Anti Q- L-Fuzzy M-Subgroups

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Abstract— **In this paper we introduced the concept of anti Q-L- fuzzy M-subgroups and discuss some of its properties.**

Keywords— **Fuzzy set, Q-fuzzy set, anti L-fuzzy M-subgroup, anti Q-L- fuzzy M-subgroup.**

A. 1. Introduction

The fundamental concept of fuzzy sets was initiated by Zadeh. L [14] in 1965 . Fuzzy set theory has been developed in many directions by many scholars and has evoked great interest among mathematicians working in different fields of mathematics such as topological spaces, functional analysis, loop, group, ring, near- ring, vector spaces, automation.The fuzzy algebraic structures play a prominent role in mathematics with wide applications in many other branches such as theoretical physics, computer science, control engineering, information science, coding theory, group theory, real analyses, hectare theory etc.In 1971, Rosenfeld. A [8] first introduced the concept of fuzzy subgroups, which was the first fuzzification of any algebraic such as fuzzy structures. The introduced the concept of L-Fuzzy Sets is given byGoguen. J.Ain 1967 [3].The notion of Algebraic fuzzy systems, fuzzy sets and systems is introduced by Swamy . U. M and ViswanadhaRaju. D in 1991[12].Katsaras . A. K. and Liu . D. B, [4] considered the Fuzzy vector spaces and fuzzy topological vector spaces. Wang-Jin Liu [13] introduce and define a new Fuzzy invariant subgroups and fuzzy sets and systems in 1981.Garrett Birkhof [2] introduce the concept of Lattice Theory. In 2008, SatyaSaibaba. G.S.V [9] Fuzzy lattice Ordered Groups. Ath.Kehages [1], the introduced the concept of the lattice of fuzzy intervals and sufficient conditions for its dis-tibutivity .In 1991, Murali. V [6], defined and studied lattice of fuzzy algebras and closure systems in I^X.Makandar. M.U andDr.A.D.Lokhande [5], introduce the new concept of AntiFuzzy Lattice Ordered M-Group in 2013. The concept of Structure Properties of M-Fuzzy Groups is given by Subramanian. S, et.al. [11] in 2012. Soaliraju.A and Nagarajan . R in 2009 [10], introduced the concept of A New structure and construction of Q- fuzzy groups. In this paper, we initiate the study of Q-fuzzy lattice M-groups. Pandiammal. P, Natarajan. R and Palaniappan. N [7],introduced the concept of anti L-fuzzy M-subgroups and anti L-fuzzy normal M-subgroups in 2010. We also introduced the concept of anti Q-L- fuzzy M-subgroups and discuss some of its properties

B. 2. Preliminaries:

In this section, we review some basic definitions and some results of anti L- fuzzy subgroup which will be used in the later sections. Through this section we mean that $(G,*)$is a group, e is the identity of G and xyas $(x * y)$.

Definition: 2.1[Zadeh. L.A (14)]

A mapping $\mu: X \to [0,1]$, where $X$is an arbitrary non-empty set and is called a fuzzy set in X.

Definition: 2.2[Rosenfeld .A (8)]

Let G be any group. A mapping $\mu: G \to [0,1]$ is a fuzzy group if

(i) $\mu(xy) \geq min\{\mu(x), \mu(y)\}$

(ii) $\mu(x^{-1}) = \mu(x)$, for all $x, y \in G$.

Definition: 2.3[Solairaju. A and Nagarajan. R (10)]

Let Q and G a set and a group respectively. A mapping $\mu: G \times Q \to [0,1]$ is called Q-fuzzy set in G. For any Q-fuzzy set μ in G and $t \in [0,1]$ we define the set $U(\mu; t) = \{x \in G \ / \mu(x,q) \geq t, q \in Q\}$ which is called an upper cut of "μ" and can be use to the characterization of μ.

Definition: 2.4[Solairaju. A and Nagarajan. R (10)]

A Q- fuzzy set A is called Q-fuzzy group of G if

(i) $A(xy, q) \geq \min\{A(x,q), A(y,q)\}$

(ii) $A(x^{-1}, q) = A(x,q)$

(iii) $A(x,q) = 1$, for all $x, y \in G$and $q \in Q$.

Definition: 2.5 [Subramanian. S, Nagarajan. R and Chellappa. B (11)]

A group with operators is an algebraic system consisting of a group G, a set M and a function defined in the product set $M \times G$and having the values in G such that if mxdenotes the element in G determined by the element $x$of G and the element m of M, and $m\varepsilon M$, then G is called M- group with operators.

Definition: 2.6 [Subramanian. S, Nagarajan. R and Chellappa. B (11)]

A subgroup A of an M- group G is said to be the fuzzy subgroup if $mx\varepsilon A$ for all $m\varepsilon M and x\varepsilon A$.

Definition: 2.7[Subramanian. S, Nagarajan. R and Chellappa. B (11)]

Let A be a fuzzy set in U and • $: G \times G \to$ G be a composition law, such that (G, •) forms M- group. If two conditions $A(m(xy)) \geq min\{A(mx), A(my)\}$ and $A(mx - 1) = A(mx)$ for all x, yin A. If the supplementary conditions $A(meG) = 1$ is also satisfied, then this M- fuzzy group is called a standardized M- fuzzy group, where e is an identity of M- group (G, •).

Definition: 2.8 [Pandiammal. P, Natarajan. R and Palaniappan. N (7)]

Let (G, .) be a M-group. A L-fuzzy subset A of G is said to be anti L-fuzzy M-subgroup (ALFMSG) of G if the following conditions are satisfied:

(i) $\mu_A(mxy) \leq \mu_A(x) \vee \mu_A(y)$,

(ii) $\mu_A(x^{-1}) \leq \mu_A(x)$, for all$x and y in G$.

Definition: 2.9 [Pandiammal. P, Natarajan. R and Palaniappan. N (7)]

Let (G, .) and (G', .) be any two M-groups. Let $f: G \to G'$ be anyfunctionandletAbeanantiL-fuzzyM-subgroupinG,Vbeananti L-fuzzy M-subgroup in f (G)

$=G'$defined by $\mu_v(y) = inf_{x\in f^{-1}(y)}\mu_A(x)$,$\forall x \in G$,$\forall y \in G'$. .Then A is called a preimage of V under f and is denoted by $f^{-1}(v)$.

