Some Results on Heronian Mean Labeling of Arbitrary Super Subdivision Graphs

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Abstract: A graph G=(V,E) with p vertices and q edges is said to be a Heronian Mean Graph if it is possible to label the vertices x ϵV with distinct labels f(x) from 1,2,3,. . . ,q+1 in such a way that in each edge e=uv is labeled with

f(e=uv)= $\left\lfloor \frac{f(u)+\sqrt{f(u)f(v)}+f(v)}{3} \right\rfloor$ (OR) $\left\lceil \frac{f(u)+\sqrt{f(u)f(v)}+f(v)}{3} \right\rceil$, then the edge labels are distinct. In this case f is called a **Heronian Mean Labeling** of G. In this paper we apply Heronian Mean Labeling technique on Arbitrary Super Subdivision of graphs.

Keywords: Super Heronian mean, Super Subdivision of graphs, Arbitrary Super Subdivision of graphs, Triangular Snake.

1. INTRODUCTION

The graphs which are used here are finite, undirected graphs. Here V(G)indicates vertices and E(G) indicates edges. For all described view of Graph Labeling we refer to J.A. Gallian [1] and we follow Harary [2] for all other standard terminology and notations in Graph Theory. We will provide short summary and definitions which are useful for the present investigation.

Definition:1.1

Let G be a graph. A graph H is called a super subdivision of G if H is obtained from G by replacing every edge e_i of G by a complete bipartite graph K_{2,m_i} for some m_i,$1\le i\le q$ is such a way that the ends of e_i are merged with two vertices part of K_{2,m_i} after removing the edge e_i from graph G.

Definition:1.2

A Super subdivision H of G is said to be an Arbitrary Super Subdivision of G if every edge of G is replaced by an arbitrary K_{2,m_i} where m_i may vary for each edge arbitrarily. It is denoted by ASS(G).

2. MAIN RESULTS

Theorem: 2.1

Arbitrary super subdivision of Triangular snake T_n is heronian mean graph.

Proof:

Let T_n be the Triangular snake graph with the vertices $u_1, u, \dots, u_n \& v_1, v_2, \dots, v_{n-1}$and let$u_i u_{i+1}, u_i v_i$, $v_i u_{i+1}$,1≤i≤n-1, be the edges of a Triangular snake T_n.

Let H be an arbitrary super subdivision of T_n, where each edge of T_n is replaced by a complete bipartite graph $K_{2,m_i}$where m_i is any integer.

$$V(H) = \{u_{1,}, u_{2,} \dots, , u_n, v_{1,} v_{2,} \dots, v_{n-1}, w_{m_1}(k), w_{m_2}(k) \dots, w_{m_{n-1}}(k), w_{1,} w_{2,} \dots, w_{m_1,} w_{m_1+1,}$$
$$w_{m_1+2,} \cdots, w_{m_1+m_2,} w_{m_1+m_2+1},, w_{m_1+m_2+2},, \cdots, w_{m_1+m_2+ \dots + m_{2n-2}}\}$$

Arbitrary Super Subdivision of T_n.

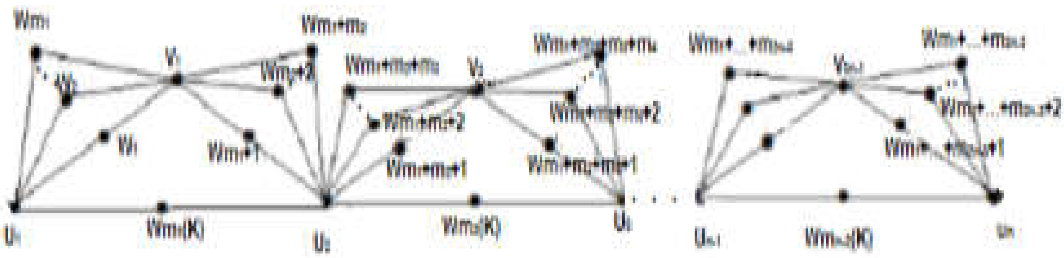

Figure :1 T_n

When m_1=m_2= . . . = m_{2n-2}= 3

An Arbitrary Super Subdivision of T_4 is given in the following figure.

