

# Some Results on Heronian Mean Labeling of Arbitrary Super Subdivision Graphs

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**Abstract:** A graph  $G=(V,E)$  with  $p$  vertices and  $q$  edges is said to be a Heronian Mean Graph if it is possible to label the vertices  $x \in V$  with distinct labels  $f(x)$  from  $1,2,3,\dots,q+1$  in such a way that in each edge  $e=uv$  is labeled with

$f(e=uv) = \left\lfloor \frac{f(u) + \sqrt{f(u)f(v)} + f(v)}{3} \right\rfloor$  (OR)  $\left\lceil \frac{f(u) + \sqrt{f(u)f(v)} + f(v)}{3} \right\rceil$ , then the edge labels are distinct. In this case  $f$  is called a **Heronian Mean Labeling** of  $G$ . In this paper we apply Heronian Mean Labeling technique on Arbitrary Super Subdivision of graphs.

**Keywords:** Super Heronian mean, Super Subdivision of graphs, Arbitrary Super Subdivision of graphs, Triangular Snake.

## 1. INTRODUCTION

The graphs which are used here are finite, undirected graphs. Here  $V(G)$  indicates vertices and  $E(G)$  indicates edges. For all described view of Graph Labeling we refer to J.A. Gallian [1] and we follow Harary [2] for all other standard terminology and notations in Graph Theory. We will provide short summary and definitions which are useful for the present investigation.

### Definition:1.1

Let  $G$  be a graph. A graph  $H$  is called a super subdivision of  $G$  if  $H$  is obtained from  $G$  by replacing every edge  $e_i$  of  $G$  by a complete bipartite graph  $K_{2,m_i}$  for some  $m_i, 1 \leq i \leq q$  in such a way that the ends of  $e_i$  are merged with two vertices part of  $K_{2,m_i}$  after removing the edge  $e_i$  from graph  $G$ .

### Definition:1.2

A Super subdivision  $H$  of  $G$  is said to be an Arbitrary Super Subdivision of  $G$  if every edge of  $G$  is replaced by an arbitrary  $K_{2,m_i}$  where  $m_i$  may vary for each edge arbitrarily. It is denoted by  $ASS(G)$ .

## 2. MAIN RESULTS

**Theorem: 2.1**

Arbitrary super subdivision of Triangular snake  $T_n$  is heronian mean graph.

**Proof:**

Let  $T_n$  be the Triangular snake graph with the vertices  $u_1, u, \dots, u_n$  &  $v_1, v_2, \dots, v_{n-1}$  and let  $u_i u_{i+1}, u_i v_i, v_i u_{i+1}, 1 \leq i \leq n-1$ , be the edges of a Triangular snake  $T_n$ .

Let H be an arbitrary super subdivision of  $T_n$ , where each edge of  $T_n$  is replaced by a complete bipartite graph  $K_{2,m_i}$  where  $m_i$  is any integer.

$$V(H) = \{u_1, u_2, \dots, u_n, v_1, v_2, \dots, v_{n-1}, w_{m_1}(k), w_{m_2}(k) \dots, w_{m_{n-1}}(k), w_1, w_2, \dots, w_{m_1}, w_{m_1+1}, w_{m_1+2}, \dots, w_{m_1+m_2}, w_{m_1+m_2+1}, w_{m_1+m_2+2}, \dots, w_{m_1+m_2+\dots+m_{2n-2}}\}$$

Arbitrary Super Subdivision of  $T_n$ .



Figure :1  $T_n$

When  $m_1 = m_2 = \dots = m_{2n-2} = 3$

An Arbitrary Super Subdivision of  $T_4$  is given in the following figure.