Definition: 2.10[Pandiammal. P, Natarajan. R and Palaniappan. N (7)]

Let A and B be two L-fuzzy subsets of sets G and H, respectively. The anti-productof A and B, denoted by $A \times B$, is defined as $A \times B$, $= \{ \langle (xy), \mu_{A\times B}(xy)\rangle$ / for all x in G, y in H }, Where $\mu_{A\times B}(xy) = \mu_A(x)\vee\mu_B(y)$, for all x in G and y in H,

Definition: 2.11[Pandiammal. P, Natarajan. R and Palaniappan. N (7)]

Let A and B be two anti L-fuzzy M-subgroups of a M-group G. Then A and B are said to be conjugate anti L-fuzzy M-subgroupsof G if for some g in G, $\mu_A(x) = \mu_B(g^{-1}xg)$for every x inG .

Definition: 2.12[SatyaSaibaba. G.S.V (9)]

A lattice ordered group is a system $G = (G,*,\leq)$ where

(i) $(G,*)$ is a group

(ii) $(G,\leq)$ is a lattice and

(iii) the inclusion is invariant under all translations $x \mapsto a * x * b$.

That is, $x \leq y \Rightarrow a * x * b \leq a * y * b$, for all $a, b \in G$.

Definition: 2.13 [Goguen. J. A (3)]

A L- Fuzzy subset λ of X is a mapping from X into L, where is a complete lattice satisfying from X meet distributive law. If L is the unit interval [0, 1] of real numbers, there are the usual fuzzy subset of X.

A L-fuzzy subset λ:X → L is said to be a nonempty, if it is not the constant map which assumes the values 0 of L

Definition: 2.14 [Goguen. J. A (3)]

Let λ:X → Lbe a L-fuzzy subset set of X. Then for $t \in L$, the set $\lambda_t = x \in X / \lambda(x) \leq t$ is called a lower t-cut or t-level set of λ.

Definition: 2.15[Goguen. J. A (3)]

Let λ, $\mu: X \rightarrow$ L be a L-fuzzy sub sets of X. If $\lambda(x) \leq \mu(x)$ for all $x \in X$, then we say that λ is contained in μ and we write $\lambda \subseteq \mu$.

Definition: 2.16 [Goguen. J. A (3)]

Let λ, $\mu: X \rightarrow$ L be a L-fuzzy sub sets of X. Define $\lambda\cup\mu$, $\lambda\cap\mu$ are L-fuzzy subsets of X by all $x \in X$, $(\lambda\cup\mu)(x) = \lambda(x)\vee\mu(x)$ and $(\lambda\cap\mu)(x) = \lambda(x)\wedge\mu(x)$. Then $\lambda \cup \mu, \lambda \cap \mu$ are called union and intersection of λ and μ respectively.

Definition: 2.17[Goguen. J. A (3)]

A L-fuzzy subset λ of X is said to have sup property if, for any subset A of X, there exists $a_o \in A$ such that $\lambda(a_o) = \vee_{a\in A}\lambda(a)$.

Definition: 2.18 [Wang-Jin Liu (13)]

Let f be any function from a set X to set Y, and let λ be any L-fuzzy subset of X. Then λ is called f- invariant if $f(x) = f(y)$ implies $\lambda(x) = \lambda(y)$, where $x, y \in X$.

Definition: 2.19 [Makandar. U. M and ,Dr.A.D.Lokhande (5)]

A L- fuzzy subset λ of G is said to be a L- fuzzy subgroup of G, If for all $x, y \in G$,

(i) $\lambda(xy) \leq \lambda(x) \vee \lambda(y)$,

(ii) $\lambda(x^{-1}) = \lambda(x)$.

Definition: 2.20[Makandar. U. M and ,Dr.A.D.Lokhande (5)]

A L- fuzzy subset λ of G is said to be an anti L- fuzzy subgroup of G, If for all $x, y \in G$,

(i) $\lambda(xy) \leq \lambda(x) \vee \lambda(y)$,

(ii) $\lambda(x^{-1}) = \lambda(x)$.

Remark:$\lambda(e) \leq \lambda(x)$, for all $x \in G$.

Definition: 2.21[Makandar. U. M and ,Dr.A.D.Lokhande (5)]

A L-fuzzy subset λ of G is said to be an anti L- fuzzy subgroup of G if and only if λ_t is a subgroup of G for all $\lambda(G) \cup t \in L$ / $\lambda(e) \leq t$.

Definition: 2.22[Makandar. U. M and ,Dr.A.D.Lokhande (5)]

Let Q and X be ant two sets and λ be a L-fuzzy subset of X. A Q-L-fuzzy subset λ of X is a mapping from of $Q \times X$ into L, where L is a complete lattice satisfying the infinite meet distributive law. If L is the unit interval [0,1] of real numbers, there are the usual Q-fuzzy subset of X. A Q- L- fuzzy subset $\lambda: X \times Q \rightarrow$ Lis said to be a nonempty, if it is not the constant map which assumes the values 0 of L.

Definition: 2.23 [Makandar. U. M and ,Dr.A.D.Lokhande (5)]

Let λ, $\mu: X \times Q \rightarrow$ L be a Q-L-fuzzy subsets of X. If $\lambda(x, q) \leq \mu(x, q)$ for all $x \in G$ and$q \in Q$ then we say that λ is contained in μ and we write $\lambda \subseteq \mu$.

Definition: 2.24[Makandar. U. M and ,Dr.A.D.Lokhande (5)]

Let λ, $\mu: X \times Q \rightarrow$ L be a Q-L-fuzzy subsets of X. . Define $\lambda \cup \mu$, $\lambda \cap \mu$ are Q- L-fuzzy subsets of X by all $x \in X$, $(\lambda \cup \mu)(x, q) = \lambda(x, q) \vee \mu(x, q)$ and $(\lambda \cap \mu)(x, q) = \lambda(x, q) \wedge \mu(x, q)$ for all $x \in G$ and$q \in Q$. Then $\lambda \cup \mu$, $\lambda \cap \mu$ are called union and intersection of λ and μ respectively.

3.ANTI Q-L-FUZZY M-SUBGROUPS:

In this section, we introduce the concept of anti Q-L-Fuzzy M-subgroup of a group and discussed some of its properties. Throughout this section, we mean that G, **.**is a group, *e* is the identify of G and mxy as $x \,.\, y$.