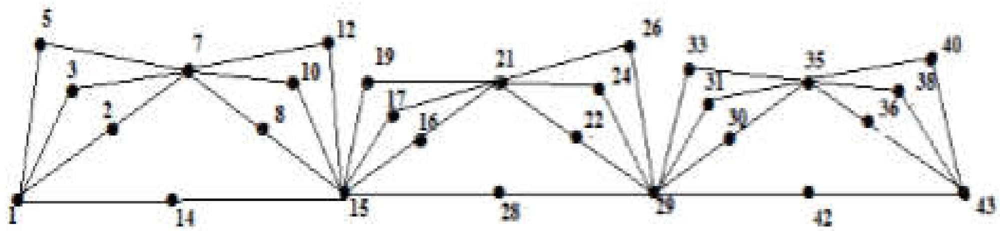

Figure : 2 T_4

Define φ : V(H) →{1,2, . . . ,q+1} by,

φ (u_i)=14i-13 ; 1≤i≤n

φ (v_i)=14i-7 ; 1≤i≤n-1

φ (w_1)=2 ;φ (w_2)=3

φ (w_i)=φ (w_{i-1})+2 ; 3≤i≤ m_1+m_2+ . . . +m_{2n-2}& i $\neq m_1 + 1$,m_1+$m_2 + 1$, . . ., $m_1 + m_2 + \dots + m_{2n-3}$+1

$$\varphi\left(w_{\sum_{i=1}^{k} m_{i+1}}\right) = \begin{cases} \varphi\left(w_{\sum_{i=1}^{k} m_i}\right) + 4\ ; k\ is\ even, k = 2,4,\dots,2n-2 \\ \varphi\left(w_{\sum_{i=1}^{k} m_i}\right) + 3\ ; k\ is\ odd,\ \ k = 1,3,\dots,2n-3 \end{cases}$$

$$\varphi\left(w_{m_i}(k)\right) = 14i; 1 \le i \le n-1$$

Then the edge labels are distinct.

In this similar manner we prove for all n's and m's ($m_1 \le 8$).

Hence arbitrary Super Subdivision of Triangular snake T_n is heronian mean graph.

Theorem:2.2

Arbitrary super subdivision of $T_n \odot K_{1,m}$, m≤5 is heronian mean graph.

Proof:

Let T_n be the Triangular snake graph with the vertices $u_1, u_2, \dots, u_n$ and $v_1, v_2, \dots, v_{n-1}$. Let $u_iu_{i+1}, u_iv_i, v_iu_{i+1}$; 1≤i≤n-1 be the edges of a Triangular snake T_n.

Let $x_1, x_2, \dots, x_m$ be the vertices of $k_{1,m}$. Here $u_n = x_1$.

Let H be an arbitrary Super subdivision of $T_n \odot K_{1,m}$ where each e_i of $Tn, \odot K_{1,m}$ in replaced by a complete bipartite graph K_{2,m_i} where m_i is any positive integer. ($m_1 \le 8$).

V(H)= $\{u_1, u_2, \dots, u_n, v_1, v_2, \dots, v_{n-1}, w_{m_1}(k), w_{m_2}(k) \dots, w_{m_{n-1}}(k), w_1, w_2, \dots, w_{m_1}, w_{m_1+1},$

$w_{m_1+2}, \cdots, w_{m_1+m_2}, w_{m_1+m_2+1}, w_{m_1+m_2+2}, \dots, w_{m_1+m_2+\dots+m_{2n-2}}, x_0, x_1, \dots, x_{r_1}, y_1, y_2, \dots, y_m\}$

An Arbitrary Super Subdivision of $T_n \odot K_{1,5}$ is given is the following figure.

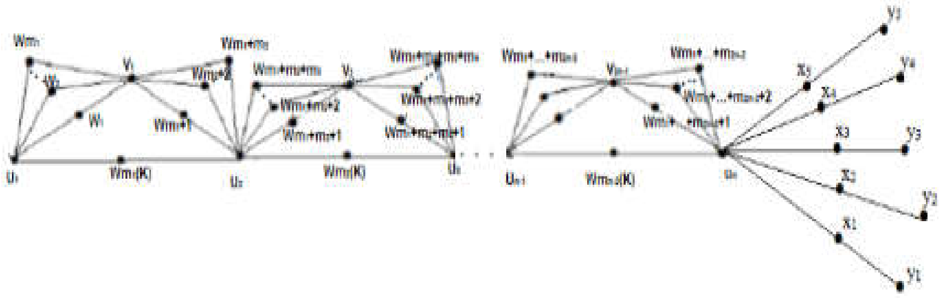

Figure : 3 $T_n \odot K_{1,5}$

When $m_1 = m_2 = \dots = m_{2n-2} = 3$

An Arbitrary Super Subdivision of $T_4 \odot K_{1,5}$ is given in the following figure.