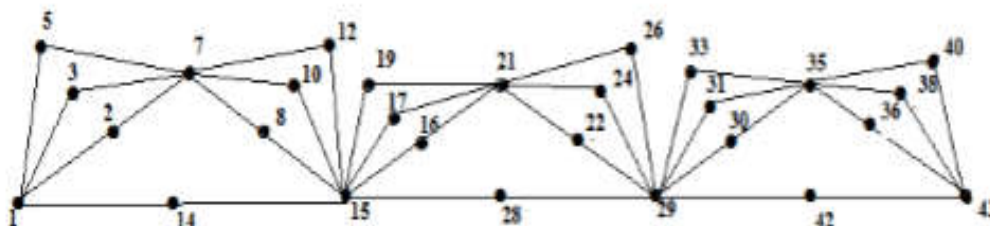


Figure : 2  $T_4$

Define  $\varphi : V(H) \rightarrow \{1, 2, \dots, q+1\}$  by,

$$\varphi(u_i) = 14i - 13 ; 1 \leq i \leq n$$

$$\varphi(v_i) = 14i - 7 ; 1 \leq i \leq n-1$$

$$\varphi(w_1) = 2 ; \varphi(w_2) = 3$$

$$\varphi(w_i) = \varphi(w_{i-1}) + 2 ; 3 \leq i \leq m_1 + m_2 + \dots + m_{2n-2} \text{ \& } i \neq m_1 + 1, m_1 + m_2 + 1, \dots, m_1 + m_2 + \dots + m_{2n-3} + 1$$

$$\varphi \left( w_{\sum_{i=1}^k m_{i+1}} \right) = \begin{cases} \varphi \left( w_{\sum_{i=1}^k m_i} \right) + 4 ; k \text{ is even}, k = 2,4, \dots, 2n - 2 \\ \varphi \left( w_{\sum_{i=1}^k m_i} \right) + 3 ; k \text{ is odd}, k = 1,3, \dots, 2n - 3 \end{cases}$$

$$\varphi \left( w_{m_i}(k) \right) = 14i ; 1 \leq i \leq n - 1$$

Then the edge labels are distinct.

In this similar manner we prove for all n's and m's ( $m_1 \leq 8$ ).

Hence arbitrary Super Subdivision of Triangular snake  $T_n$  is heronian mean graph.

**Theorem:2.2**

Arbitrary super subdivision of  $T_n \odot K_{1,m}$ ,  $m \leq 5$  is heronian mean graph.

**Proof:**

Let  $T_n$  be the Triangular snake graph with the vertices  $u_1, u_2, \dots, u_n$  and  $v_1, v_2, \dots, v_{n-1}$ . Let  $u_i u_{i+1}, u_i v_i, v_i u_{i+1}$ ;  $1 \leq i \leq n-1$  be the edges of a Triangular snake  $T_n$ .

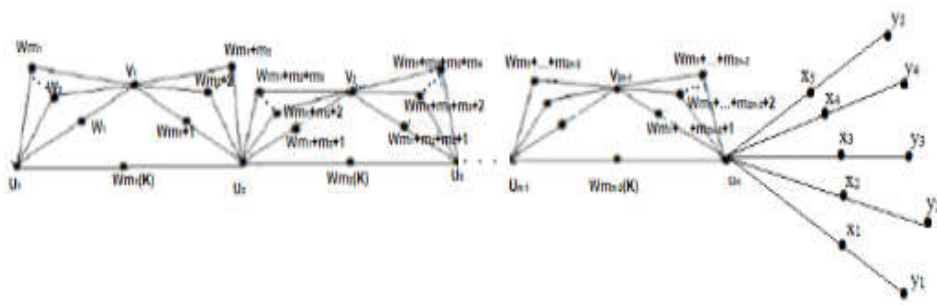
Let  $x_1, x_2, \dots, x_m$  be the vertices of  $K_{1,m}$ . Here  $u_n = x_1$ .

Let H be an arbitrary Super subdivision of  $T_n \odot K_{1,m}$  where each  $e_i$  of  $T_n \odot K_{1,m}$  is replaced by a complete bipartite graph  $K_{2,m_i}$  where  $m_i$  is any positive integer. ( $m_i \leq 8$ ).

$$V(H) = \{u_1, u_2, \dots, u_n, v_1, v_2, \dots, v_{n-1}, w_{m_1}(k), w_{m_2}(k) \dots, w_{m_{n-1}}(k), w_1, w_2, \dots, w_{m_1}, w_{m_1+1},$$

$$w_{m_1+2}, \dots, w_{m_1+m_2}, w_{m_1+m_2+1}, w_{m_1+m_2+2}, \dots, w_{m_1+m_2+\dots+m_{2n-2}}, x_0, x_1, \dots, x_{r_1}, y_1, y_2, \dots, y_m\}$$

An Arbitrary Super Subdivision of  $T_n \odot K_{1,5}$  is given is the following figure.