Definition: 3.1

Let (G, **.**) be a M-group. A Q-L-fuzzy subset A of G is said to be **anti Q- L-fuzzy M-subgroup** (AQLFMSG) of G if the following conditions are satisfied:

(i) $\alpha_A(mxy, q) \leq \alpha_A(x, q) \vee \mu_A(y, q)$

(ii) $\alpha_A(x^{-1}, q) \leq \alpha_A(x, q), \forall x, y \in Gand\forall q \in Q$.

Definition: 3.2

Let (G, .) and (G', .) be any two M-groups. Let f: G → G' be anyfunctionandletAbeananti Q-L-fuzzyM-subgroupinG,Vbeananti Q-L-fuzzyM-subgroup in f

(G) =G', defined by $\mu_v(y,q) = inf_{x\in f^{-1}(y,q)}\mu_A(x,q)$,$\forall x \in G$,$\forall y \in G'and\forall q \in Q$.Then A is called a preimage of V under f and is denoted by $f^{-1}(v)$.

Definition: 3.3

Let A and B be two Q- L-fuzzy subsets of sets G and H, respectively. The **anti-product** of A and B, denoted by $A \times B$, is defined as $A \times B$ = { ⟨(xy,q), $\alpha_{A\times B}(xy$, q)⟩ / for all x in G, y in H and for all in $qinQ$ }, Where$\alpha_{A\times B}(xy$, q) = $\alpha_A(x,q)\vee\alpha_B(y,q)$, for all x in G and y in H, and for all in $qinQ$

Definition: 3.4

Let A and B be two anti Q- L-fuzzy M-subgroups of a M-group G. Then A and B are said to be **conjugate anti Q-L-fuzzy M-subgroups** of G if for some g in G, $\alpha_A(x,q)$ = α_B($g^{-1}xg$, q), for every x inG and q in Q.

Theorem:3.5

IfAisananti Q - L-fuzzyM-subgroupofaM-group(G,·),then

$\alpha_A(x^{-1},q)$= $\alpha_A(x,q)$, for all x in G and $\alpha_A(x,q)$≥ α_A (e, q), for x inG and q in Q , where e is the identity element inG.

Proof:

For$x$ in G and e is the identify element inG .

Now, $\alpha_A(x,q)$=α_A ((x^{-1},q)$^{-1}$)

$$\leq \alpha_A(x^{-1},q) \leq \alpha_A(x,q).$$

Therefore, $\alpha_A(x^{-1},q) = \alpha_A(x,q), \forall x \in Gand\forall q \in Q$.

Now, $\alpha_A(e,q) = \alpha_A(xx^{-1},q)$

$$\leq \alpha_A(x,q)\vee\alpha_A(x^{-1},q)$$

$$=\alpha_A(x,q).$$

Therefore, $\alpha_A(e,q) \leq \alpha_A(x,q)$, $\forall x \in G$ and$\forall q \in Q$.

Theorem: 3.6

If Aisananti Q - L-fuzzyM-subgroupofaM-group(G,·),then$\alpha_A(xy^{-1},q) = \alpha_A(e,q)$ gives $\alpha_A(x,q) = \alpha_A(y,q)$, for x,y in G, and q in Q, where e is the identity element inG.

Proof:

Let x in G, q in Q and $e$be the identity element in G.

Now,$\alpha_A(x,q) = \alpha_A(xy^{-1}y,q)$

$$\leq \alpha_A(xy^{-1},q)\vee\alpha_A(y,q)$$

$$= \alpha_A(\mathrm{e,q})\vee\alpha_A(\mathrm{y,q})$$

$$=\alpha_A(\mathrm{y,q})$$

$$\alpha_A(y,q) = \alpha_A(yx^{-1}x,q)$$

$$\leq \alpha_A(yx^{-1},\mathrm{q})\vee\alpha_A(x,q)$$

$=\alpha_A(\mathrm{e,q})\vee\alpha_A(x,q)$

$=\alpha_A(x,q)$

Therefore. $\alpha_A(x,q) = \alpha_A(y, q)$, for all x and y in G, and q in Q.

Theorem: 3.7

A is an anti Q- L-fuzzy M-subgroup of a M-group (G, ·) if and only if $\alpha_A(mxy^{-1},q) \leq \alpha_A(x,q) \vee \alpha_A(y,q)$, for all x, and y in G, q in Q and m in M.

Proof:

Assume that A is an anti Q-L-fuzzy M-subgroup of a M-group (G, .)

We have, $\alpha_A(mxy^{-1},q) \leq \alpha_A(x,q) \vee \alpha_A(y^{-1},q)$,

$\leq \alpha_A(x,q) \vee \alpha_A(y,q)$, since A is an AQLFMSG of G.

Therefore, $\alpha_A(mxy^{-1},q) \leq \alpha_A(x,q) \vee \alpha_A(y,q)$, for all x, y in G, q in Q and for all $m \in M$

Conversely,

if $\alpha_A(mxy^{-1},q) \leq \alpha_A(x,q)$, replace y by x , then

$\alpha_A(x,q) \geq \alpha_A(me, q) \geq \alpha_A(e, q)$, for all x, y in G, q in Q and for all $m \in M$

Now, $\alpha_A(x^{-1}, q) = \alpha_A(ex^{-1}, q)$

$\leq \alpha_A(e, q) \vee \alpha_A(x,q)$

$= \alpha_A(x,q)$.

Therefore, $\alpha_A(ex^{-1}, q) \leq \alpha_A(x,q)$,

for all x, y in G and for all $q in Q$

It follows that, $\alpha_A(xy,q) = \alpha_A(x(y^{-1})^{-1}, q)$

$\leq \alpha_A(x,q) \vee \alpha_A(y^{-1}, q)$

$\leq \alpha_A(x,q) \vee \alpha_A(y,q)$.

Therefore, $\alpha_A(xy,q) \leq \alpha_A(x,q) \vee \alpha_A(y,q)$,

for all x, y in G and for all $q in Q$

Clearly $\alpha_A(mx,q) \leq \alpha_A(x,q)$.

Hence A is an anti Q- L-fuzzy M-subgroup of a M-group (G, ·).

Theorem:3.8

LetAbeaL-fuzzysubsetofaM-group(G,·).Ifα_A(e, q)=0and $\alpha_A(mxy^{-1},q) \leq \alpha_A(x,q) \vee \mu_A(y,q)$, for all x and y in G and q in Q , then A is an anti Q- L-fuzzy M-subgroup of a M-group G, where e is the identity element of G.