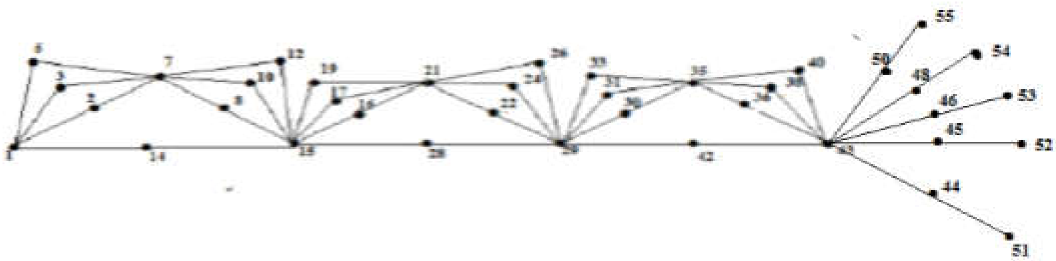

Figure : 4 $T_4 \odot K_{1,5}$

Define φ : V(H) $\rightarrow$ {1,2, . . . ,q+1} by,

φ (u_i)=14i-13 ; 1≤i≤n

φ (v_i)=14i-7 ; 1≤i≤n-1

φ (w_1)=2 ;φ (w_2)=3

φ (w_i)=φ (w_{i-1})+2 ; 3≤i≤ $m_1+m_2+ \ldots +m_{2n-2}$& i $\neq m_1+1, m_{1+}m_2+1, \ldots, m_1+m_2+\ldots+m_{2n-3}$+1

$$\varphi\left(w_{\sum_{i=1}^{k} m_{i+1}}\right) = \begin{cases} \varphi\left(w_{\sum_{i=1}^{k} m_i}\right) + 4\,; k \text{ is even}, k = 2,4,\ldots,2n-2 \\ \varphi\left(w_{\sum_{i=1}^{k} m_i}\right) + 3\,; k \text{ is odd}, \ k = 1,3,\ldots,2n-3 \end{cases}$$

$$\varphi\left(w_{m_i}(k)\right) = 14i; 1 \leq i \leq n-1$$

φ (x_0)=φ (u_n)=14n-13

φ (x_i)=φ (u_n)+1 ;1≤i≤3

φ (x_i)=φ (u_n)+2i-3 ; 4≤i≤r_1

φ (y_i)=φ (x_{r_1})+i ; 1≤i≤m.

Then the edge labels are distinct. In this similar manner we can prove for all n's and m_1's (m_1≤8).

Hence Arbitrary Super Subdivision of $T_n \odot K_{1,m}$, m≤5 is heronian mean graph.

Theorem:2.3

Arbitrary super subdivision of $C_n \odot T_m$ is Heroniam mean graph.

Proof:

Let C_n be acycle with consecutive vertices $u_1,u_2, \ldots ,u_n$ and let e_i denote the edge $u_{i-1}u_i$of C_n for1≤i≤n.Let T_m be the triangular snake with the vertices $w_1, w_2, \ldots, w_m$and $x_1, x_2, \ldots, x_{m-1}$.

Let $w_iw_{i+1}, w_ix_i, x_iw_{i+1,}$ 1≤i≤m-1, be the edges of a Triangular snake graph T_m.

Let H be an arbitrary super subdivision of $C_n \odot T_m$, where each edge of $C_n \odot T_m$ is replaced by a complete bipartite graph K_{2,m_i} where m_i is any positive integer.

V(H)={ $u_{1,}, u_{2,} \ldots, , u_n, v_{1,} v_{2,} \ldots, v_{m_1}, v_{m_1+1}, v_{m_1+2}, \ldots, v_{m_1+m_2}, v_{m_1+m_2+1}, \ldots,$

$v_{m_1+m_2+\ldots+m_{n-1}}, v_{m_1+m_2+\ldots+m_{n-1}+1}, w_{1,} w_{2,} \ldots, w_m$,

$x_1, x_2, \ldots, x_{m-1}, y_1, y_2, \ldots, y_{r_1}, y_{r_1+1}, \ldots, y_{r_1+r_2+\ldots+r_{2m-2}}, w_{r_1}(k), w_{r_2}(k), \ldots, w_{r_{m-1}}(k)$}

An Arbitrary Super Subdivision of $C_n \odot T_m$.