**Figure : 3**  $T_n \odot K_{1,5}$

When  $m_1 = m_2 = \dots = m_{2n-2} = 3$

An Arbitrary Super Subdivision of  $T_4 \odot K_{1,5}$  is given in the following figure.



**Figure : 4**  $T_4 \odot K_{1,5}$

Define  $\varphi : V(H) \rightarrow \{1, 2, \dots, q+1\}$  by,

$$\varphi(u_i) = 14i - 13; 1 \leq i \leq n$$

$$\varphi(v_i) = 14i - 7; 1 \leq i \leq n-1$$

$$\varphi(w_1) = 2; \varphi(w_2) = 3$$

$$\varphi(w_i) = \varphi(w_{i-1}) + 2; 3 \leq i \leq m_1 + m_2 + \dots + m_{2n-2} \& i \neq m_1 + 1, m_1 + m_2 + 1, \dots, m_1 + m_2 + \dots + m_{2n-3} + 1$$

$$\varphi\left(w_{\sum_{i=1}^k m_{i+1}}\right) = \begin{cases} \varphi\left(w_{\sum_{i=1}^k m_i}\right) + 4; k \text{ is even}, k = 2, 4, \dots, 2n-2 \\ \varphi\left(w_{\sum_{i=1}^k m_i}\right) + 3; k \text{ is odd}, k = 1, 3, \dots, 2n-3 \end{cases}$$

$$\varphi(w_{m_i}(k)) = 14i; 1 \leq i \leq n-1$$

$$\varphi(x_0) = \varphi(u_n) = 14n - 13$$

$$\varphi(x_i) = \varphi(u_n) + 1; 1 \leq i \leq 3$$

$$\varphi(x_j) = \varphi(u_n) + 2i - 3; 4 \leq i \leq r_1$$

$$\varphi(y_i) = \varphi(x_{r_1}) + i; 1 \leq i \leq m.$$

Then the edge labels are distinct. In this similar manner we can prove for all n's and m<sub>1</sub>'s (m<sub>1</sub> ≤ 8).

Hence Arbitrary Super Subdivision of  $T_n \odot K_{1,m}$ , m ≤ 5 is heronian mean graph.

**Theorem:2.3**

Arbitrary super subdivision of  $C_n \odot T_m$  is Heronian mean graph.

**Proof:**

Let  $C_n$  be acycle with consecutive vertices  $u_1, u_2, \dots, u_n$  and let  $e_i$  denote the edge  $u_{i-1}u_i$  of  $C_n$  for  $1 \leq i \leq n$ . Let  $T_m$  be the triangular snake with the vertices  $w_1, w_2, \dots, w_m$  and  $x_1, x_2, \dots, x_{m-1}$ .

Let  $w_i w_{i+1}, w_i x_i, x_i w_{i+1}, 1 \leq i \leq m-1$ , be the edges of a Triangular snake graph  $T_m$ .

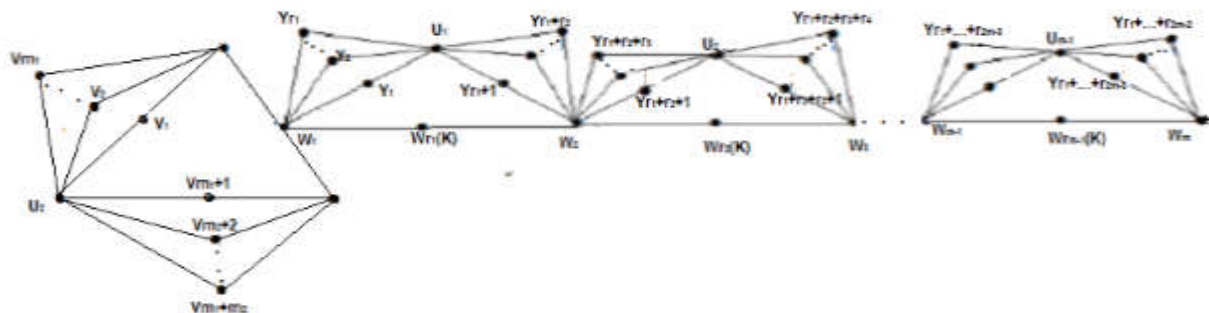
Let H be an arbitrary super subdivision of  $C_n \odot T_m$ , where each edge of  $C_n \odot T_m$  is replaced by a complete bipartite graph  $K_{2,m_i}$  where  $m_i$  is any positive integer.