Proof:

Let e be identity element of G and x and y in G and q in Q.

$\alpha_A(x^{-1}, q) = \alpha_A(ex^{-1}, q)$

$\leq \alpha_A(e,q) \vee \alpha_A(x,q)$

$= 0 \vee \alpha_A(x,q)$,

$= \alpha_A(x, q)$.

Therefore, $\alpha_A(x^{-1}, q) \leq \alpha_A(x, q)$, for all x in G and q in Q.

$\alpha_A(mxy, q) = \alpha_A(mx(y^{-1})^{-1}, q)$

$\leq \alpha_A(x, q) \vee \alpha_A(y^{-1}, q)$

$\leq \alpha_A(x, q) \vee \alpha_A(y, q)$.

Therefore, $\alpha_A(mxy, q) \leq \alpha_A(x, q) \vee \alpha_A(y, q)$,

for all x and y in G and q in Q.

Hence A is an anti Q- L-fuzzy M-subgroup of a M-group G.

Theorem: 3.9

If A is an anti Q-L-fuzzy M-subgroup of a M-group (G, ·), then H = $\{x, q / x \in G, q \in Q : \alpha_A(x, q) = 0\}$ is either empty or is a M-subgroup of a M-group G.

Proof:

If no element satisfies this condition, then H isempty.

If x and y in H and $q in Q$, then

$$\alpha_A(mxy^{-1}, q) \leq \alpha_A(x, q) \vee \alpha_A(y^{-1}, q)$$

$\leq \alpha_A(x, q) \vee \alpha_A(y, q)$, since A is an AQLFMSG of a M-group G

$= 0 \vee 0 = 0$.

Therefore, $\alpha_A(mxy^{-1}, q) = 0$,

for all x and y in G and $q in Q$.

We get, mxy^{-1} in H.

Therefore, H is a M-subgroup of a M-group G.

Hence H is either empty or is a M-subgroup of a M-group G.

Theorem: 3.10

If A is an anti Q- L-fuzzy M-subgroup of a M-group (G, ·), then H = $\{x \in G \text{ and } q \in Q : \alpha_A(x, q) = \alpha_A(e, q)\}$ is either empty or is a M-subgroup of G, where e is the identity element ofG.

Proof:

If no element satisfies this condition, then H is empty. If x and y satisfy this condition, then

$$\alpha_A(x^{-1}, q) = \alpha_A(x, q) = \alpha_A(e, q), \text{ for all } x \text{ in G and q in Q by Theorem 3.7}$$

Therefore,

$\alpha_A(x^{-1}, q) = \alpha_A(x, q) \forall x \in$ G and q ∈ Q.

Hence $x^{-1} \in$ H.

Now, $\alpha_A(mxy^{-1}, q) \leq \alpha_A(x, q) \vee \alpha_A(y^{-1}, q)$

$\leq \alpha_A(x, q) \vee \alpha_A(y, q)$, since A is an AQLFMSG of a M-group G

$= \alpha_A(e,q) \vee \alpha_A(e,q)$,

$= \alpha_A(e,q)$

Therefore, $\alpha_A(mxy^{-1},q) \leq \alpha_A(e,q)$ ------------------------ (1).

And, $\alpha_A(e,q) = \alpha_A(mxy^{-1},q)(mxy^{-1},q)^{-1})$

$\leq \alpha_A(mxy^{-1},q) \vee \alpha_A(mxy^{-1},q)^{-1}$

since A is an AQLFMSG of a M-group G

$\leq \alpha_A(mxy^{-1},q) \vee \alpha_A(mxy^{-1},q)$

$= \alpha_A(mxy^{-1},q)$

Therefore, $\alpha_A(e,q) \leq \alpha_A(mxy^{-1},q)$------------------------ (2).

From (1) and (2), we get $\alpha_A(e,q) = \alpha_A(mxy^{-1},q) \forall x,y \in$ G and q ∈ Q.
Therefore, mxy^{-1} in H and q in Q.

Hence H is a M-subgroup of a M-group G.

Theorem:3.11

LetAbeananti Q-L-fuzzyM-subgroupofaM-group(G,·). If $\alpha_A(xy^{-1},q) = 0$, then$\alpha_A(x) = \alpha_A(y)$, for all $x$and$y$ in G and q ∈ Q.

Proof:

Let x and x in G and q ∈ Q

Now$\alpha_A(x,q) = \alpha_A(xy^{-1}y,q)$

$\leq \alpha_A(xy^{-1},q) \vee \alpha_A(y,q)$ },since A is an AQLFMSG of a M-group G

$= 0 \vee \alpha_A(y,q)$

$= \alpha_A(y,q)$

$= \alpha_A(y^{-1},q)$, since A is an AQLFMSG of a M-group G

$= \alpha_A(x^{-1}xy^{-1},q)$

$\leq \alpha_A(x^{-1},q) \vee \alpha_A(xy^{-1},q)$

$= \alpha_A(x^{-1},q) \vee 0$

$= \alpha_A(x^{-1},q)$

$= \alpha_A(x,q)$.

Therefore,$\alpha_A(x,q) = \alpha_A(y,q)$, for all$xandyinG$ and $q \in Q$.

Theorem: 3.12

LetAbeananti Q-L-fuzzyM-subgroupofaM-group(G,·).If

$\alpha_A(xy^{-1},q) = 1$, then either $\alpha_A(x,q) = 1$ or $\alpha_A(y,q) = 1$, for all $x$and$y$ in G and $q \in Q$.

Proof:

Let x and y in G and and $q \in Q$

By the definition

$\alpha_A\,(xy^{-1},q) \leq \alpha_A\,(x,q) \vee \alpha_A\,(y,q)$

which implies that $1 \leq \alpha_A\,(x,q) \ \vee \alpha_A\,(y,q)$.

Therefore, either $\alpha_A\,(x,q) = 1$ or $\alpha_A\,(y,q) = 1$, for all x and y in G and $q \in Q$.

Theorem: 3.13

Let (G, ·) be a M-group. If A is an anti Q- L-fuzzy M-subgroup of G, then $\alpha_A(xy,q) = \alpha_A(x,q) \vee \alpha_A(y,q)$, for each x and y in G and $q \in Q$ with $\alpha_A(x,q) \neq \alpha_A(y,q)$.