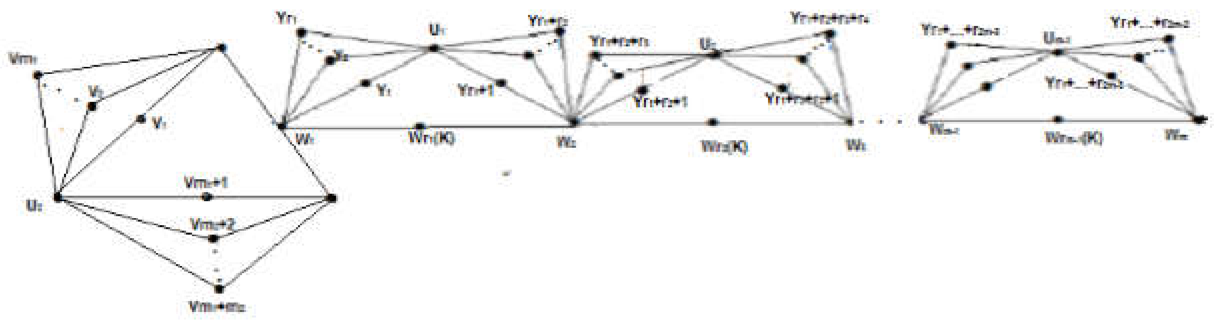

Figure : 5 $C_n \odot T_m$

When m_1=m_2= . . . = m_{n-1}= 3 and

r_1=r_2= . . . = r_{2m-2}= 3

An Arbitrary Super Subdivision of $C_3 \odot T_4$ is given is the following figure.

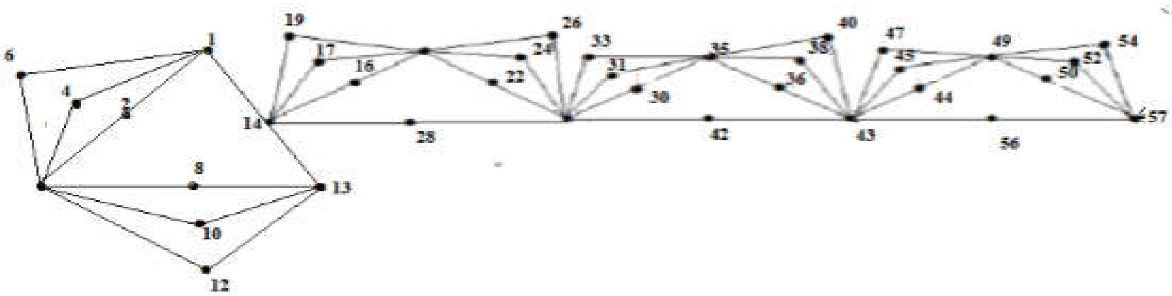

Figure :6 $C_3 \odot T_4$

Define φ :V(H)→{1,2, . . . ,q+1} by,

$\varphi(u_1)=1$

$\varphi(u_i)=6i-5$; $2\leq i\leq n$

$\varphi(v_i)=2i$; $1\leq i\leq m_1+m_2+\ldots+m_{n-1}$

$\varphi(v_{m_1+m_2+\ldots+m_{n-1}}+1)=\varphi(u_n)+1$

$\varphi(w_1)=\varphi(v_{m_1+m_2+\ldots+m_{n-1}}+1)$

$\varphi(w_i)=\varphi(w_1)+14(i-1)+1$; $2\leq i\leq m$

$\varphi(x_i)=14i+7$; $1\leq i\leq m-1$

$\varphi(y_1)=\varphi(w_1)+2$

$\varphi(y_2)=\varphi(y_1)+1$

$\varphi(y_i)=\varphi(y_{i-1})+2$; $3\leq i\leq r_1+r_2+\ldots+r_{2m-2}$ & $i \neq r_1+1, (r_1+r_2)+1, \ldots, (r_1+r_2+\ldots+r_{2m-3})+1$

$$\varphi\left(y_{\sum_{i=1}^{k} r_{i+1}}\right)=\begin{cases}\varphi\left(y_{\sum_{i=1}^{k} m_i}\right)+4\,; k \text{ is even,}\\ \varphi\left(y_{\sum_{i=1}^{k} m_i}\right)+3\,; k \text{ is odd,}\end{cases}$$

$$\varphi\left(w_{r_i}(k)\right)=14i; 1\leq i\leq m-1$$

Then the edge labels are distinct. In this similar manner we can prove for all n's and m_i's ($m_1\leq 4$).
Hence Arbitrary Super Subdivision of $C_n \odot T_m$ is heronian mean graph.

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