$$V(H) = \{ u_1, u_2, \dots, u_n, v_1, v_2, \dots, v_{m_1}, v_{m_1+1}, v_{m_1+2}, \dots, v_{m_1+m_2}, v_{m_1+m_2+1}, \dots,$$

$$v_{m_1+m_2+\dots+m_{n-1}}, v_{m_1+m_2+\dots+m_{n-1}+1}, w_1, w_2, \dots, w_m,$$

$$x_1, x_2, \dots, x_{m-1}, y_1, y_2, \dots, y_{r_1}, y_{r_1+1}, \dots, y_{r_1+r_2+\dots+r_{2m-2}}, w_{r_1}(k), w_{r_2}(k), \dots, w_{r_{m-1}}(k) \}$$

An Arbitrary Super Subdivision of  $C_n \odot T_m$ .



**Figure : 5**  $C_n \odot T_m$

When  $m_1 = m_2 = \dots = m_{n-1} = 3$  and

$$r_1 = r_2 = \dots = r_{2m-2} = 3$$

An Arbitrary Super Subdivision of  $C_3 \odot T_4$  is given is the following figure.

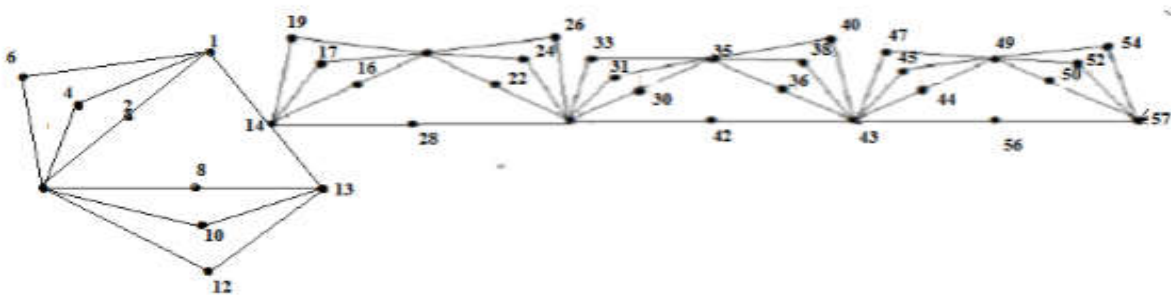


Figure :6  $C_3 \odot T_4$

Define  $\varphi : V(H) \rightarrow \{1, 2, \dots, q+1\}$  by,

$$\varphi(u_1) = 1$$

$$\varphi(u_i) = 6i - 5; 2 \leq i \leq n$$

$$\varphi(v_i) = 2i; 1 \leq i \leq m_1 + m_2 + \dots + m_{n-1}$$

$$\varphi(v_{m_1 + m_2 + \dots + m_{n-1} + 1}) = \varphi(u_n) + 1$$

$$\varphi(w_i) = \varphi(v_{m_1 + m_2 + \dots + m_{n-1} + 1})$$

$$\varphi(w_i) = \varphi(w_i) + 14(i-1) + 1; 2 \leq i \leq m$$

$$\varphi(x_i) = 14i + 7; 1 \leq i \leq m-1$$

$$\varphi(y_1) = \varphi(w_1) + 2$$

$$\varphi(y_2) = \varphi(y_1) + 1$$

$$\varphi(y_i) = \varphi(y_{i-1}) + 2; 3 \leq i \leq r_1 + r_2 + \dots + r_{2m-2} \& i \neq r_1 + 1, (r_1 + r_2) + 1, \dots, (r_1 + r_2 + \dots + r_{2m-3}) + 1$$

$$\varphi(y_{\sum_{i=1}^k r_{i+1}}) = \begin{cases} \varphi(y_{\sum_{i=1}^k m_i}) + 4; k \text{ is even,} \\ \varphi(y_{\sum_{i=1}^k m_i}) + 3; k \text{ is odd,} \end{cases}$$

$$\varphi(w_{r_i}(k)) = 14i; 1 \leq i \leq m - 1$$

Then the edge labels are distinct. In this similar manner we can prove for all n's and m\_i's ( $m_i \leq 4$ ).

Hence Arbitrary Super Subdivision of  $C_n \odot T_m$  is heronian mean graph.

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