Proof:

Let x and y belong to G and $q \in Q$

Assume that $\alpha_A(x,q) < \alpha_A(y,q)$.

$$\begin{aligned}
\text{Now, } \alpha_A(y,q) &= \alpha_A(x^{-1}xy, q) \\
&\leq \alpha_A(x^{-1},q) \vee \alpha_A(xy,q) \\
&\leq \alpha_A(x,q) \vee \alpha_A(xy,q) \\
&= \alpha_A(xy,q) \\
&\leq \alpha_A(x,q) \vee \alpha_A(y,q) \\
&= \alpha_A(y,q).
\end{aligned}$$

Therefore, $\alpha_A(xy) = \alpha_A(y) = \alpha_A(x) \vee \alpha_A(y)$, for all x and y in G and $q \in Q$.

Theorem: 3.14

If A and B are two anti Q- L-fuzzy M-subgroups of a M-group (G, ·), then their union $A \cup B$ is an anti Q-L-fuzzy M-subgroup of G.

Proof:

Let x and y belong to G and $q \in Q$

$A = \{\langle (x,q), \alpha_A(x,q) \rangle \,/\, x \in G \text{ and } q \in Q\}$ and

$B = \{\langle (x,q), \alpha_B(x,q) \rangle \,/\, x \in G \text{ and } q \in Q\}$.

$Let\ C = A \cup B$ and $C = \{\langle (x,q), \alpha_C(x,q) \rangle \,/\, x \in G \text{ and } q \in Q\}$.

$$\begin{aligned}
(i)\ \alpha_C(mxy) &= \alpha_A(mxy,q) \vee \alpha_B(mxy,q) \\
&\leq \{\alpha_A(x,q) \vee \alpha_A(y,q)\} \vee \{\alpha_B(x,q) \vee \alpha_B(y,q)\} \\
&\leq \{\alpha_A(x) \vee \alpha_B(x)\} \vee \{\alpha_A(y) \vee \alpha_B(y)\} \\
&= \alpha_C(x) \vee \alpha_C(y).
\end{aligned}$$

Therefore, $\alpha_C(xy) \leq \alpha_C(x) \vee \alpha_C(y)$, for all x and y in G.

$$\begin{aligned}
(ii)\ \alpha_C(x^{-1},q) &= \alpha_A(x^{-1},q) \vee \alpha_B(x^{-1},q) \\
&\leq \alpha_A(x,q) \vee \alpha_B(x,q) \\
&= \alpha_C(x,q).
\end{aligned}$$

Therefore, $\alpha_C(x^{-1},q) \leq \alpha_C(x,q)$, for all x in G.

Hence $A \cup B$ is an anti Q- L-fuzzy M-subgroup of a M-group G.

Theorem: 3.15

The union of a family of anti Q-L-fuzzy M-subgroups of a M-group (G, ·) is an anti Q-L-fuzzy M-subgroup of a M-groupG.

Proof:

Let $\{ A_i \}_{i \in I}$ be a family of anti L-fuzzy M-subgroups of a M-group G and let $A = \cup A_i$.

Then for x and y belong to G and $q \in Q$. we have

(i) $\alpha_A(mxy, q) = \text{Sup}_{i\in I}\, \alpha_{Ai}(mxy, q)$

$\leq Sup_{i\in I}\{ \alpha_{Ai}(x,q) \vee \alpha_{Ai}(y, q) \}$

$= Sup_{i\in I}(\alpha_{Ai}(x, q) \vee Sup_{i\in I}(\alpha_{Ai}(y, q)$

$= \alpha_A(x, q) \vee \alpha_A(y, q)$.

Therefore, $\alpha_A(mxy, q) \leq \alpha_A(x, q) \vee \alpha_A(y, q)$, for all x and y in G and q in Q.

(ii) $\alpha_A(x^{-1}, q) = Sup_{i\in I}\, \alpha_{Ai}(x^{-1}, q)$

$\leq Sup_{i\in I}\, \alpha_{Ai}(x, q)$

$= \alpha_A(x, q)$.

Therefore, $\alpha_A(x^{-1}, q) \leq \alpha_A(x, q)$for all x in G and q in Q

Hence the union of a family of anti Q- L-fuzzy M-subgroups of a M-group G is an anti Q-L-fuzzy M-subgroup ofG.

Theorem: 3.16

IfAisananti Q-L-fuzzyM-subgroupofaM-group(G,·),then

$\alpha_A(xy, q) = \alpha_A(yx, q)$ if and only if$\alpha_A(x, q) = \alpha_A(y^{-1}xy)$, for x and y in G and q in Q.

Proof:

Let x and$y$ in G and q in Q.

Assume that

$\alpha_A(xy, q) = \alpha_A(yx, q)$.

Now,

$\alpha_A(y^{-1}xy, q) = \alpha_A(y^{-1}yx, q)$

$= \alpha_A(ex, q)$

$= \alpha_A(x, q)$.

Therefore

$\alpha_A(x, q) = \alpha_A(y^{-1}xy, q)$, for all x and y in G and q in Q

Conversely, assume that $\alpha_A(x,q) = \alpha_A(y^{-1}xy,q)$.

$$Now, \alpha_A(xy,q) = \alpha_A(xyxx^{-1},q)$$
$$= \alpha_A(yx,q).$$

Therefore, $\alpha_A(xy,q) = \alpha_A(yx,q)$, for all x and y in G and q in Q.

Theorem: 3.17

LetAbeanantiQ-L-fuzzyM-subgroupofaM-group(G,·).
If $\alpha_A(x,q) < \alpha_A(y,q)$, for some x and y in G and q in Q, then $\alpha_A(xy,q) = \alpha_A(y,q) = \alpha_A(yx,q)$, for all x and y in G and q in Q.

Poof:

Let A be an anti Q-L-fuzzy M-subgroup of a M-group (G, ·).
Given $\alpha_A(x,q) < \alpha_A(y,q)$, for some x and y in G,

$\alpha_A(xy,q) \leq \alpha_A(x,q) \vee \alpha_A(y,q)$, as A is an AQLFMSG of G

$= \alpha_A(y,q)$; and

$$\alpha_A(y,q) = \alpha_A(x^{-1}xy,q)$$
$$\leq \alpha_A(x^{-1},q) \vee \alpha_A(xy,q)$$

$\leq \alpha_A(x,q) \vee \alpha_A(xy,q)$, as A is an ALFMSG of G

$$= \alpha_A(xy,q).$$

Therefore, $\alpha_A(xy,q) = \alpha_A(y,q)$, for all x and y in G and q in Q.

