)\}$.
$M^+(x,q) \geq \min\{M^+(x + y, q), M^+(y,q)\}$,

Definition 3.2. An interval-valued anti Q −fuzzy right (left, lateral) ideal $\widetilde{M} = [M^-, M^+]$ of a ternary semiring S is said to be an interval-valued anti Q −fuzzy right (left, lateral) k −ideal of S if for all $x, y \in S, q \in Q,$
$M^+(x,q) \leq max\{M^+(x + y, q), M^+(y,q)\}$,
$M^-(x,q) \leq max\{M^-(x + y, q), M^-(y,q)\}$.

Definition 3.3. An interval-valued intuitionistic Q −fuzzy right (left, lateral) ideal $A = (\widetilde{M}_A, \widetilde{N}_A)$ in S is called an interval-valued intuitionistic Q −fuzzy right (left, lateral) k −ideal of S if

1. $\widetilde{M}_A(x,q) \geq \min\{\widetilde{M}_A(x + y, q), \widetilde{M}_A(u,q)\}$
2. $\widetilde{N}_A(x,q) \leq max\{\widetilde{N}_A(x + y, q), \widetilde{N}_A(u,q)\}$ for all $x, y \in S, q \in Q$.

Theorem 3.6. If an interval-valued Q-fuzzy subset $\widetilde{M}$ is an interval-valued Q −fuzzy right (left, lateral) k-ideal of a ternary semiring S if and only if $\widetilde{M}^c$ is an interval-valued anti Q-fuzzy right (left, lateral) k −ideal of S.

Proof. Let $\widetilde{M}$ be an interval-valued Q −fuzzy right k −ideal of a ternary semiring S. By Theorem 2.2, $\widetilde{M}^c$ is an interval-valued anti Q-fuzzy right ideal of S.

Let $x, y \in S, q \in Q$.

Then $\widetilde{M}(x,q) \geq \min^i\{\widetilde{M}(x + y, q), \widetilde{M}(y,q)\}$

$\Rightarrow\ -\widetilde{M}(x,q) \leq -\overset{i}{\min}\{\widetilde{M}(x + y, q), \widetilde{M}(y,q)\}$

$\Rightarrow\ 1 - \widetilde{M}(x,q) \leq 1 - \overset{i}{\min}\{\widetilde{M}(x + y, q), \widetilde{M}(y,q)\}$

$\Rightarrow\ 1 - \widetilde{M}(x,q) \leq max\{1 - \widetilde{M}(x + y, q), 1 - \widetilde{M}(y,q)\}$

$\Rightarrow\ \widetilde{M}^c(x,q) \leq \overset{i}{\max}\{\widetilde{M}^c(x + y, q), \widetilde{M}^c(y,q)\}$

Therefore $\widetilde{M}^c$ is an interval-valued anti Q −fuzzy right ideal of S.

Similarly, we can prove the converse.

Theorem 3.7 An interval-valued intuitionistic Q-fuzzy set A in S is an interval-valued intuitionistic Q-fuzzy right (left, lateral) k −ideal of S if and only if any level set $S_A^{(s,t)}$ is a right (left, lateral) k −ideal of S for $s, t \in [0,1]$ whenever nonempty.

Proof:

Let A be an interval-valued intuitionistic Q-fuzzy right k −ideal of S.

Let $S_A^{(s,t)}$ is a right ideal of S.

If there exists $x, y \in S, q \in Q$ such that $x + y,\ y \in S_A^{(s,t)}$ and $x \notin S_A^{(s,t)}$, then $\min^i\{\widetilde{M}_A\{(x + y, q), \widetilde{M}_A(u,q)\} \geq s > \widetilde{M}_A(x,q)$ and
$\max^i\{\widetilde{N}_A(x + y, q), \widetilde{N}_A(u,q)\} \geq t > \widetilde{N}_A(x,q)$, which is a contradiction.

Therefore $S_A^{(s,t)}$ is a right k-ideal in S.

Conversely,

if there exists $x, y \in S, q \in Q$ such that $\widetilde{M}_A(x,q) < s = \min\{\widetilde{M}_A(x+y,q), \widetilde{M}_A(y,q)\}$ and $\widetilde{N}_A(x,q) > t = \max\{\widetilde{N}_A(x+y,q), \widetilde{N}_A(y,q)\}$ then $x+y, y \in S_A^{(s,t)}$ and $x \notin S_A^{(s,t)}$ which is a contradiction.

Therefore by Theorem 2.3,

$A = (\widetilde{M}_A, \widetilde{N}_A)$ is an interval-valued intuitionistic Q-fuzzy right k −ideal of S.