And, $\alpha_A(yx,q) \leq \alpha_A(y,q) \vee \alpha_A(x,q)$, as A is an AQLFMSG ofG

$= \alpha_A(y,q)$; and

$$\alpha_A(y,q) = \alpha_A(yxx^{-1},q)$$
$$\leq \alpha_A(yx,q) \vee \alpha_A(x^{-1},q)$$

$\leq \alpha_A(yx,q) \vee \alpha_A(x,q)$,as A is an AQLFMSG of G

$$= \alpha_A(yx,q).$$

Therefore, $\alpha_A(yx,q) = \alpha_A(y,q)$,

for all x and y in G and q in Q.

Hence $\alpha_A(xy,q) = \alpha_A(y,q) = \alpha_A(yx,q)$,

for all x and y in G and q in Q.

Theorem: 3.18

LetAbeananti Q-L-fuzzyM-subgroupofaM-group(G,·).If

$\alpha_A(x,q) > \alpha_A(y,q)$,for some x and y in G and q in Q, then $\alpha_A(xy,q) = \alpha_A(x,q) = \alpha_A(yx,q)$, for all x and y in G and q in Q.

Proof:

Let A be an anti Q-L-fuzzy M-subgroup of a M-group G. Given $\alpha_A(y,q) < \alpha_A(x,q)$, for some x and y in G and q in Q

$\alpha_A(xy,q) \leq \alpha_A(x,q) \vee \alpha_A(y,q)$, as A is an AQLFMSG of G

$= \alpha_A(x,q)$; and

$\alpha_A(x,q) = \alpha_A(xy\,y^{-1},q)$

$\leq \alpha_A(xy,q) \vee \alpha_A(y^{-1},q)$

$\leq \alpha_A(xy,q) \vee \alpha_A(y,q)$, as A is an AQLFMSG of G

$= \alpha_A(xy,q)$.

Therefore, $\alpha_A(xy,q) = \alpha_A(x,q)$, for all x and $y$in G and q in Q.

And, $\alpha_A(yx,q) \leq \alpha_A(y,q) \vee \alpha_A(x,q)$, as A is an AQLFMSG of G

$= \alpha_A(x,q)$;and

$\alpha_A(x,q) = \alpha_A(y^{-1}yx,q)$

$\leq \alpha_A(y^{-1},q) \vee \alpha_A(yx,q)$

$\leq \alpha_A(y,q) \vee \alpha_A(yx,q)$, as A is an AQLFMSG

$= \alpha_A(yx,q)$.

Therefore, $\alpha_A(yx,q) = \alpha_A(x,q)$, for all x and y in G and q in Q

Hence $\alpha_A(xy,q) = \alpha_A(x,q) = \alpha_A(yx,q)$,

for all x and y in G and q in Q.

Theorem: 3.19

Let A be an anti Q- L-fuzzy M-subgroup of a M-group (G, ·) such that $\mathrm{Im}\alpha_A = \{t_1, q\}$, where$t_1$in L and q in Q. If $A = B \cap C$, where B and C are anti Q- L-fuzzy M-subgroups of a M-group G, then either B ⊆ C or C ⊆B.

Proof:

Let A = B∩C = { $\langle (x,q), \alpha_A(x,q) \rangle / x \in G$, q$\in Q$},

$B = \{\langle (x,q), \alpha_B(x,q)\rangle / x \in G$, q $\in Q$ }and

$C = \{\langle (x,q), \alpha_C(x,q)\rangle / x \in G$, q $\in Q$ }.

Assume that $\alpha_B(x,q) > \alpha_C(x,q)$, for some x and y in G.

Then, $(t_1,q) = \alpha_A(x,q)$

$= \alpha_{B\cap C}(x,q)$

$= \alpha_B(x,q) \wedge \alpha_C(x,q)$

$= \alpha_C(x,q) < \alpha_B(x,q)$.

Therefore, $(t_1,q) < \alpha_B(x,q), \forall x \in G$.

And, $(t_1,q) = \alpha_A(y,q)$

$= \alpha_{B\cap C}(y,q)$

$= \alpha_B(y,q) \wedge \alpha_C(y,q)$

$= \alpha_B(y,q) < \alpha_C(y,q)$.

Therefore, $(t_1, q) < \alpha_C(y,q)$, for all y in G and q in G

So that, $\alpha_C(y,q) > \alpha_C(x,q)$ and $\alpha_B(x,q) > \alpha_B(y,q)$.

Hence$\alpha_B(xy,q) = \alpha_B(x,q)$ and

$\alpha_C(xy,q) = \alpha_C(y,q)$,byTheorem 3.17 and Theorem 3.18.

Butthen, $(t_{1,}, q) = \alpha_A(xy,q)$

$= \alpha_{B\cap C}(xy,q)$

$= \alpha_B(xy,q) \wedge \alpha_C(xy,q)$

$= \alpha_B(x,q) \wedge \alpha_C(y,q)$

$> (t_{1,}, q)$ ------------------(1).

It is a contradiction by (1).

Therefore, either B ⊆ C or C ⊆B.is true.

Theorem: 3.20

If A is an anti Q- L-fuzzy M-subgroup of a M-group (G, ·)and if there is a sequence $\{x_n\}$ in G such that $\lim_{n\to\infty}\{\alpha_A(x_n,q) \vee \alpha_A(x_n,q)\}$ $\alpha_A(e,q)$ = 0, where e is the identity in G .

Proof:

Let A be an anti Q-L-fuzzy M-subgroup of a M-group G with e as its identity element.

Let x in G and q in Q be an arbitrary element.

We have x in G impliesx^{-1}in G and

hence$(xx^{-1}, q) = (e, q)$.

Then, we have $\alpha_A(e,q) = \alpha_A(xx^{-1}, q)$

$\leq \alpha_A(x,q) \vee \alpha_A(x^{-1}, q)$

$\leq \alpha_A(x,q) \vee \alpha_A(x,q)$.

Therefore, for each x, we have

$\alpha_A(e,q) \leq \alpha_A(x,q) \vee \alpha_A(x,q)$.

Since$\alpha_A(e,q) \leq \lim_{n\to\infty}\{\alpha_A(x_n,q) \vee \alpha_A(x_n,q)\}$=0

Therefore $\alpha_A(e,q) = 0$.

Theorem: 3.21

If A and B are anti Q-L-fuzzy M-subgroups of the M-groups G and H, respectively, then$A \times B$ is an anti Q-L-fuzzy M-subgroup of$G \times H$.

Proof:

Let A and B be anti Q- L-fuzzy M-subgroups of the M-groups G and H respectively.Let x_1 and x_2 be in G, $y_1$and y_2 be in H and q in Q.

Then $(x_1, y_1)\ and\ (x_2, y_2)$ are in $G \times H$.

Now,

$$\alpha_{A\times B}\,[\, m((x_1,y_1)(x_2,y_2)^{-1},q)\,] = \alpha_{A\times B}\,((mx_1{x_2}^{-1}\,, my{y_2}^{-1}), q)$$
$$= \alpha_A(mx_1{x_2}^{-1}, q\,) \vee \alpha_B(my_1{y_2}^{-1}, q\,)$$
$$\leq \{\alpha_A(x_1\,, q) \vee \alpha_A(x_2, q\,)\} \vee \{\alpha_B(y_1, q) \vee \alpha_B(y_2, q)\}$$
$$= \{\alpha_A(x_1\,, q)\ \vee \alpha_B(y_1, q)\ \}\{\,\alpha_A(x_2, q\,) \vee \alpha_B(y_2, q)\,\}$$
$$= \alpha_{A\times B}\,(x_1, y_1) \vee \alpha_{A\times B}\,((x_2, y_2).$$

Therefore,

$$\alpha_{A\times B}\big[m\big((x_1,y_1)(x_2,y_2)^{-1},q\big)\big] \leq \alpha_{A\times B}\,((x_1,y_1),q) \vee \alpha_{A\times B}\,((x_2,y_2),q)$$

for all $x_1$andx_2 be in G, y_1 andy_2 be in H and q in Q.

Hence anti-product $A \times B$ is an anti Q-L-fuzzy M-subgroup of $G \times H$.

Theorem: 3.22

Let A be a Q-L-fuzzy subset of a M-group (G, .) and V be the anti-strongest Q-L-fuzzy relation of G. Then A is an anti Q-L-fuzzy M-subgroup of G if and only if V is an anti Q-L-fuzzy M-subgroup of$G \times G$.

Proof:

Suppose that A is an anti Q- L-fuzzy M-subgroup of G. Then for any $X = (x_1, x_2)$ and $Y = (y_1, y_2)$ in $G \times G$ and q in Q

We have, $\alpha_V(xy^{-1}, q) = \alpha_V[\,(x_1,x_2)(y_1,y_2)^{-1}\,, q]$

$$= \alpha_V(\,(x_1{y_1}^{-1}\,, x_2{y_2}^{-1}), q)$$
$$= \alpha_A(x_1{y_1}^{-1}, \mathrm{q}) \vee \alpha_A(x_2{y_2}^{-1}, \mathrm{q})$$
$$\leq \{\,\alpha_A(x_1, q \vee \alpha_A(y_1,\quad q)\} \vee \{\alpha_A(x_2, q) \vee \alpha_A(y_2, q)\}$$
$$= \{\alpha_A(x_1, q) \vee \alpha_A(\,x_2, q)\,\} \vee \{\alpha_A(y_1, q) \vee \alpha_A(y_2, q)\}$$
$$= \alpha_V((x_1,x_2), q) \vee \alpha_V((y_1,y_2), q)$$
$$= \alpha_V(X, q) \vee \alpha_V(Y, q).$$

Therefore, $\alpha_V(XY^{-1}, q) \leq \alpha_V(X, q) \vee \alpha_V(Y, q)$,

for all X and Y in $G \times G$ and q in Q.

This proves that V is an anti Q-L-fuzzy M-subgroup of $G \times G$.

Conversely,

Assume that V is an anti Q-L-fuzzy M-subgroup of $G \times G$, then for any $X = (x_1, x_2)$ and $Y = (y_1, y_2)$ in $G \times G$wehave

$$\alpha_A(x_1{y_1}^{-1}, \mathrm{q}) \vee \alpha_A(x_2{y_2}^{-1}, \mathrm{q})$$

$$= \alpha_V((x_1y_1{}^{-1}, x_2y_2{}^{-1}), q)$$

$$= \alpha_V[(x_1, x_2)(y_1, y_2)^{-1}, q]$$
$$= \alpha_V(XY^{-1}, q)$$
$$\leq \alpha_V(X, q) \vee \alpha_V(Y, q)$$
$$= \alpha_V((x_1, x_2), q) \vee \alpha_V((y_1, y_2), q)$$
$$= \{\alpha_A(x_1, q) \vee \alpha_A(x_2, q) \} \vee \{\alpha_A(y_1, q) \vee \alpha_A(y_2, q) \}.$$

If we put, $x_2 = y_2 = 1$

we get $\alpha_A(x_1y_1{}^{-1}, \text{q}) \leq \alpha_A(x_1, q) \vee \alpha_A(y_1, q)$

for all x and y in G and q in Q.

Hence A is an anti Q-L-fuzzy M-subgroup of G.

Theorem: 3.23

Let an anti Q-L-fuzzy M-subgroup A of a M-group G be conjugate to an anti Q-L-fuzzy M-subgroup M of G and an anti Q- L-fuzzy M-subgroup B of a M-group H be conjugate to an anti Q- L-fuzzy M-subgroup N of H. Then an antiQ- L-fuzzy M-subgroup $A \times B$ of a M-group $G \times H$ is conjugate to an anti Q-L-fuzzy M-subgroup $M \times N$ of $G \times H$.

Proof:

Let A and B be anti Q-L-fuzzy M-subgroups of the M-groups G and H respectively.

Let x, x^{-1} and f be in G and y, y^{-1} and g be in H and then q in Q.

Then (x, y) , (x^{-1}, y^{-1}) and (f, g) are in $G \times H$.

Now $\alpha_{A\times B}((f, g), q) = \alpha_A(f, q) \vee \alpha_B(g, q)$

$$= \alpha_M(xf\ x^{-1}, q) \vee \alpha_N(yg\ y^{-1}, q),$$
$$= \alpha_{M\times N} [(xf\ x^{-1}, yg\ y^{-1}), q]$$
$$= \alpha_{M\times N}[(x, y)(f, g)(x^{-1}, y^{-1}), q]$$
$$= \alpha_{M\times N}[(x, y)(f, g)(x, y)^{-1}, q].$$

Therefore, $\alpha_{A\times B} ((f, g), q) = \alpha_{M\times N}[(x, y)(f, g)(x, y)^{-1}, q]$,

for all x in G and y in H and q in Q. Hence an anti Q-L-fuzzy M-subgroup $A \times B$ of a M-group $G \times H$ is conjugate to an anti Q-L-fuzzy M-subgroup $M \times N$ of $G \times H$.