Corollary 3.1

An interval-valued intuitionistic fuzzy set $\mathrm{A} = (\widetilde{\mathrm{M}}_{\mathrm{A}}, \widetilde{\mathrm{N}}_{\mathrm{A}})$ in S is an interval-valued intuitionistic Q −fuzzy right (left, lateral) k −ideal of S if and only if for every $\tilde{s}, \tilde{t} \in \mathrm{D}[0;\ 1]$ such that $\tilde{s} + \tilde{t} \leq 1$ all non-empty $\mathrm{U}(\widetilde{\mathrm{M}}_{\mathrm{A}}, \tilde{\mathrm{s}})$ and $\mathrm{L}(\widetilde{\mathrm{N}}_{\mathrm{A}}, \tilde{\mathrm{t}})$ are right (left, lateral) k-ideals of S.

Theorem 3.8. An interval-valued intuitionistic fuzzy set $\mathrm{A} = (\widetilde{\mathrm{M}}_{\mathrm{A}}, \widetilde{\mathrm{N}}_{\mathrm{A}})$ in S is an interval-valued intuitionistic Q-fuzzy right (left, lateral) k −ideal of S if and only if the interval-valued Q-fuzzy subsets $\widetilde{\mathrm{M}}_{\mathrm{A}}$ and $\widetilde{\mathrm{N}}_{\mathrm{A}}^{\mathrm{c}}$ are interval-valued Q −fuzzy right (left, lateral) k-ideals of S.

Proof. If $A = (\widetilde{\mathrm{M}}_{\mathrm{A}}, \widetilde{\mathrm{N}}_{\mathrm{A}})$ is an interval-valued intuitionistic Q-fuzzy right k-ideal of S, then clearly $\widetilde{\mathrm{M}}_{\mathrm{A}}$ is an interval-valued Q −fuzzy right k-ideal of S.

For all $x, y \in S, q \in Q$,

$$\widetilde{N}_A^c(x,q) = 1 - g_A(x,\mathrm{q}) \geq 1 - \max{}^{\mathrm{i}}\{\widetilde{N}_A(x+y,q), \widetilde{N}_A(y,q)\}$$
$$= \min{}^{i}\{1 - \widetilde{N}_A(x+y,q), 1 - \widetilde{N}_A(y,q)\}$$
$$= \min\{\widetilde{N}_A^c(x+y,q), \widetilde{N}_A^c(y,q)\}.$$

Thus $\widetilde{N}_A^c$ is an interval-valued intuitionistic Q-fuzzy right k-ideal of S.

Conversely assume that $\widetilde{\mathrm{M}}_A$ and $\widetilde{\mathrm{N}}_A^c$ are interval-valued Q-fuzzy right k −ideals of S, then by Theorem 2.4, $A = (\widetilde{\mathrm{M}}_{\mathrm{A}}, \widetilde{\mathrm{N}}_{\mathrm{A}})$ is an interval-valued intuitionistic Q-fuzzy right ideal of S. Now for all $x, y \in S, q \in Q$,

$$1 - \widetilde{\mathrm{N}}_A(x,q) = \widetilde{\mathrm{N}}_A^c(x,q) \geq \min\{\widetilde{\mathrm{N}}_A^c(x+y,q), \widetilde{\mathrm{N}}_A^c(y,q)\} = \min\{1 - \widetilde{\mathrm{N}}_A(x+y,q), 1 - \widetilde{\mathrm{N}}_A(y,q)\}$$
$$= 1 - \max\{\widetilde{\mathrm{N}}_A(x+y,q), \widetilde{\mathrm{N}}_A(y,q)\}$$

which implies $-\widetilde{\mathrm{N}}_A(x,q) \geq -\max\{\widetilde{\mathrm{N}}_A(x+y,q), \widetilde{\mathrm{N}}_A(y,q)\}$ implies

$\widetilde{\mathrm{N}}_A(x,q) \leq \max\{\widetilde{\mathrm{N}}_A(x+y,q), \widetilde{\mathrm{N}}_A(y,q)\}$.

Therefore $A = (\widetilde{\mathrm{M}}_A, \widetilde{\mathrm{N}}_A)$ is an interval-valued intuitionistic Q-fuzzy right k −ideal of S.

Corollary 3.2. Let $\mathrm{A} = (\widetilde{\mathrm{M}}_{\mathrm{A}}, \widetilde{\mathrm{N}}_{\mathrm{A}})$ be an interval-valued intuitionistic fuzzy set in S. Then an interval-valued intuitionistic Q-fuzzy set $\mathrm{A} = (\widetilde{\mathrm{M}}_{\mathrm{A}}, \widetilde{\mathrm{N}}_{\mathrm{A}}))$ is an interval-valued intuitionistic Q-fuzzy right (left, lateral) k −ideal of S if and only if interval-valued intuitionistic Q-fuzzy set $\mathrm{A}_1 = (\widetilde{\mathrm{M}}_{\mathrm{A}}, \widetilde{\mathrm{M}}_{\mathrm{A}}^{\mathrm{c}})$ and IQFS $\mathrm{A}_2 = (\widetilde{\mathrm{N}}_{\mathrm{A}}^{\mathrm{c}}, \widetilde{\mathrm{N}}_{\mathrm{A}})$ are interval-valued intuitionistic Q-fuzzy right (left, lateral) k −ideals of S.

Proof: It is straightforward by Theorem 3.6 and Theorem 3.8

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