Theorem: 3.24

Let A and B be Q-L-fuzzy subsets of the M-groups G and H, respectively. Suppose that e and e' are the identity elements of G and H, respectively. If the anti-product $A \times B$ is an anti Q- L-fuzzy M-subgroup of $G \times H$, then at least one of the following two statements musthold.

(i) $\alpha_B(e', q) \leq \alpha_A(x, q)$, for all x inG and q in Q

(ii) $\alpha_A(e, q) \leq \alpha_B(y, q)$, for all y inH and q in Q

Proof:

Let the anti-product $A \times B$ be an anti Q-L-fuzzy M-subgroup of $G \times H$. By contraposition, suppose that none of the statements (i) and (ii) holds.

Then we can find a in G and b in H such that

$\alpha_A(a, q) < \alpha_B(e', q)$ and $\alpha_B(b, q) < \alpha_A(e, q)$.

We have $\alpha_{A\times B}((a, b), q) = \alpha_A(a, q) \vee \alpha_B(b, q)$

$$< \alpha_A(e, q) \vee \alpha_B(e', q),$$

$$= \alpha_{A\times B}((e, e'), q)$$

Thus $A \times B$ is not an anti Q-L-fuzzy M-subgroup of $G \times H$.

Hence either $\alpha_B(e', q) \leq \alpha_A(x, q)$, for all x in G or $\alpha_A(e, q) \leq \alpha_B(y, q)$, for all y in H and q in Q.

Theorem: 3.25

Let A and B be Q-L-fuzzy subsets of the M-groups G and H, respectively and anti-product $A \times B$ is an anti Q-L-fuzzy M-subgroup of $G \times H$. Then the following aretrue:

(i) if $\alpha_A(x, q) \vee \alpha_B(e', q)$, then A is an anti Q-L-fuzzy M-subgroup of G,

(ii) if $\alpha_B(x, q) \vee \alpha_A(e, q)$, then B is an anti Q- L-fuzzy M-subgroup of H,

(iii) either A is an anti Q- L-fuzzy M-subgroup of G or B is an anti Q- L-fuzzy M-subgroup of H, where e, e' are the identity elements of G and H and q in Q, respectively.

Proof:

Let anti-product $A \times B$ be an anti Q-L-fuzzy M-subgroup of $G \times H$, x and y in G. Then (x, e') and (y, e') are in $G \times H$.

Now, using the property $\alpha_A(x, q) \vee \alpha_B(e', q)$, for all x in G and q in Q,

we get,

$$\alpha_A(xy^{-1}, q) = \alpha_A(xy^{-1}, q) \vee \alpha_B(e'e', q)$$

$$= \alpha_{A\times B}[(xy^{-1}, q), (e'e', q)]$$

$$= \alpha_{A\times B}[(x, e')(y^{-1}, e'), q]$$

$$\leq \alpha_{A\times B}((x, e'), q) \vee \alpha_{A\times B}((y^{-1}, e'), q)$$

$$= \{\alpha_A(x, q) \vee \alpha_B(e', q),\} \vee \{\alpha_A(y^{-1}, q) \vee \alpha_B(e', q)\}$$

$$= \alpha_A(x, q) \vee \alpha_A(y^{-1}, q)$$

$$\leq \alpha_A(x, q) \vee \alpha_A(y, q).$$

Therefore, $\alpha_A(xy^{-1}, q) \leq \alpha_A(x, q) \vee \alpha_A(y, q)$.,

for all x and y in G and q in Q.

Clearly $\alpha_A(mx, q) \leq \alpha_A(x, q)$.

Hence A is an anti L-fuzzy M-subgroup of G.

Thus (i) is proved.

Now, using the property $(x, q) \vee \alpha_A(e, q)$, for all x in G and q in Q

weget, $\alpha_B(xy^{-1}, q) = \alpha_B(xy^{\alpha_B - 1}, q) \vee \alpha_A(ee, q)$

$= \alpha_{A\times B}((ee, q), (xy^{-1}, q))$

$= \alpha_{A\times B}[(e, x)(e, y^{-1}), q]$

$\leq \alpha_{A\times B}((e, x), q) \vee \alpha_{A\times B}((e, y^{-1}), q)$

$= \{\alpha_B(x, q) \vee \alpha_A(e, q)\} \vee \{\alpha_B(y^{-1}, q) \vee \alpha_A(e, q)\}$

$= \alpha_B(x, q) \vee \alpha_B(y^{-1}, q)$

$\leq \alpha_B(x, q) \vee \alpha_B(y, q)$.

Therefore, $\alpha_B(xy^{-1}, q) \leq \alpha_B(x, q) \vee \alpha_B(y, q)$, for all x and y in G and q in Q.

Clearly $\alpha_B(mx, q) \leq \alpha_B(x, q)$

Hence B is an anti Q- L-fuzzy M-subgroup of H.

Thus (ii) is proved.

And (iii) is clear.

Theorem: 3.26

Let G be a M-group. A is a Q-L-fuzzy M-subgroup of the M-group (G, .) if and only if A^c is an anti Q-L-fuzzy M-subgroup of a M-group G.

Proof:

Suppose A is a Q- L-fuzzy M-subgroup ofG.

For all x and y in G and q in Q,

we have, $\alpha_A(mxy^{-1}, q) \geq \alpha_A(x, q) \wedge \alpha_A(y, q)$

implies that $1 - \alpha_{A^c}(mxy^{-1}, q) \geq (1 - \alpha_{A^c}(x, q)) \wedge (1 - \alpha_{A^c}(y, q))$,

impliesthat$\alpha_{A^c}(mxy^{-1}, q) \leq 1 - \{(1 - \alpha_{A^c}(x, q)) \wedge (1 - \alpha_{A^c}(y, q))\}$,

impliesthat$\alpha_{A^c}(mxy^{-1}, q) \leq \alpha_{A^c}(x, q) \quad \alpha_{A^c}(y, q)$.

Thus A^c is an anti Q- L-fuzzy M-subgroup of a M-group G.
Converse also can be proved similarly.

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